Human capital accumulation through recurrent education *

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Abstract

Population aging is one of the most important policy issues in many industrialized countries. Without appropriate policy prescriptions, it would have a serious negative impact on potential growth through reducing working age population and increasing retired age population. Under declining population, human capital accumulation is a key factor in enhancing potential growth. However, to achieve a desirable human capital accumulation, an aging society may need a different education policy than a society with larger young workers. When young population is large, it is likely that education for the young plays a key role for human capital accumulation. However, human capital accumulated in the young may become obsolete for elder workers. Thus, in an aging society where young workers are scarcer than old workers, we need recurrent education for human capital accumulation to sustain economic growth. But, it is not clear whether laissez-faire leads to enough recurrent education to sustain economic growth in an aging society.

To answer this question, this paper analyzes an OLG model where human capital accumulates through recurrent education. We focus not only on education for the young which depends on learning from their parents and tertiary education but also on recurrent education. Regardless of the level of recurrent education, declining mortality rate increases the level of tertiary education. However, the effect of mortality rate on recurrent education depends on how recurrent education affects the elasticity of human capital to tertiary education. If the recurrent education increases the elasticity, declining mortality rate increases recurrent education as well, which promotes human capital accumulation. In contrast, if the recurrent education decreases the elasticity, declining mortality rate decreases recurrent education, which may result in decline of human capital accumulation. In the latter case, laissez-faire may not lead to enough recurrent education to sustain economic growth in an aging society.
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1. Introduction

Population aging is one of the most important policy issues in many industrialized countries. Without appropriate policy prescriptions, it would have a serious negative impact on potential growth through reducing working age population and increasing retired age population. For example, Figure 1 depicts the relationship between the aging rate in 1991 and the real GDP average growth rate of OECD countries from 1991 to 2000. The figure shows that aging countries tend to face lower real GDP average growth rates.

Under declining population, human capital accumulation is a key factor in enhancing potential growth. However, to achieve a desirable human capital accumulation, an aging society may need a different education policy than a society with larger young workers. When young population is large, it is likely that education for the young plays a key role for human capital accumulation. However, human capital accumulated in the young may become obsolete for elder workers. Thus, in an aging society where young workers are scarcer than old workers, we need recurrent education or adult learning for human capital accumulation to sustain economic growth.

For instance, Figure 2-1 depicts the relationship between the participation rate in recurrent education from 25 to 64 years old in 2012 and the real GDP average growth rate from 1991 to 2000 in aging OECD counties where the aging rate exceeds 14%. The figure 2-1 shows that there is a positive correlation between the participation rate in recurrent education and the real GDP growth rate in an aging society. In contrast, in non-aging OECD counties, where the aging rate is less than 14%, Figure 2-2 depicts the relationship between the participation rate in recurrent education from 25 to 64 years old in 2012 and the real GDP average growth rate from 1991 to 2000. The figure 2-2 shows that there is a weak negative correlation between the participation rate in recurrent education and the real GDP growth rate in a non-aging society. Figure 2-1 and Figure 2-2 suggest that promoting recurrent education has a special importance in enhancing economic growth in an aging society.

The role of recurrent education in an aging society is in marked contrast with that of general education. For example, Figure 3-1 depicts the relationship between the enrollment rate in education from 15 to 19 years old in 2012 and the real GDP average growth rate from 1991 to 2000 in aging OECD counties. The figure 3-1 shows that there is a weak positive correlation between the enrollment rate in education and the real GDP growth rate in an aging society. Similarly, in non-aging OECD counties, Figure 3-2 depicts the relationship between the enrollment rate in education from 15 to 19 years old in 2012 and the real GDP average growth rate from 1991 to 2000. The figure 3-2 shows that there is a weak positive correlation between the enrollment rate in education and the real GDP growth rate in a non-aging society. However, it should be noted that the coefficient of the enrollment rate in education for non-aging countries is bigger than that for aging countries. Figure 3-1 and Figure 3-2 suggest that promoting education may enhance economic growth in any society, but more largely in a non-aging society rather than in an aging society.

However, it is not clear whether laissez-faire leads to enough recurrent education to sustain economic growth in an aging society. For example, Figure 4 depicts the relationship between the aging rate in 1991 and the participation rate in recurrent education from 25 to 64 years old in 2012 in OECD countries. The figure shows that there is only a weak positive correlation between the aging rate and the participation rate in recurrent education in OECD countries. In particular, there are some countries which are aging but have lower participation rate in the figure. In OECD countries, aging countries do not necessarily promote recurrent education, even though recurrent education enhances human capital accumulation. Does laissez-faire really lead to enough recurrent education to sustain economic growth in an aging society?

To answer this question, we explore a role of recurrent education in a simple two-period overlapping generation model when fertility rate and mortality rate change to investigate the effects of recurrent education on human capital accumulation in an aging society. In the model, each agent is educated by his parents through learning from their human capital. He is enrolled in tertiary education when young, from which firms can enjoy higher human capital in the next period. A key feature in our model is that old workers can take recurrent education. When old workers take recurrent education, firms can enjoy higher human capital instantaneously. Since smaller human capital is a serious concern in an aging society, recurrent education is potential underpinning to secure high quality workforces. Hence, recurrent education is critical in understanding a source of economic growth in an aging society. This paper analyzes such a model by focusing on recurrent education.

