Emotion and the Market: A Model of Anticipated Regret, Investor Behavior and Market Turbulence☆

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Abstract

Based on Regret Theory, this paper examines the effects of regret on investor behavior and market turbulence in a model where investors not only regret wrong actions, but also regret inaction. We demonstrate that regret aversion can cause investors to ride a bubble, exit and reenter the market, or choose non-trading. As a result, herds and partial herds can occur in the market, and the stronger is regret over inaction, the easier it is for herds to occur. The model predicts that order volume and order imbalance tend to have positive (negative) correlation when a bubble (crash) is forming.

JEL classification: D81, G12

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Abstract

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1. Introduction

Regret is a painful feeling caused by "counterfactual thinking" that compares the true outcome of a choice with "what might have been."[1] In financial markets, it is natural for investors to have counterfactual thinking because investors can easily compare the performance of their portfolios with other assets and calculate "how much they might have earned." For example, Harry Markowitz vividly described how anticipated regret affected his choice of a pension plan. "I should have computed the historical co-variance of the asset classes and drawn an efficient frontier. Instead, I visualized my grief if the stock market went way up and I wasn't in it – or if it went

[1] See Kahneman and Miller (1986), Landman (1987b), Kahneman (1995), Roese (1997) and others.

way down and I was completely in it. My intention was to minimize my future regret. So I split my contributions 50/50 between bonds and equities" (As quoted in Zweig (2007), pp 4.).

Studies of psychology and neuroscience, such as Camille et al. (2004) and Coricelli et al. (2005), provide strong evidence that regret can influence decision making.[2] Furthermore, the effects of emotions on decision making are hard to control by rational thinking. As neuroscientist J. LeDoux noted, "While conscious control over emotions is weak, emotions can flood consciousness. This is so because the wiring of the brain at this point in our evolutionary history is such that connections from the emotional systems to the cognitive systems are stronger than connections from the cognitive systems to the emotional systems" (LeDoux (1996), pp 19.). Thus, not only inexperienced investors but also professional investors such as fund managers can be affected by regret. Because regret is neither observable nor verifiable, it is difficult to contract it away. Arbitrage trading against the distortions caused by regret is difficult too, not only because there are limits of arbitrage as noted by Shleifer and Vishny (1997) but also because arbitragers themselves are not immune to the influence of emotions. A typical case that illustrates the influence of regret on the aggregate market may be the formation of a bubble; when the market booms, individuals rush to buy overvalued assets because they do not want later regret about missing a bull market. Such behaviors fuel the bubble and finally result in market fluctuations at the aggregate level.[3]

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[2] The next section is a brief review of studies of regret in psychology and neuroscience. For detailed reviews, see Gilovich and Medvec (1995), Barbey et al. (2009), and Crespi et al. (2012).

[3] See, for example, Nofsinger (2012) for household behaviors in bubbles and crashes.

The influence of regret on economic decisions was first mentioned by L. J. Savage as a rationale for the "minimax rule" of decision making under uncertainty (Savage (1954), pp 163). Regret Theory, a theoretical model of regret and decision making, was developed independently by Bell (1982, 1983) and Loomes and Sugden (1982, 1987). Until the 1990s, this theory attracted more attention in neuroscience than in economics; however, in recent years, the number of studies about regret and economic decision making increased quickly. For example, Skiadas (1997) shows that conditional utilities including regret can be aggregated to form an unconditional utility; Filiz-Ozbay and Ozbay (2007) apply Regret Theory to auctions; Sarver (2008) demonstrates that fewer options lead to higher utility for regret-avoidant individuals; and Hayashi (2008) and Bikhchandani and Segal (2014) propose alternative approaches to model regret. Regret Theory has also been applied to studies of financial markets; for example, Fogel and Berry (2006) examine the relationship between regret and the disposition effect, Muermann et al. (2006) analyze the effects of regret on pension schemes, and Michenaud and Solnik (2008) use regret aversion to explain currency-hedging decisions.

Based on Regret Theory, the present paper constructs a theoretical model that aims to illustrate the effects of regret on market fluctuations. The basic structure of the model is a sequential trading model that follows Glosten and Milgrom (1985), where informed traders and noise traders sequentially trade a risky asset with a market maker. However, instead of assuming risk-neutral agents, we assume that an informed trader's utility depends on his conditional expectation of return as well as on his

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See also Kindleberger and Aliber (2005) for a description of bubbles in history.

anticipated regret, where regret is measured by a "regret function" that compares the factual outcome from the chosen option with foregone payoffs from unchosen options.

A distinctive feature of the model developed by this paper is that it not only includes "regret over action", the regret of a bad investment, but also incorporates "regret over inaction", the regret of a missed opportunity. Psychology studies such as Kahneman and Miller (1986) and Gilovich and Medvec (1994) suggest that people regret wrong actions more in the short term but tend to have more regrets about missed opportunities in the long term. Furthermore, in experimental settings, Büchel et al. (2011) illustrate that regret of missed opportunities leads to more risk taking in the future, and Steiner and Redish (2014) suggest that the relationship between regret of missed opportunities and risking taking may have deep roots in mammals' nervous systems.[4] In financial markets, if asset prices keep rising and individuals strongly regret missed opportunities, it is quite likely that they will try to correct their mistake by investing heavily in already overheated markets and thus cause a bubble to grow even larger. Based on this intuition as well as experimental evidence, the present paper extends the original Regret Theory by incorporating regret over inaction. In the model, a parameter z controls the magnitude of regret over inaction. By changing the value of z, we illustrate how regret over inaction affects investor behavior and the aggregate market. Previous studies on regret and decision making do not distinguish regret over inaction from regret over action. To the knowledge of the author, the present paper provides the first theoretical model that explicitly illustrates the effects of regret over

[4] In the experiment conducted by Steiner and Redish (2014), rats encounter a serial sequence of tasks that can create "regret-inducing" situations. The activities of rats' orbitofrontal cortex and ventral stratum suggest that rats regret bad decisions. Moreover, when rats are regretting a missed opportunity, they become impatient and

inaction on decision making.

Using this model, we first demonstrate that investors may ride a bubble to avoid anticipated regret. In the market, for traders who observe negative signals, the price of the asset is higher than the conditional expectation of the asset value; however, if the price keeps rising, these traders will choose to buy the overvalued asset to avoid regret. The phenomenon where informed traders choose to ride a bubble even after they are aware of overpricing has been addressed by influential studies such as De Long et al. (1990), Allen et al. (1993), and Abreu and Brunnermeier (2003). In these studies, informed traders ride the bubble either because they expect that they can sell the overpriced asset at an even higher price to "greater fools", or because they expect that the bubble will not burst soon.[5] The present paper proposes an additional explanation: informed traders ride, instead of trading against, a bubble because they try to avoid anticipated regret. This explanation is appealing in the sense that it is intuitive and does not require investors' to have specific "irrational" beliefs about the market; moreover, the structure of the model is much simpler than many rival models.