In literature, there have been a number of theoretical studies that explored how population growth is related with human capital accumulation in the context of economic growth. For example, de la Croix and Doepke (2003) develop a model in which there is a trade-off between education and fertility decisions, and show that a higher fertility rate among the poor lowers average education level and hence growth rate. Nakamura and Seoka (2014) also focus on the fertility differential and show that a lower fertility rate still leads to the poverty trap, unless human capital accumulation among the rich is sufficiently large.

However, few of them focused on the role of recurrent education for enhancing human capital under population aging[1]. Stoikov (1973) and Nishimura et al. (2004) are few ex-

[1] There have been some empirical studies that analyzed the role of recurrent education or adult education for enhancing human capital in developed countries. For example, Vignoles et al. (2004) focus on work related training to analyze its impact on wage growth in the UK. They find that if workers participate in the training, their wages significantly increase, but, at the same time, firms select the workers who are likely to increase their productivities through the training, and give them an opportunity to participate in the training. Stenberg (2005) compares the effects of recurrent educational program in Sweden on unemployment by comparing the Adult Education Initiative (AEI), which offers the

ceptional studies that analyzed the role of recurrent education. Stoikov (1973) examines the losses in present values of the investment in human capital, defined as the differences between the present value of the immediate investment in young workers and the present values of the two types of education programs, i.e. postponement of educational activity of young workers to a later stage of life or investment in old workers, respectively. The paper numerically shows that the loss from postponement of education of the young is greater than the loss from investing in recurrent education under reasonable parameters. Nishimura et al. (2004) characterize various patterns of human capital development by constructing a dynamic optimization model of each worker's decision as to whether he works or accumulates human capital, and show that a worker may alternate between working and recurrent education if earning wages is relatively human capital intensive, human capital depreciates rapidly, and the worker is sufficiently young. However, unlike our model, the model of Stoikov is static and does not consider each agent's decision, while that of Nishimura et al. (2004) is a dynamic model of identical agents. Allowing heterogeneous agents in the model, we study intergenerational effects of recurrent education, as well as learning from their parents and tertiary education, on human capital accumulation in an aging society.

Our model is similar to de la Croix and Michel (2002) in the sense that there exists the trade-off between studying and working in the young. However, we apply this framework to the case where there exists recurrent education besides basic education and tertiary education. A key feature in our model is that we focus on demographic impact on human capital accumulation through recurrent education. In the analysis, we explored how fertility rate, mortality rate, and recurrent education affect the dynamics of human capital accumulation.

Unlike the previous studies which have focused on the impact of decreasing fertility rate on basic education, this paper emphasizes the impact of declining mortality rate on recurrent education rather than declining fertility rate in an aging society. Regardless of the level of recurrent education, declining mortality rate increases the level of tertiary education. However, the effect of mortality rate on recurrent education depends on how recurrent education affects the elasticity of human capital to tertiary education. If the recurrent education increases the elasticity, declining mortality rate increases recurrent education as well, which promotes human capital accumulation. In contrast, if the recurrent education decreases the elasticity, declining mortality rate decreases recurrent education, which may result in decline of human capital accumulation. In the latter case, laissez-faire may not

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unemployed elementary or upper secondary level education, with the vocational part of traditional Labor Market Training (LMT), and concludes that the AEI lowers unemployment incidence but prolongs unemployment duration. Stenberg et al. (2012) reveal that adult education at the compulsory or upper secondary level for the middle-aged workers in Sweden has no significant impact on the timing of retirement, and propose that proponents of adult education for the old workers should argue its potentially positive effects on individual productivity in the labor market or social issues such as fairness and equity, and so on.

lead to enough recurrent education to sustain economic growth.

This paper is organized as follows. Section 2 describes the model. Section 3 derives the equilibrium conditions. Section 4 analyzes the effects of declining mortality rate on recurrent education. Section 5 examines the effects of declining mortality rate on human capital accumulation. Section 6 shows some numerical examples. Section 7 presents the conclusion.

2. The Model

Time is discrete and goes from 0 to ∞. At each period t, a physical good is produced from human capital. We take the good produced at each period as the numeraire. In each period t, N_t persons are born, the fertility rate n is constant, where $n > -1$. Each agent lives for two periods with probability $1 - q$, while he lives for only one period and die at the beginning of the second period with probability q. We call q the mortality rate.

In our model, human capital is accumulated through learning in the young, tertiary education, and recurrent education. Each young individual who is born at period t takes basic education and accumulates h_t. For simplicity, this education is free because the government covers all of the costs. By this education, a fraction δ of human capital in the old is learned to the endowment of the young in the next generation.

In addition to the learning in the young, there exist two types of education to upgrade human capital in the old. One is the human capital accumulated through tertiary education and the other is the human capital accumulated through recurrent education.

In the young, each individual allocates $\lambda_t h_t$ to increase human capital in the old and $(1 - \lambda_t)h_t$ to earn wage income in the young, where $0 < \lambda_t < 1$. When he uses $\lambda_t h_t$ in the young, his human capital is upgraded in the old. Besides, he decides the level of recurrent education. If he takes recurrent education, $\epsilon_{t+1} = e > 0$; otherwise, $\epsilon_{t+1} = 0$. While it takes one period for the young to upgrade his human capital, the old become productive as soon as they take recurrent education. Although the old do not pay a fee for recurrent education, they experience disutility associated with taking recurrent education. Depending on the levels of tertiary education λ_t and recurrent education ϵ_{t+1}, the human capital of the young h_t is upgraded to $\psi(\lambda_t, \epsilon_{t+1})h_t$ in the old.