Second, we show that regret over inaction plays an important role in the formation of bubbles. In the model, investors who noticed the overpricing may temporarily leave the market in the early stage of a bubble; however, after the price rises to an even higher level, these sideliners will reenter the market to join the buy-herd.[6] It is shown

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tend to choose risky options in subsequent tasks.

[5] See also Conlon (2004), Doblas-Madrid (2012), and Matsushima (2013). For a survey of the literature, see Brunnermeier and Oehmke (2013).

[6] This paper is related to the literature of informational cascades but with an important difference: in standard informational cascades models, informed traders do not buy overvalued assets or sell undervalued assets; they herd because of informational externality. For surveys of studies on informational cascades, see Bikhchandani and Sharma (2001) and Hirshleifer and Teoh (2003).

that the stronger is investors' regret over inaction, the easier it is for a bubble to occur. This result is in accordance with experimental findings that regret of missed opportunities leads to risk-taking in subsequent decision making. It also provides a theoretical explanation for the observation that anticipated regret of missed opportunities can force otherwise rational persons to join the herd during a bubble.[7] By analogous arguments, the model in this paper can also explain "fire selling" during a crash.

Finally, the model predicts that when the market is moving toward a bubble, order volume (the total number of orders from traders) tends to have a positive correlation with order imbalance (the ratio between the number of buy orders and the number of sell orders). Conversely, if the market is moving toward a crash, order imbalance and volume tend to have a negative correlation. The model also suggests that trading activity will be lower after a crash, whereas the probability of market turbulence will become higher after a long-continued bull market.

The remainder of the paper is organized as follows. Section 2 reviews the related literature in psychology and neuroscience. Section 3 describes the framework of the model and derives investors' trading strategies in a special case. Section 4 analyzes investors' trading strategies in a more general setting. Section 5 examines the effects of regret on market fluctuations. Section 6 discusses testable implications of the model. Section 7 presents the study's conclusions.

2. Regret and Regret Theory

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[7] For example, Kindleberger and Aliber (2005, pp30) describe that a "follow-the-leader process" forms in a bubble because "There is nothing as disturbing to one's well-being

In economics, decision making has been treated as a pure cognitive process for many years. Nevertheless, a growing number of psychologists, neuroscientists, and economists in the emerging field of neuroeconomics started to explore the effects of emotions on decision making.[8] For example, Damasio (1994) and Bechara and Damasio (2005) proposed a "somatic marker hypothesis," which states that individuals can "feel" and react to risk even before they consciously know about it. Mellers et al. (1997, 1999) proposed a "decision affect theory," which examines the effects of counterfactual comparisons and surprisingness on decision making. Loewenstein et al. (2001) proposed a "risk-as-feelings hypothesis," which describes how immediate emotions and anticipated emotions affect decision making. For reviews of research on emotions and economic decision making, see Cohen (2005), Elster (1998), Loewenstein and Lerner (2003), Phelps (2009), and Rick and Loewenstein (2008).

The present paper focuses on the effects of regret on decision making in financial markets. According to psychology studies such as Kahneman and Tversky (1982), Kahneman and Miller (1986), Landman (1987b), Landman and Manis (1992), Roese (1997), and Roese and Morrison (2009), regret is a painful feeling caused by "counterfactual thinking" that compares the true outcome of a choice with "what might have been." Regret can easily be confused with disappointment because disappointment also arises from counterfactual thinking. However, they are different emotions: regret is felt when the result of the chosen option is worse than the result of the foregone option, whereas disappointment is felt when the result of a decision is

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and judgment as to see a friend get rich."

[8] See Camerer et al. (2005), Glimcher at al. (2009), and Innocenti and Sirigu (2012) for surveys of neuroeconomics.

worse than expectation.[9] Moreover, biologically speaking, regret and disappointment involve different nervous systems, a fact demonstrated by neuroimaging studies carried out in recent years by Coricelli et al. (2005) and others. Regret may also be confused with risk because both are related to bad outcomes of decisions. To observe the difference, consider the case that an investor invests all his savings on riskless assets; this investor has no risk, but he may regret this decision in the future, especially when he faces a booming stock market. For reviews of studies on regret in psychology, see Landman (1993) and Gilovich and Medvec (1995).

Regret Theory is a theoretical model of regret and decision making developed independently by Bell (1982, 1983) and Loomes and Sugden (1982, 1987). They propose the following "modified utility function":

$$u(r_i, r_j) = r_i + f(r_i - r_j), \tag{1}$$

$$u(r_j, r_i) = r_j + f(r_j - r_i) \tag{2}$$

where r_i is the uncertain outcome of asset-i, r_j is the outcome of asset-j, and $f(\cdot)$ is a strictly increasing function with $f(0) = 0$. For an individual who has chosen asset-i, his utility is increasing in r_i, the actual return on asset-i, and is decreasing in r_j, the counterfactual return he would have obtained if he had chosen asset-j. The function $f(\cdot)$ measures the individual's regret or rejoice of a decision and is known as the "regret-rejoice function" by Loomes and Sugden (1982). Regret Theory assumes that, faced with a choice between asset-i and asset-j, an individual acts to maximize his "expected modified utility"; that is, the individual strictly prefers asset-i if

[9] For detailed discussions on the difference between regret and disappointment, see Zeelenberg et al. (1998) and Zeelenberg et al. (2000). See also Bell (1985) and Loomes and Sugden (1986) for studies on the effects of disappointment on economic decisions.

$$\mathrm{E}[u(r_i, r_j)] > E[u(r_j, r_i)]. \tag{3}$$

Under the assumption that $f(0) = 0$, $f'(x) > 0$, and $f''(x) > f''(-x)$ for $x > 0$, these authors show that Regret Theory can explain the Allais Paradox and some other experimental findings that violate the expected utility theory.[10]

Supportive evidence for Regret Theory has been obtained in experiments carried out by neuroscientists. For example, Camille et al. (2004) compare behavior of healthy people with that of patients with orbitofrontal cortex (OFC) lesions in an experiment where subjects make a choice between two gambling tasks. It is found that healthy subjects feel regret when they find that the outcome of the chosen gamble is worse than the outcome of the unchosen gamble; furthermore, they learn from their emotional experiences and try to minimize anticipated regret in future decisions. In contrast, OFC patients neither feel regret after making a bad choice nor anticipate regret in future decisions. In an experiment with similar gambling tasks, Coricelli et al. (2005) use functional magnetic resonance imaging (fMRI) to investigate the brain regions involved in regret. They find that subjects exhibit increasing avoidance to anticipated regret after repeatedly experiencing regret and that this learning process is accompanied by increased activity in the OFC and amygdala. These results are confirmed by Chandrasekhar et al. (2008) and others. For surveys of related studies in neuroscience, see Coricelli et al. (2007), Barbey et al. (2009), and Crespi et al. (2012). Studies of experimental economics also provide support for Regret Theory. See Zeelenberg (1999) and Zeelenberg et al. (2000) for a review.