It thus holds that

$$h_{t+1} = \delta(1 - q)\psi(\lambda_t, \epsilon_{t+1})h_t. \tag{1}$$

Figure 5 describes the process of human capital accumulation.

The function ψ is assumed to be increasing in both λ_t and ϵ_{t+1}, and concave in λ_t. We assume interior solutions, i.e. $0 < \lambda_t < 1$.

In the young, each individual uses $\lambda_t h_t$ to increase human capital in the old and $(1-\lambda_t)h_t$ to earn wage income in the young. The young consume all the wage income. The budget constraint of the young in period t is thus written as

$$(1-\lambda_t)w_t h_t = c_t, \tag{2}$$

where w_t is the wage per unit of human capital, c_t is the consumption of the young, and h_t is the human capital of the old.

Since the old consume the wage income, the budget constraint of the old is as follows:

$$\begin{aligned} &d_{t+1} = w_{t+1}\psi(\lambda_t, e)h_t,\ \text{if taking recurrent education,} \\ &d_{t+1} = w_{t+1}\psi(\lambda_t, 0)h_t,\ \text{otherwise.} \end{aligned} \tag{3}$$

The life cycle utility function is assumed to be logarithmic:

$$\begin{aligned} &U(c_t, d_{t+1}) = \ln c_t + \beta(1-q)\ln d_{t+1} - \phi(e),\ \text{if taking recurrent education,} \\ &U(c_t, d_{t+1}) = \ln c_t + \beta(1-q)\ln d_{t+1},\ \text{otherwise.} \end{aligned} \tag{4}$$

where $\phi(e)$ is the disutility from taking recurrent education.

The production function is given as follows.

$$F(H_t) = AH_t. \tag{5}$$

Aggregate human capital H_t is the sum of human capital in the young $(1-\lambda_t)\delta(1-q)\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}$ and in the old $\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}$ in each period. Thus, it holds that

$$\begin{aligned} &H_t = (1-q)N_{t-1}\psi(\lambda_{t-1}, \epsilon_t)h_{t-1} + N_t(1-\lambda_t)\delta(1-q)\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}, \\ &= (1-q)N_{t-1}\{1+(1+n)(1-\lambda_t)\delta\}\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}. \end{aligned} \tag{6}$$

3. The equilibrium conditions

Substituting equations (2) and (3) into equation (4), the optimal level of tertiary education λ_t^* which maximizes the life cycle utility is given as follows.

$$\frac{\psi(\lambda_t, \epsilon_{t+1})}{(1-\lambda_t)\psi_\lambda(\lambda_t, \epsilon_{t+1})} = \beta(1-q). \tag{7}$$

The optimal level of tertiary education λ_t^* is determined by equation (7), given the level of recurrent education ϵ_{t+1}^*. This equation implies that fertility rate n has no impact on λ_t^* and ϵ_{t+1}^*.

In contrast, equations (2), (3), and (4) imply that the optimal level of recurrent education ϵ_{t+1}^* which maximizes the life cycle utility is given as follows.

$$\begin{aligned} &\epsilon_{t+1} = e \text{ if } \ln\frac{\psi(\lambda_t, e)}{\psi(\lambda_t, 0)} \geq \phi(e), \\ &\epsilon_{t+1} = 0 \text{ if } \ln\frac{\psi(\lambda_t, e)}{\psi(\lambda_t, 0)} < \phi(e). \end{aligned} \tag{8}$$

This equation implies that the optimal level of recurrent education ϵ_{t+1}^* depends on the level of tertiary education λ_t^*.

By solving equations (7) and (8) simultaneously, the optimal pair of $(\lambda_t^*, \epsilon_{t+1}^*)$ is given as follows.

$$\frac{\psi(\lambda_t^*, \epsilon_{t+1}^*)}{(1-\lambda_t^*)\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*)} = \beta(1-q), \text{ and } \epsilon_{t+1}^* = e \text{ or } 0. \tag{9}$$

Under the assumption, the optimal level of tertiary education satisfies $0 < \lambda_t^* < 1$.

The wage rate w_t is given as follows.

$$w_t = A. \tag{10}$$

The intertemporal equilibrium

An intertemporal equilibrium is a sequence $\{c_t, d_{t+1}, \lambda_t, \epsilon_{t+1}, h_t, w_t\}$ of the endogenous variables that attains equilibrium each period t. Each agent chooses the optimal levels of tertiary education λ_t^* and recurrent education, i.e. $\epsilon_{t+1}^* = e$ or 0, according to equations (7) and (8), and chooses the optimal consumption levels (c_t^*, d_{t+1}^*) according to equations (2) and (3). The wage rate w_t satisfies equation (10). The human capital h_t accumulates according to equation (1), and the aggregate human capital is given by equation (6).

4. Effects of declining mortality rate on recurrent education

This section examines the effects of declining mortality rate q on tertiary education and recurrent education. We firstly analyze the effect on tertiary education.