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[10] See Kahneman and Tversky (1979) for experimental evidence about the Allais

3. A Special Case

Following Regret Theory, the present paper assumes that investors are regret-averse; however, unlike the original model of Regret Theory, where investors buy either asset-i or asset-j, the present paper allows investors to choose inaction as well. To illustrate the basic idea, the analysis begins with a special case where investors have same attitude toward regret over action and regret over inaction; a more general version of the model is provided in the next section, where regret over inaction and regret over action are treated differently.

The framework of the model is as follows. In the market, a risky asset with uncertain value V is traded whose prior distribution is

$$V = \begin{cases} 1, & with\, probalility\, \mu \\ 0, & with\, probalility\, 1-\mu \end{cases} \tag{4}$$

where $0 < \mu < 1$. Each investor can choose among three positions: long, short, and not trading. Let $x \in \{1, 0, -1\}$ represent an investor's choices: $x = 1$ if the investor buys one unit of the asset, $x = -1$ if he sells one unit of the asset, and $x = 0$ if he decides not to trade. With p representing the price of the asset and $r = r(x)$ the return on position x, an investor's utility maximizing problem can be expressed as follows:

$$\max_x E[u(r(x))] \tag{5}$$

where

$$r(x) = \begin{cases} V-p, & if\ x = 1 \\ 0, & if\ x = 0 \\ p-V, & if\ x = -1. \end{cases} \tag{6}$$

Following Bell (1982), Loomes and Sugden (1982), and Quiggin (1994), we assume that $u(\cdot)$ is a "modified utility function" which not only includes the utility of

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Paradox and other behaviors that are inconsistent with the theory of expected utility.

investment return but also reveals the disutility of regret.

Assumption 1 An investor's preference is described by the following modified utility function:

$$u(r) = r - f(\max\{r\} - r), \tag{7}$$

where

$$f(r) = \eta\sqrt{r}, \tag{8}$$

$\max\{r\} = \max\{r(1),\ r(-1)\}$, and $\eta > 0$.

This assumption reveals an investor's counterfactual thinking, which compares realized return r with the best possible outcome

$$\max\{r\} = \begin{cases} 1-p, & if\ V = 1 \\ p, & if\ V = 0. \end{cases} \tag{9}$$

Regret caused by counterfactual thinking is measured by the "regret function" $f(\cdot)$ in equation (8), where parameter η controls the magnitude of regret aversion. Assumption 1 is consistent with previous studies of Regret Theory in the sense that the disutility of regret is an increasing function of the difference between the factual result and "what might have been".[11] The square root function in equation (8) is adopted because it largely simplifies the calculation while still preserving most of the important

[11] The modified utility function proposed by Bell (1982) and Loomes and Sugden (1982) is pairwise. Quiggin (1994) extends Regret Theory to multiple choices and proposes a modified utility function that takes the form $\phi(r_{in}, max\{r_{in}\})$ where r_{in} is the return on the chosen option-i in state-n, and $\max\{r_{in}\}$ is the best possible result among all options in state-n. Following Muermann et al. (2006), Michenaud and Solnik (2008), and Sarver (2008) among others, the present paper uses Quiggin's version of the modified utility function.

features of Regret Theory.[12]

The trading mechanism follows the sequential trading model developed by Glosten and Milgrom (1985). The risky asset is traded at date $t = 1, 2, \cdots, T$ among informed traders of a mass ϕ, noise traders of a mass $1 - \phi$, and a single market maker who receives all orders. At date $T+1$, the true value of V will be revealed to the public and all positions will be liquidated accordingly. Each informed trader observes a conditionally independent signal $s \in \{0, 1\}$ at $t = 0$ with $q \equiv \Pr\{s = V|V\} > 0.5$. At each date t, each informed trader can submit one order to buy or sell one unit of the asset, or he can choose to do nothing; each noise trader randomly submits a buy order or a sell order of one unit of the asset with equal probability. Trading takes place following the mechanism below.

1. The market maker announces a price p_t at which he is willing to buy and sell the asset.
2. Informed traders and noise traders submit orders to the market maker.
3. The market maker randomly picks one piece of the order from the pool of orders and executes at price p_t. Let x_t denote the sign of the order executed at date t: if it is a buy order, $x_t = 1$; if it is a sell order, $x_t = -1$.
4. x_t is announced to the public after the order has been executed.

The history of previous trading rounds, denoted by $h_t \equiv \{ (p_\tau, x_\tau), \tau = 1,2, \cdots, t-1\}$

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[12] The regret function in Assumption 1 can be extended to the case where there is a continuum of options. For example, we can assume that an investor chooses $x \in [-1,1]$ with $r(x) = (v - p)x$ and $\max\{r\} = \max\{r(x)|x \in [-1,1]\}$. However, a difficulty in this case is that inaction does not differ from buying or selling an infinitely small amount of the asset. Because an important purpose of the present paper is to illustrate the effects of regret over inaction, we adopt the choice set $\{1, 0, -1\}$ where inaction significantly differs from actions.

with $h_1 = \Phi$, is public information available to all market participants. The expected value of the asset conditional on this public information, denoted by $\mu_t \equiv \Pr\{V = 1|h_t\}$ with $\mu_1 = \mu$, is called "public belief", which is a sufficient statistic of public information. At the end of each trading round, public belief is updated through Bayesian learning; $\mu_{t+1} = Pr\{V = 1|h_t, x_t\}$.

The market maker sets the price of the asset equal to the conditional expectation of asset value:

$$p_t = \mu_t. \tag{10}$$

Different from the original model of Glosten and Milgrom (1985), bid-ask spread is assumed zero to simplify the analysis.

Each informed trader calculates his expected return conditioned on both public information and private information. Because of this information advantage, an informed trader who has received good news $s = 1$ has a positive expected return $E[V - p_t|h_t, s = 1] > 0$ on a long position; therefore, he has an incentive to buy the asset and can be called a "bullish trader". In contrast, an informed trader who has received bad news $s = 0$ can be called a "bearish trader" because he can earn a positive expected return $E[p_t - V|h_t, s = 0] > 0$ by taking a short position. Let $E_t[\cdot] = E[\cdot\,|h_t]$ denote the expectation conditioned on public information, $E_t^1[\cdot] = E[\cdot\,|h_t, s = 1]$ the expectation conditioned on public information and positive signal $s = 1$, and $E_t^0[\cdot] = E[\cdot\,|h_t, s = 0]$ the expectation conditioned on public information and negative signal $s = 0$. According to the Bayesian rule, a bullish trader's expectation about asset value, denoted by $\mu_t^1 \equiv E_t^1[V]$, and a bearish trader's expectation, denoted by $\mu_t^0 \equiv E_t^0[V]$, are as follows.

$$\mu_t^1 = \frac{\mu_t q}{\mu_t q + (1-\mu_t)(1-q)} = \frac{\lambda_t \lambda_q}{1+\lambda_t \lambda_q}, \tag{11}$$

$$\mu_t^0 = \frac{\mu_t (1-q)}{\mu_t (1-q) + (1-\mu_t) q} = \frac{\lambda_t / \lambda_q}{1+\lambda_t / \lambda_q} \tag{12}$$

where $\lambda_t \equiv \frac{\mu_t}{1-\mu_t}$ and $\lambda_q \equiv \frac{q}{1-q}$ are likelihood ratios.

Next, consider the decision-making problem faced by an informed trader, whose current position on the asset is zero.[13] If this trader builds position x at date t, his expected utility will be $U_t^s(x) \equiv E_t^s[u(r(x))]$, where $u(\cdot)$ is the modified utility function in equations (7)-(8), $r(x) = (V - p_t)x$, and $x \in \{1, 0, -1\}$. Table 1 illustrates $r(x)$ and $u(r(x))$ for $x \in \{1, 0, -1\}$. It is easy to see that

$$U_t^s(1) = \mu_t^s(1 - p_t) - (1 - \mu_t^s)\left(p_t + \eta\sqrt{2p_t}\right), \tag{13}$$

$$U_t^s(0) = -\eta\mu_t^s\sqrt{1 - p_t} - \eta(1 - \mu_t^s)\sqrt{p_t}, \tag{14}$$

$$U_t^s(-1) = \mu_t^s\left(p_t - 1 - \eta\sqrt{2(1 - p_t)}\right) + (1 - \mu_t^s)p_t\ . \tag{15}$$

Table 1 Return and utility of each position.

	$r(1)$	$r(0)$	$r(-1)$	$\max\{r\}$	$u(r(1))$	$u(r(0))$	$u(r(-1))$
$V = 1$	$1 - p_t$	0	$-(1 - p_t)$	$1 - p_t$	$1 - p_t$	$-\eta\sqrt{(1 - p_t)}$	$p_t - 1 - \eta\sqrt{2(1 - p_t)}$
$V = 0$	$-p_t$	0	p_t	p_t	$-p_t - \eta\sqrt{2p_t}$	$-\eta\sqrt{p_t}$	p_t

It is worth noting that $U_t^s(x)$ can be explained as an informed trader's expected utility if he builds position x at date t and keeps this position unchanged until the final date. At first glance, it seems that the informed trader can have higher expected utility if he dynamically adjusts his position in future trading rounds. However, we argue that when a trader is choosing position x at date t, he need not consider future

[13] At any date t, while the whole population is a continuum of mass one, informed traders who have already built a non-zero positon in previous trading rounds have mass zero. Thus, without loss of generality, we can neglect these traders and restrict

dynamic trading, because the probability that he can successfully change his position in future trading rounds is too small.[14] Therefore, without loss of generality, we can assume that at each date t an informed trader with signal $s \in \{0, 1\}$ will act in the following way,

$$x_t^1 = \begin{cases} 1, & if\ U_t^1(1) \geq \max\{U_t^1(0), U_t^1(-1)\} \\ 0, & if\ U_t^1(0) > \max\{U_t^1(1), U_t^1(-1)\} \\ -1, & otherwise; \end{cases} \tag{16}$$

$$x_t^0 = \begin{cases} -1, & if\ U_t^0(-1) \geq \max\{U_t^1(1), U_t^1(0)\} \\ 0, & if\ U_t^0(0) > \max\{U_t^0(1), U_t^0(-1)\} \\ 1, & otherwise \end{cases} \tag{17}$$

where $x_t^s = 1$ means the trader submits a buy order, $x_t^s = -1$ means he submits a sell order, and $x_t^s = 0$ means he chooses inaction at date t. Below, Lemma 1 compares buy and sell, and Lemma 2 compares inaction and action.

Lemma 1 Comparing a long position and a short positon, it is found that there exist

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our analysis to investors whose current position is zero.

[14] To see this, consider an investor who builds position x at date t and follows a dynamic trading strategy in the future. Let w_τ denote the probability that the investor's order submitted at date τ is executed by the market maker, and let $\bar{w} \equiv \max\{w_t: t = 1,2, \dots, T\}$. The probability that the investor can change his position in future trading rounds is $1 - \prod_{\tau=t+1}^{T}(1 - w_\tau) < (T - t)\bar{w}$. On the other hand, because there are T trading rounds in total and the size of each order is one unit, the greatest possible return an investor can earn is less than T. Thus, even if a trader obtains the best possible result, his utility is less than T. Therefore, if an investor with position x at date t decides to keep this position until the liquidation date, the expected utility is $U_t^s(x)$; if he decides to adjust his position in the future by following a dynamic trading strategy, the increment in his expected utility is less than $\left(T - U_t^s(x)\right)(T - t)\bar{w}$. Note that the present paper adopts the framework of sequential trading model, where there is a continuum of traders whereas only one order is executed in each trading round. Thus, $\bar{w}$ is infinitely small. As a result, for the investor who holds position x at date t, whether he will keep this position until $T + 1$ or he will apply a dynamic trading strategy in future trading rounds, the expected utility is the same. Therefore, when an informed trader is choosing position x at date t, he need not consider future dynamic trading; he only need to choose the position x that maximizes $U_t^s(x)$.