As showed in Section 3, the optimal level of tertiary education is given as follows.

$$\frac{\psi(\lambda_t^*, \epsilon_{t+1}^*)}{(1-\lambda_t^*)\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*)} = \beta(1-q). \tag{11}$$

Define the following implicit function:

$$J(\lambda_t^*) \equiv \beta(1-q)(1-\lambda_t^*)\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*) - \psi(\lambda_t^*, \epsilon_{t+1}^*) = 0. \tag{12}$$

This expression enables us to analyze the effect of declining mortality rate q on the level of tertiary education λ_t^* by using the implicit function theorem. Given the optimal level of recurrent education ϵ_{t+1}^*, we have the following equation:

$$\frac{d\lambda_t^*}{dq} = -\frac{\partial J(\lambda_t^*)}{\partial q} \Big/ \frac{\partial J(\lambda_t^*)}{\partial \lambda_t^*} = -\frac{-\beta(1-\lambda_t^*)\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*)}{\beta(1-q)\{-\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*) + (1-\lambda_t^*)\psi_{\lambda\lambda}(\lambda_t^*, \epsilon_{t+1}^*)\} - \psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*)} < 0. \tag{13}$$

This implies that the level of tertiary education λ_t^* is negatively correlated with q. In other words, as mortality rate q decreases, the level of tertiary education λ_t^* increases. This is because when mortality rate declines, human capital investment becomes more attractive to cover the expenditure in the old. Hence, regardless of the level of recurrent education, a decline in mortality rate increases the level of tertiary education.

Next, we analyze the effect on recurrent education. As showed in Section 3, the optimal level of recurrent education is determined by equation (8). The equation implies that mortality rate affects recurrent education through the level of tertiary education λ_t^*. Then, we are going to examine the effect of an increase in the level of tertiary education λ_t^* on the level of recurrent education ϵ_{t+1}^* by utilizing equation (8).

The term $\ln \frac{\psi(\lambda_t,e)}{\psi(\lambda_t,0)}$ in equation (8) is increasing in λ_t iff the following equation holds.

$$1 < \frac{\psi_\lambda(\lambda_t^*, e)/\psi(\lambda_t^*, e)}{\psi_\lambda(\lambda_t^*, 0)/\psi(\lambda_t^*, 0)}. \tag{14}$$

This equation means that the recurrent education increases the elasticity of human capital to tertiary education. We call the case "the complementary case". Hence, equation (8) implies that as mortality rate declines, the old are more likely to take recurrent education in the complementary case.

In contrast, $\ln \frac{\psi(\lambda_t,e)}{\psi(\lambda_t,0)}$ is decreasing in λ_t iff the following equation holds.

$$0 < \frac{\psi_\lambda(\lambda_t^*, e)/\psi(\lambda_t^*, e)}{\psi_\lambda(\lambda_t^*, 0)/\psi(\lambda_t^*, 0)} < 1. \tag{15}$$

This equation means that the recurrent education decreases the elasticity of human capital to tertiary education. We call the case "the substitute case". Hence, equation (8) implies that as mortality rate declines, the old are less likely to take recurrent education in the substitute case.

5. Effects of declining mortality rate on human capital accumulation

This section analyzes the effects of declining mortality rate on human capital accumulation. Equation (1) implies that the growth rate of human capital $\frac{h_{t+1}}{h_t}$ is given as follows.

$$\frac{h_{t+1}}{h_t} = \delta(1-q)\psi(\lambda_t^*, \epsilon_{t+1}^*). \tag{16}$$

Since neither λ_t^* nor ϵ_{t+1}^* depend on δ, the equation (16) implies that the growth rate of human capital increases, as the learning in the young δ increases. It should be noted, however, a decline in mortality rate q does not necessarily increase the growth rate of human capital because λ_t^* and ϵ_{t+1}^* depend on q.

The effect of declining mortality rate on human capital growth rate through the level of tertiary education λ_t^* is given as follows.

$$\frac{d(h_{t+1}/h_t)}{dq} = \delta(1-q)\{\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*) + \psi_\epsilon(\lambda_t^*, \epsilon_{t+1}^*)\frac{d\epsilon_{t+1}^*}{d\lambda_t^*}\}\frac{d\lambda_t^*}{dq}. \tag{17}$$

As showed in equation (13), $\frac{d\lambda_t^*}{dq} < 0$. As assumed in Section 2, both $\psi_\lambda(\lambda_t^*, \epsilon_{t+1}^*)$ and $\psi_\epsilon(\lambda_t^*, \epsilon_{t+1}^*)$ are positive. However, the sign of $\frac{d\epsilon_{t+1}^*}{d\lambda_t^*}$ is not clear. It is nonnegative, that is $\frac{d\epsilon_{t+1}^*}{d\lambda_t^*} \geq 0$, in the complementary case, while it is nonpositive, that is $\frac{d\epsilon_{t+1}^*}{d\lambda_t^*} \leq 0$, in the substitute case. Hence, a decline in mortality rate always increases the growth rate of human capital in the complementary case, but may decrease it in the substitute case. We are going to describe possible paths of human capital, and analyze the effects of declining mortality rate on human capital accumulation by utilizing equation (8) and the implication of equation (17).

If the growth rate of human capital satisfies $\delta(1-q)\psi(\lambda_t^*, 0) > 1$, the sequence $\{h_t\}_{t\geq 0}$ is increasing and tends toward infinity, even if the cost of taking recurrent education exceeds the benefit. In this case, human capital always increases over time whether taking recurrent education or not. On the other hand, if the growth rate of human capital satisfies $\delta(1-q)\psi(\lambda_t^*, e) < 1$, the sequence $\{h_t\}_{t\geq 0}$ is decreasing and converges to 0, even if the benefit of taking recurrent education exceeds the cost. In this case, human capital always decreases over time whether taking recurrent education or not. More interestingly, if the growth rate of human capital satisfies $\delta(1-q)\psi(\lambda_t^*, 0) < 1 < \delta(1-q)\psi(\lambda_t^*, e)$, the sequence $\{h_t\}_{t\geq 0}$ is increasing and tends toward infinity if the old take recurrent education, while it may be decreasing and converge to 0 if the old do not take recurrent education.