$\underline{\mu} \in (0,0.5)$ and $\overline{\mu} \in (0.5,1)$ such that

$$\begin{cases} U_t^1(1) < U_t^1(-1) & for\ \mu_t \in (0,\underline{\mu}) \\ U_t^1(1) = U_t^1(-1) & for\ \mu_t = \underline{\mu} \\ U_t^1(1) > U_t^1(-1) & for\ \mu_t \in (\underline{\mu},1) \end{cases} \tag{18}$$

holds for bullish traders and

$$\begin{cases} U_t^0(1) < U_t^0(-1) & for\ \mu_t \in (0,\overline{\mu}) \\ U_t^0(1) = U_t^0(-1) & for\ \mu_t = \overline{\mu} \\ U_t^0(1) > U_t^0(-1) & for\ \mu_t \in (\overline{\mu},1) \end{cases} \tag{19}$$

holds for bearish traders.

See Appendix A for a proof. Lemma 1 illustrates that an informed trader may not always follow his private signal. For a bullish trader, although expected return on a long position is always higher than on a short position, if public belief μ_t is low enough, the anticipated regret of buying a valueless asset will be so high that the investor prefers sell to buy. Symmetrically, a bearish trader prefers buy to sell if the anticipated regret of selling a valuable asset is large enough.

Lemma 2 Comparing inaction with action, it is found that for bullish traders,

$$\begin{cases} U_t^1(0) < U_t^1(-1) & for\ \ \mu_t \in (0,\underline{\mu}) \\ U_t^1(0) < U_t^1(1) & for\ \ \mu_t \in [\underline{\mu},1); \end{cases} \tag{20}$$

and for bearish traders,

$$\begin{cases} U_t^0(0) < U_t^0(-1) & for\ \ \mu_t \in (0,\overline{\mu}] \\ U_t^0(0) < U_t^0(1) & for\ \ \mu_t \in (\overline{\mu},1). \end{cases} \tag{21}$$

See Appendix A for a proof. Lemma 2 states that neither bullish traders nor bearish traders will choose not trading. To understand this result, note that if an investor chooses inaction, he will undoubtedly regret this decision: he will regret not having bought the asset when the true value of the asset is found to be $V = 1$ at the final date, whereas he will regret not having sold the asset if the true value of the asset is revealed to be $V = 0$.

Following Lemma 1 and Lemma 2, investors' optimal trading strategies are obtained.

Proposition 1 Under Assumption 1, investors' optimal trading strategies are as follows:

$$x_t^1 = \begin{cases} -1 & for\ \mu_t \in (0, \underline{\mu}) \\ 1 & for\ \mu_t \in [\underline{\mu}, 1); \end{cases} \tag{22}$$

$$x_t^0 = \begin{cases} -1 & for\ \mu_t \in (0, \overline{\mu}) \\ 1 & for\ \mu_t \in [\overline{\mu}, 1) \end{cases} \tag{23}$$

where $0 < \underline{\mu} < 0.5 < \overline{\mu} < 1$.

The proof for Proposition 1 is abbreviated as it directly follows from Lemma 1 and Lemma 2. Recall that the market price of the asset is set at $p_t = \mu_t$. Therefore, Proposition 1 implies that bearish traders will buy if the price is high enough, whereas bullish traders will sell if the price drops to a low enough level. In the next section, we will generalize this proposition and discuss its implications with more details. Below, Table 2 provides some numerical examples where $\overline{\mu}$ and $\underline{\mu}$ are defined as in Lemma 1. z is a parameter that controls the magnitude of regret over inaction. In this section, z is set equal to one; a more general model is provided in the next section, where z can take different values. To facilitate comparisons with results obtained in the next section, Table 2 also includes thresholds $\underline{\kappa}$, $\underline{\nu}$, $\overline{\nu}$, and $\overline{\kappa}$, which are defined in the proof of Lemma 2 in Appendix A.

Table 2 Numerical examples when $z = 1$.

Parameters	$\underline{\kappa}$	$\underline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\overline{\mu}$	$\overline{\kappa}$
$z = 1,\ \eta = 10,\ q = 0.7$	0.0275	0.1371	0.4692	0.5309	0.8629	0.9725
$z = 1,\ \eta = 20,\ q = 0.7$	0.0289	0.1458	0.4931	0.5069	0.8542	0.9711
$z = 1,\ \eta = 10,\ q = 0.8$	0.0092	0.0488	0.2129	0.7871	0.9512	0.9908

In Table 2, we can see that $\underline{\mu}$ is increasing in η, the magnitude of regret aversion, but is decreasing in q, the precision of informed traders' signal. $\overline{\mu}$ depends on η and q in a symmetric manner. The following corollary confirms this observation.

Corollary 1 $\overline{\mu}$ and $\underline{\mu}$ depend on η and q with

(i) $\frac{\partial \underline{\mu}}{\partial \eta} > 0$, $\frac{\partial \overline{\mu}}{\partial \eta} < 0$;

(ii) $\frac{\partial \underline{\mu}}{\partial q} < 0$, $\frac{\partial \overline{\mu}}{\partial q} > 0$.

See Appendix A for a proof. Statement (i) states that region $[\underline{\mu}, 1)$, where bullish traders buy, and region $(0,\ \underline{\mu}]$, where bearish traders sell, shrink as η increases. The intuition of this result is straightforward: the larger is the magnitude of regret aversion, the stronger is an informed trader's tendency to ignore his private information. Statement (ii) suggests that regions $[\underline{\mu}, 1)$ and $(0,\ \underline{\mu}]$ expand as information precision q increases. This result is also intuitive: the more precise is private information, the higher expected return an informed trader can obtain by following his private information, and the stronger is the incentive to do so.

When informed traders act according to the strategy shown in Proposition 1, it is easy to see that the price follows a martingale with respect to public information: $E[p_{t+1}|h_t] = E[E[V|h_{t+1}]|h_t] = E[V|h_t] = p_t$. However, the market maker's pricing rule in equation (10) is assumed exogenously. This setting, which is in line with the sequential trading model of Glosten and Milgrom (1985), facilitates the analysis of bubble riding, herding, and other results addressed in Sections 4 and 5, but it also limits the model's ability to explain price movements. To examine the effects of regret on equilibrium price during bubbles and crashes, it is important to endogenize the pricing rule. One approach is to adopt other trading mechanisms, such as batch trading. This interesting and challenging task is left for our future research. Below, we consider a case where the pricing rule is slightly more general than the one in equation (10).

Note that the market maker faces adverse selection in the following situations: $x_t^1 = 1$ and $x_t^0 = -1$; $x_t^1 = 1$ and $x_t^0 = 0$; $x_t^1 = 0$ and $x_t^0 = -1$. Because in equation (10) the market maker is assumed to set price equal to the expected asset value, he will have a negative expected return due to adverse selection. However, as shown in Corollary 2 below, the model can be extended to the case where the market maker sets a bid-ask spread when he faces adverse selection. Let p_t^b denote the bid price, p_t^a the ask price, and e_t the bid-ask spread.

Corollary 2. $\exists \bar{e} > 0$, such that when bid price and ask price are set at

$$p_t^a = \begin{cases} \mu_t + \frac{e_t}{2} & if \;\; x_t^1 > x_t^0 \\ \mu_t & otherwise; \end{cases} \tag{24}$$

$$p_t^b = \begin{cases} \mu_t - \frac{e_t}{2} & if \;\; x_t^1 > x_t^0 \\ \mu_t & otherwise \end{cases} \tag{25}$$

where $0 \le e_t < \bar{e}$ and $0 < p_t^a \le p_t^b < 1$, informed traders will act according to the trading strategy given by equations (22)-(23).

A proof for this corollary is provided in Appendix A. In equations (24)-(25), we require that the bid-ask spread is zero when there is no adverse selection; this assumption is consistent with the spirit of Glosten and Milgrom (1985), who establish the relationship between bid-ask spread and adverse selection.[15] Obviously, asset price $\{p_t^a, p_t^b\}$ in equations (24)-(25) and trading strategy $\{x_t^1, x_t^0\}$ in equations (16)-(17) depend on each other. Hence, an equilibrium should be defined as a pair of pricing rule and trading strategy such that, given the market maker's pricing rule, investors' trading strategy satisfies equations (16)-(17) and, given investors' trading strategy, the market maker's pricing rule satisfies (24)-(25) and $0 < p_t^a \le p_t^b < 1$. In Appendix A, we prove that such equilibriums do exist, where informed traders act according to the trading strategy in equations (22)-(23).

Corollary 2 illustrates that results obtained in this section hold in the case where a bid-ask spread exists but is small. However, the existence of a bid-ask spread makes

[15] In the original model of Glosten and Milgrom (1985), the market maker always faces adverse selection, and the bid-ask spread is set equal to the market maker's expected loss due to adverse selection. In the present paper, because informed traders may ignore their private information, the market maker does not always face adverse selection. For this reason, in Corollary 2, we assume that the market maker can set a non-zero bid-ask spread only when he faces adverse selection. Moreover, because it is beyond the scope of this paper to examine the relationship between bid-ask spread and regret aversion, we do not require the bid-ask spread to be exactly equal to the expected loss arising from adverse selection; this simplification enables us to avoid linking the bid-ask spread to regret aversion η.

the model less tractable. Therefore, we will restrict our attention to the situation where there is no bid-ask spread and keep the assumption of equation (10) for the remainder of this paper.

4. Investor Behavior

In the previous section, we assume that an investor has same attitude toward regret over action and regret over inaction. However, psychology studies indicate that regret over inaction differs from regret over action. In their seminal studies about regret, Kahneman and Tversky (1982) and Kahneman and Miller (1986) show that regret over action tends to be stronger than regret over inaction. One experiment by Kahneman and his colleagues is as follows (Quoted from Kahneman and Miller (1986), pp. 145).

> Mr. Paul owns shares in Company A. During the past year he considered switching to stock in company B, but he decided against it. He now finds out that he would have been better off by $1,200 if he had switched to the stock of company B. Mr. George owned shares in company B. During the past year he switched to stock in company A. He now finds that he would have been better off by $1200 if he had kept his stock in company B. Who feels greater regret?
>
> Mr. Paul 8% Mr. George 92% (N=138)

Obviously, most subjects thought that regret over action is stronger than regret over inaction. Kahneman and Miller (1986) suggest that a possible reason for this phenomenon is that inaction is generally thought to be more normal than action. Experiments by Landman (1987a) obtained similar results. Zhou et al. (2010), who measure subjects' brain activities when they regret, also find that emotional responses after action are stronger than emotional responses after inaction. On the other hand, Gilovich and Medvec (1994) suggest that regret over action and regret over inaction have a time pattern. In the short term, people tend to regret more strongly actions such as something they have done wrongly. However, in the long term, most people more strongly regret inactions such as missed educational opportunities, a failure to seize the moment and so on.[16] For a review of related studies, see Gilovich and Medvec (1995), Kahneman (1995), and Anderson (2003).

Based on these findings, in this section we extend Regret Theory, which treats regret over action only, into the dimension where regret over inaction differs from regret over inaction.

Assumption 2 An investor's modified utility function takes the following form:

$$u\big(r(x)\big) = \begin{cases} r(x) - \eta\sqrt{\max\{r\} - r(x)} & for\ x \in \{1, -1\} \\ r(x) - \eta z\sqrt{\max\{r\} - r(x)} & for\ x = 0 \end{cases} \tag{26}$$

where $\eta > 0$ and $z > 0$.

Parameter z is newly introduced into the modified utility function to reflect investors' different attitudes toward action and inaction: when $z < 1$, regret over inaction tends to be weaker than regret over action; when $z > 1$, the situation is opposite; when $z = 1$, Assumption 2 reduces to Assumption 1, which has been analyzed in the previous section as a special case of our model. Below, Proposition 2 derives

[16] Kahneman disagrees with Gilovich and Medvec, arguing that lifetime regret comes from wishful thinking rather than counterfactual thinking. See Gilovich, Medvec and

investors' optimal trading strategy in the case where regret over inaction is relatively strong.

Proposition 2 If $z \geq \frac{\sqrt{2}}{2}$, informed traders' optimal strategies are as follows:

$$x_t^1 = \begin{cases} -1 & for\ \mu_t \in (0, \underline{\mu}) \\ 1 & for\ \mu_t \in [\underline{\mu}, 1); \end{cases} \tag{27}$$

$$x_t^0 = \begin{cases} -1 & for\ \mu_t \in (0, \overline{\mu}) \\ 1 & for\ \mu_t \in [\overline{\mu}, 1) \end{cases} \tag{28}$$

where $0 < \underline{\mu} < 0.5 < \overline{\mu} < 1$.

See Appendix B for a proof. Key results are illustrated by Figure 1, where thresholds $\underline{\kappa}$, $\underline{\mu}$, $\underline{\nu}$, $\overline{\nu}$, $\overline{\mu}$, and $\overline{\kappa}$ and functions $K^1(\mu_t)$, $M^1(\mu_t)$, $G^1(\mu_t)$, $K^0(\mu_t)$, $M^0(\mu_t)$, and $G^0(\mu_t)$ are defined in Appendix B. In this proposition, $\frac{\sqrt{2}}{2}$ is a threshold for z because $\underline{\kappa} = \underline{\mu} = \underline{\nu}$ and $\overline{\nu} = \overline{\mu} = \overline{\kappa}$ hold at $z = \frac{\sqrt{2}}{2}$. As shown in the proof, $0 < \underline{\kappa} < \underline{\mu} < \underline{\nu} < 1$ and $0 < \overline{\nu} < \overline{\mu} < \overline{\kappa} < 1$ hold for $\frac{\sqrt{2}}{2} < z < \sqrt{2}$, which ensures the holding of equations (27) and (28). When $z \geq \sqrt{2}$, investors' regret over inaction is so strong that $U_t^s(0)$, the expected utility of not trading, is less than $U_t^s(1)$ and $U_t^s(-1)$ for any $\mu_t \in (0,1)$, which also implies equations (27) and (28). Obviously, Proposition 2 is a generalization of Proposition 1.