6. Numerical examples

This section examines the effects of declining mortality rate on the paths of human capital accumulation by utilizing equations (8), (11), and (16). When $\ln \frac{\psi(\lambda_0, e)}{\psi(\lambda_0, 0)} = \phi(e)$, the old are initially indifferent whether taking recurrent education or not. In this case, when mortality rate q declines, the old take recurrent education in the complementary case, while they do not take recurrent education in the substitute case. Utilizing this property, we investigate the effects of mortality rate q on human capital accumulation h_t for three alternative cases.

In the experiment, we assume that $\psi(\lambda_t^*, \epsilon_{t+1}^*) \equiv A(\lambda_t^{*\rho} + \epsilon_{t+1}^{*\rho})^{\frac{1}{\rho}}$ for simplicity, where $\rho < 1$. This CES function is concave since its Hessian is negative semidefinite. Note that as ρ goes to 1, tertiary education and recurrent education are prefect substitutes, while as ρ goes to $-\infty$, tertiary education and recurrent education are prefect complements. We assume $\alpha = 0.3$, $\beta = 0.8$, $A = 5$, $\delta = 0.9$, $h_0 = 1$ and $e = 1$. The parameter β is set to equal to $0.99^{20} \approx 0.8$, given that a quarterly psychological discount factor of 0.99 for 20 years. In the following subsections, we will examine the effects of mortality rate q on human capital accumulation. In the experiments, we only consider the case where the condition that $\delta(1-q)\psi(\lambda_t^*, 0) > 1$ is always satisfied.

6.1. Effects of declining mortality rate on human capital accumulation

This subsection compares the effects of mortality rate q on human capital accumulation between the complementary case and substitute case under the condition that $\delta(1-q)\psi(\lambda_t^*, 0) > 1$. For three alternative values of $q = 0.01$, 0.02, and 0.03, Figure 6 and Figure 7 depict the paths of human capital accumulation cases of the complementary case and substitute case, respectively.

Assuming the case where $\rho = -5$, Figure 6 depicts the paths of human capital accumulation for 3 periods in the complementary case. In the experiment, we assume that $\ln \frac{\psi(\lambda_0,e)}{\psi(\lambda_0,0)} > \phi(e)$ for $q = 0.01$, 0.02, and 0.03. This means that the old always take recurrent education for $q = 0.01$, 0.02, and 0.03. In Figure 6, the dashed, solid and dotted lines depict the graphs of $q = 0.01$, $q = 0.02$, and $q = 0.03$, respectively. The graphs show that as mortality rate declines, human capital increases. This implies that declining mortality rate promotes human capital accumulation.

In contrast, assuming the case where $\rho = 0.9$, Figure 7 depicts the paths of human capital accumulation for 3 periods in the substitute case. In the experiment, we assume that $\ln \frac{\psi(\lambda_0,e)}{\psi(\lambda_0,0)} < \phi(e)$ for $q = 0.01$, 0.02, and 0.03. This means that the old never take recurrent education for $q = 0.01$, 0.02, and 0.03. In Figure 7, the dashed, solid and dotted lines depict the graphs of $q = 0.01$, $q = 0.02$, and $q = 0.03$, respectively. The graphs show that as mortality rate declines, human capital increases. This implies that although declining mortality rate decreases recurrent education, it may promote human capital accumulation.

Hence, declining mortality rate promotes human capital accumulation both in the complementary case and the substitute case under the condition that $\delta(1-q)\psi(\lambda_t^*, 0) > 1$. Thus, in an aging society, laissez-faire leads to human capital accumulation in both cases. But, in an aging society, it may not lead to enough human capital accumulation in the substitute case.

6.2. Structural change under declining mortality rate

As showed in Section 5, a decline in mortality rate may decrease the growth rate of human capital in the substitute case due to no recurrent education. In this subsection, we demonstrate that a structural change of human capital accumulation may occur when mortality rate declines. To show a structural change, we assume that $\ln \frac{\psi(\lambda_0,e)}{\psi(\lambda_0,0)} > \phi(e)$ for $q = 0.02, 0.03$, while $\ln \frac{\psi(\lambda_0,e)}{\psi(\lambda_0,0)} < \phi(e)$ for $q = 0.01$. This means that the old take recurrent education when $q = 0.03$ and 0.02, while they do not take recurrent education when $q = 0.01$. In this case, a structural break appears when mortality declines from $q = 0.02$ to $q = 0.01$ because the old take recurrent education when $q = 0.02$ and they do not take recurrent education when $q = 0.01$.

Assuming the case where $\rho = 0.9$, Figure 8 describes a possible structural change of human capital accumulation. The figure depicts the paths of human capital accumulation for 3 periods in the substitute case. In Figure 8, the dashed, solid and dotted lines depict the graphs of $q = 0.01$, $q = 0.02$, and $q = 0.03$, respectively. As in Figures 6 and 7, the graph of $q = 0.02$ lies above that of $q = 0.03$. However, Figure 8 shows that the graph of $q = 0.01$ lies below those of $q = 0.02$ and 0.03. This implies that a structural break occurs when mortality rate declines from $q = 0.02$ to $q = 0.01$. Hence, when a structural change occurs, declining mortality rate may decrease the growth rate of human capital in the substitute case, even though tertiary education increases. In other words, in an aging society, laissez-faire may lead to a serious negative impact on human capital in the substitute case when a structural change occurs.

7. Conclusion

This paper analyzed an OLG model where human capital accumulates not only through learning in the young and tertiary education but also through recurrent education. In the analysis, we explored how fertility rate, mortality rate, and recurrent education affect the dynamics of human capital accumulation.

We obtained the following results. Regardless of the level of recurrent education, declining mortality rate increases the level of tertiary education. However, the effect of mortality rate on recurrent education depends on how recurrent education affects the elasticity of human capital to tertiary education. If the recurrent education increases the elasticity, declining mortality rate increases recurrent education as well, which promotes human capital accumulation. In contrast, if the recurrent education decreases the elasticity, declining mortality rate decreases recurrent education, which may result in decline of human capital accumulation. In the latter case, laissez-faire may not lead to enough recurrent education to sustain economic growth.

The next step is to explore policy implications. For analytical simplicity, our model did not discuss the role of government in enhancing human capital. But it is critical not only for learning in the young and tertiary education but also for recurrent education. Recurrent education is voluntary and expenses tend to be paid by participants, while education for the young tends to be compulsory and covered at public expense. Peston (1981) reviews the general problems pertaining to the government involvement with the finance of recurrent education. He points out that if recurrent education generates technological advancement and productivity improvement, financing should not be restricted by resource constraints, while if recurrent education becomes a target of social policy, the finance should be a part of general public finance. Related to Peston (1981), OECD (2003 and 2005) survey the surrounding environment and policy implications for adult learning in OECD countries. OECD (2003) emphasizes the importance of improving the participation rate in recurrent education and points out that there exist inequities among adults in undertaking adult learning. Along this line, OECD (2005) proposes the guidelines for policy actions to expand the equal learning opportunities especially for low-skilled adults who tend to face constraints, and reviews the corresponding successful policies and practices adopted in OECD countries.

We do not examine the financing problems, but focus on the case where recurrent education has positive externalities in the development of human capital. Incorporating these features in our model is our future research agenda.

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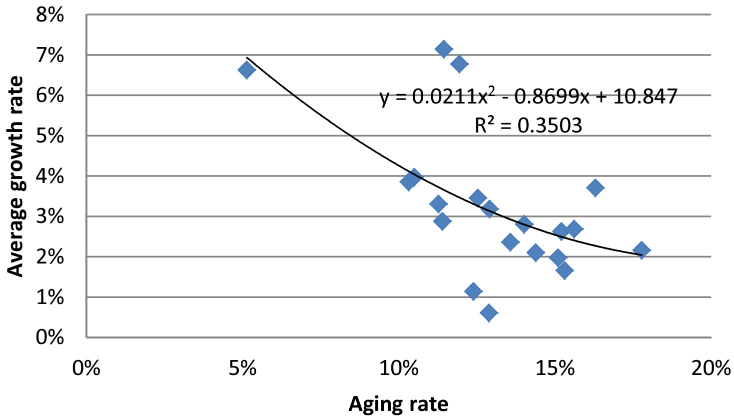

Figure 1: The aging rate and the real GDP growth rate of OECD countries
Note: The aging rate is the proportion of population aged 65 years and above in 1991. The real GDP average growth rate is the average of annual GDP growth rates of OECD countries from 1991 to 2000. The countries include Australia, Austria, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, Netherlands, Norway, Poland, Slovak Republic, Spain, Sweden, and United States.
Source: WDI

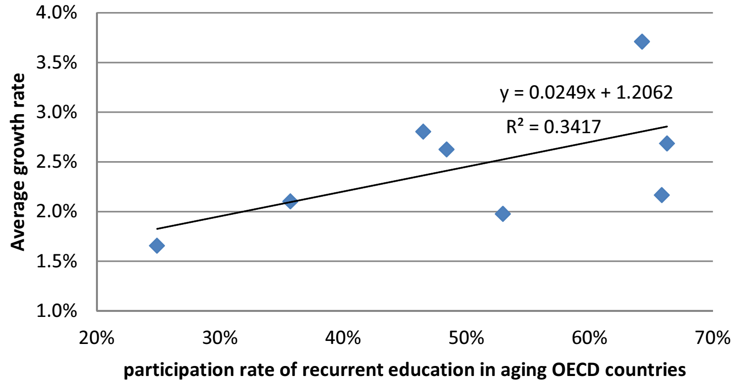

Figure 2-1: The participation rate and the average growth rate in aging OECD countries
Note: The real GDP average growth rate is the average of annual GDP growth rates from 1991 to 2000. Among the OECD members, the aging countries where the aging rate exceeds 14% include Austria, Denmark, France, Germany, Italy, Norway, Spain, and Sweden in 1991.
Source for the real GDP average growth rate: WDI
Source for the participation rate in recurrent education in 2012: The part of "Participated in formal and/or non-formal education" of Table C6.4 in OECD (2014)