Investors' trading strategies in the case of $z < \frac{\sqrt{2}}{2}$ are addressed by Propositions 3 and 4, where $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$ is a threshold for z because $\underline{\kappa} = \overline{\kappa}$ holds when $z =$

Kahneman (1998) for an interesting discussion on this issue among these researchers.

$\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$. Note that $z > 0$ by assumption, while $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} > 0$ if and only if $\eta > \frac{\lambda_q - 1}{2}$. Thus, the case of $z < \frac{\sqrt{2}}{2}$ has three subcases: (i) $\eta \le \frac{\lambda_q - 1}{2}$ and $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \le 0 < z < \frac{\sqrt{2}}{2}$; (ii) $\eta > \frac{\lambda_q - 1}{2}$ and $0 < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \le z < \frac{\sqrt{2}}{2}$; and (iii) $\eta > \frac{\lambda_q - 1}{2}$ and $0 < z < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$. Below, Proposition 3 analyzes subcases (i) and (ii), and Proposition 4 treats subcase (iii).

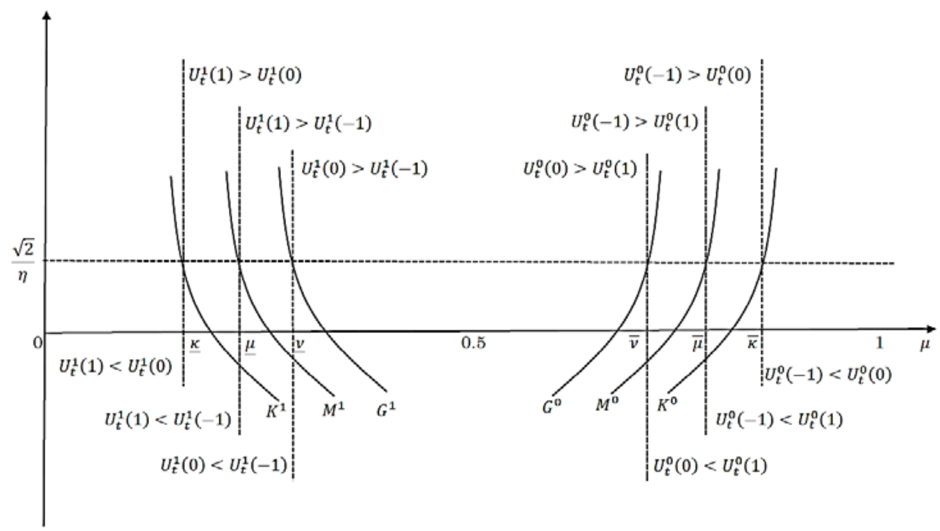

Figure 1

Proposition 3 If $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \le z < \frac{\sqrt{2}}{2}$, informed traders' optimal strategies are as follows:

$$x_t^1 = \begin{cases} -1 & for\ \mu_t \in (0, \underline{\nu}] \\ 0 & for\ \mu_t \in (\underline{\nu}, \underline{\kappa}) \\ 1 & for\ \mu_t \in [\underline{\kappa}, 1); \end{cases} \tag{29}$$

$$x_t^0 = \begin{cases} -1 & for\ \mu_t \in (0, \overline{\kappa}] \\ 0 & for\ \mu_t \in (\overline{\kappa}, \overline{\nu}) \\ 1 & for\ \mu_t \in [\overline{\nu}, 1) \end{cases} \tag{30}$$

where $0 < \underline{\nu} < \underline{\kappa} \leq 0.5 \leq \overline{\kappa} < \overline{\nu} < 1$.

See Appendix B for a proof. In this case, $0 < \underline{\nu} < \underline{\kappa} \leq 0.5 \leq \overline{\kappa} < \overline{\nu} < 1$ holds as shown in Figure 2. As a result, bullish traders choose to exit the market when $\mu_t \in (\underline{\nu}, \underline{\kappa})$, while bearish traders choose to leave when $\mu_t \in (\overline{\kappa}, \overline{\nu})$.

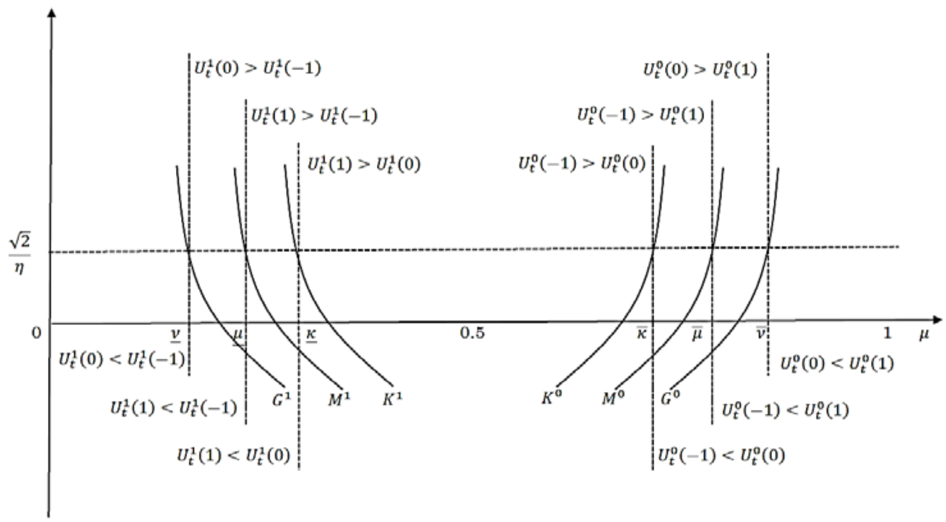

Figure 2.