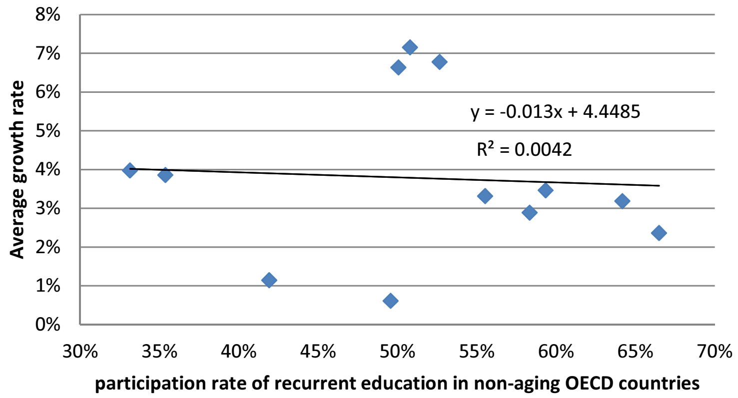

Figure 2-2: The participation rate and the average growth rate in non-aging OECD countries
Note: Among the OECD members, the non-aging countries where the aging rate is less than 14% include Australia, Canada, Czech Republic, Estonia, Finland, Ireland, Japan, Korea, Netherlands, Poland, Slovak Republic, United States in 1991.
Source: The same as in Figure 2-1.

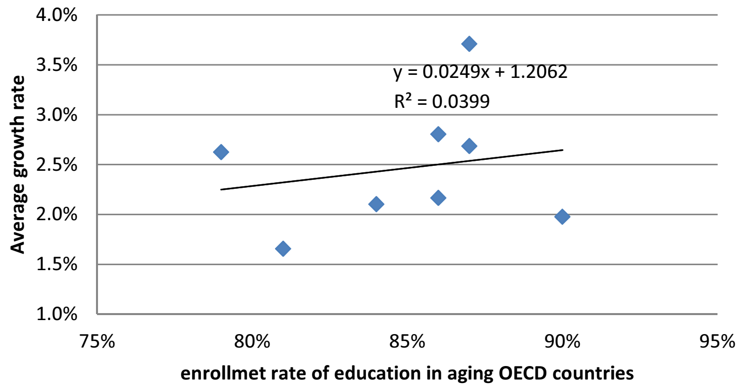

Figure 3-1: The enrollment rate and the average growth rate in non-aging OECD countries
Note: The real GDP average growth rate is the average of annual GDP growth rates from 1991 to 2000. Among the OECD members, the aging countries where the aging rate exceeds 14% include Austria, Denmark, France, Germany, Italy, Norway, Spain, and Sweden in 1991.
Source for the real GDP growth rate: WDI
Source for the enrollment rate in education in 2012: The part of "Students as a percentage of the population of a specific age group (Ages 15-19, M+W)" of Table C1.1a in OECD (2014)

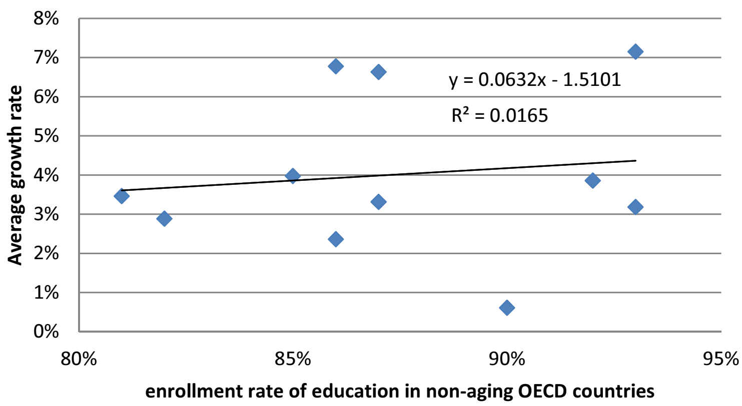

Figure 3-2: The enrollment rate and the average growth rate in non-aging OECD countries
Note: Among the OECD members, the non-aging countries where the aging rate is less than 14% include Austria, Denmark, France, Germany, Italy, Norway, Spain, and Sweden in 1991.
Source: The same as in Figure 3-1.

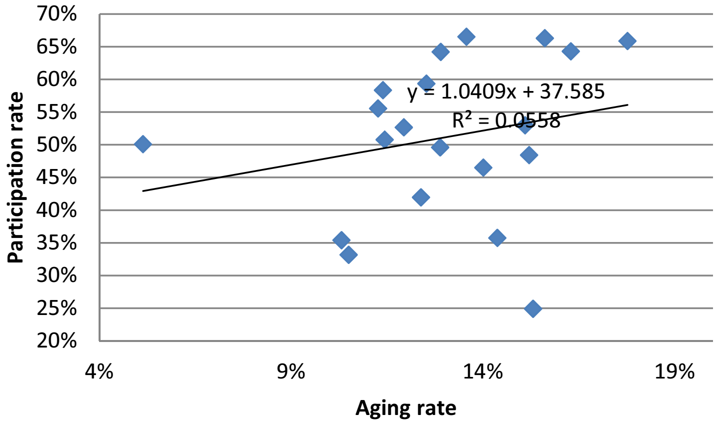

Figure 4: The aging rate and the participation rate in OECD countries
Note: The aging rate is the proportion of population aged 65 years and above in 1991. Source for the aging rate: WDI Source for the participation rate in recurrent education in 2012: The part of "Participated in formal and/or non-formal education" of Table C6.4 in OECD (2014)

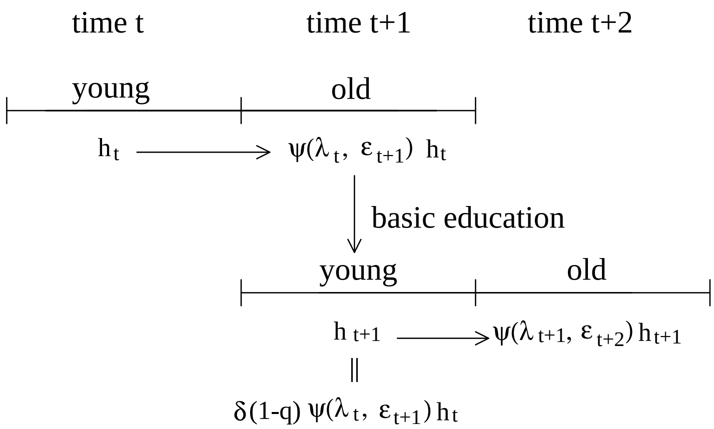

Figure 5: The process of human capital accumulation
Note: The figure describes the process of human capital accumulation.

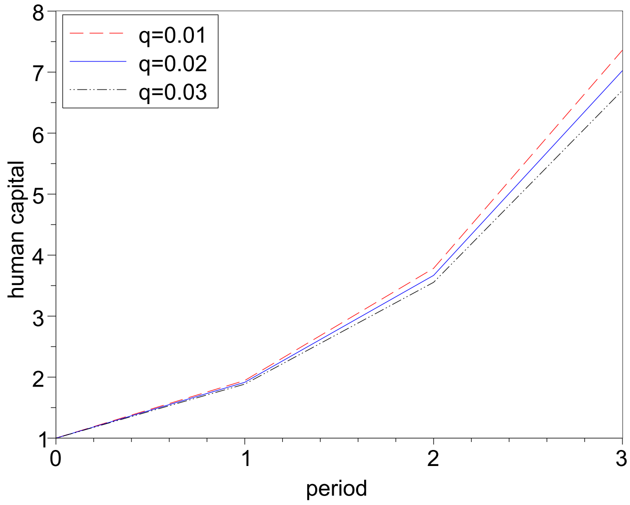

Figure 6: The effect of declining mortality rate in the complementary case
Note: The figure depicts the paths of human capital accumulation for 3 periods in the complementary case. In the figure, the dashed, solid and dotted lines depict the graphs of $q = 0.01$, $q = 0.02$, and $q = 0.03$, respectively.

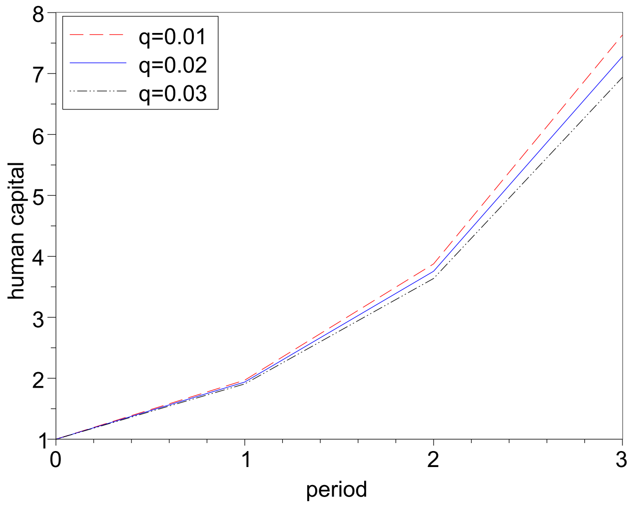

Figure 7: The effect of declining mortality rate in the substitute case
Note: The figure depicts the paths of human capital accumulation for 3 periods in the substitute case. In the figure, the dashed, solid and dotted lines depict the graphs of $q = 0.01$, $q = 0.02$, and $q = 0.03$, respectively.

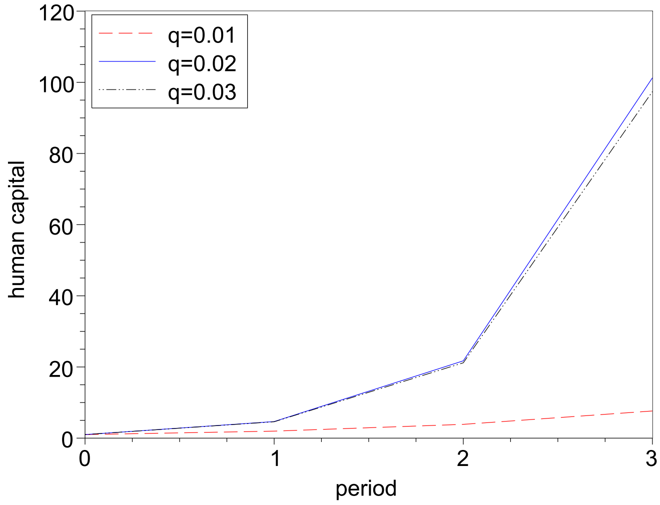

Figure 8: A possible structural change in the substitute case
Note: The figure depicts the paths of human capital accumulation for 3 periods in the substitute case. In the figure, the dashed, solid and dotted lines depict the graphs of $q = 0.01$, $q = 0.02$, and $q = 0.03$, respectively.