Proposition 4 If $\eta > \frac{\lambda_q - 1}{2}$ and $z < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$, informed traders' optimal strategies are as follows:

$$x_t^1 = \begin{cases} -1 & for\ \mu_t \in (0, \underline{\nu}] \\ 0 & for\ \mu_t \in (\underline{\nu}, \underline{\kappa}) \\ 1 & for\ \mu_t \in [\underline{\kappa}, 1); \end{cases} \tag{31}$$

$$x_t^0 = \begin{cases} -1 & for\ \mu_t \in (0, \overline{\kappa}] \\ 0 & for\ \mu_t \in (\overline{\kappa}, \overline{\nu}) \\ 1 & for\ \mu_t \in [\overline{\nu}, 1) \end{cases} \tag{32}$$

where $0 < \underline{\nu} < \overline{\kappa} < 0.5 < \underline{\kappa} < \overline{\nu} < 1$.

See Appendix B for a proof. In this case, regret over inaction is so weak such that $\underline{\nu} < \overline{\kappa} < \underline{\kappa} < \overline{\nu}$ holds. As exhibited in Figure 3, in this case, there is a "non-trading region" $(\overline{\kappa}, \underline{\kappa})$ where both bullish traders and bearish traders choose inaction.

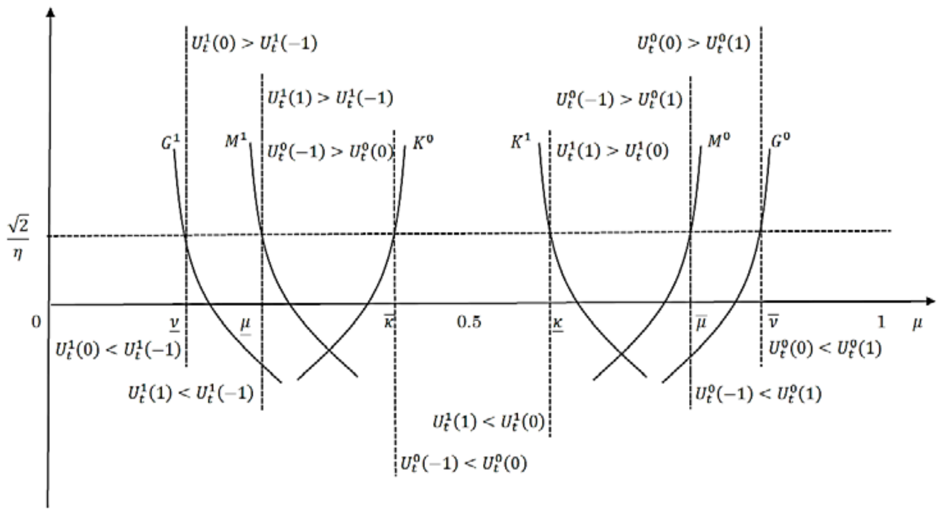

Figure 3

It is clear from Propositions 2-4 that investors' trading strategies are sensitive to the magnitude of regret over inaction. Nevertheless, one result is commonly obtained in all cases. That result is that, if the price is high enough, bearish traders are willing to buy the asset even though they know that the asset is overpriced; conversely, if the price is low enough, bullish traders will sell the asset even though they know that the asset is underpriced. We argue that this result provides a theoretical explanation of "bubble riding" and "fire selling". Because the structure of the model is symmetric, in the discussion below, we focus on "bubble riding" behavior by informed traders.

During a bubble, informed traders who are aware of the overpricing may choose to ride the bubble instead of trading against it; this phenomenon has been extensively studied in the literature of asset bubbles. For example, Abreu and Brunnermeier (2003) illustrate that incomplete information and lack of synchronization among traders can explain this behavior. In their model, an investor who learns of the overpricing of the asset does not know how many other traders are aware of the bubble; hence, to maximize expected return, the investor will choose to ride the bubble for a certain period. This model is extended by Doblas-Madrid (2012) and Matsushima (2013). Doblas-Madrid (2012) show that investors will ride the bubble for a longer period if they can hide their orders among orders from noise traders; Matsushima (2013) illustrates that even when all investors get to know the bubble simultaneously, an investor may still ride the bubble if he is not sure whether other investors are rational. Another explanation is that informed investors may ride a bubble in the hope that they can sell the asset at a higher price to "greater fools" who are unaware of the bubble. For example, De Long et al. (1990) show that rational informed traders may buy an overvalued asset hoping to sell it to positive-feedback traders in the next period. In the model of Allen et al. (1993), because the occurrence of a bubble is not common knowledge, an investor who has noticed the bubble may still hold the asset hoping that some greater fool in the market will buy the asset at a higher price. Conlon (2004) extends this model by showing that even when the occurrence of a bubble is common knowledge, the bubble may still be prolonged.[17]

The present paper provides a complementary explanation for bubble riding: regret avoidance. In our model, the price of the asset reveals all information that has been accumulated in past trading rounds; the higher is the price, the higher is the

[17] There are other approaches too. For example, DeMarzo et al. (2008) develop an overlapping generations model where rational investors invest in overvalued assets because of relative wealth concerns. Allen and Gorton (1993) show that fund managers may buy overpriced assets because of agency relationships. For a detailed review of the

probability that the true value of the asset is high. Hence, when the price is high, if a trader sells the asset, there is a high probability that he will regret later that he has bet on the wrong side; if he chooses not trading, the anticipated regret over inaction is also high. In contrast, the anticipated regret of buying is relatively low in this case because there is a high probability that the true value of the asset is high. For a bearish trader who observes a negative signal, although the expected return of selling is positive, if the price is high enough, anticipated regret of selling or inaction will be so strong that he will choose buy to avoid regret.

In some sense, the interpretation of bubble riding presented in the this paper illustrates a struggle between emotion and rational calculation. We argue that this interpretation is intuitive and compelling. In the real world, it is quite plausible that people ride a bubble because they fear that they will regret missed opportunities later. For example, in their famous book about the history of bubbles, Kindleberger and Aliber (2005, pp30) describe how a bubble forms. "A follow-the-leader process develops as firms and households see that the speculators are making a lot of money. 'There is nothing as disturbing to one's well-being and judgment as to see a friend get rich'. Unless it is to see a non-friend get rich. Similarly banks may increase their loans because they are reluctant to lose market share to other lenders. More and more firms and households begin to participate in the scramble for high profits. Making money never seemed easier. Speculation for capital gains leads from normal, rational behavior to what has been described as a 'mania' or a 'bubble'." Obviously, emotion is a strong driver for this "follow-the-leader process."

As shown in Propositions 3 and 4, the model in this paper can explain why

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literature, see Brunnermeier and Oehmke (2013).

investors exit and reenter the market and why non-trading can occur. Moreover, as discussed in Section 6, some testable implications can be derived from this model. These results also distinguish our approach from rival theories of bubbles and crashes.

5. Market Turbulence

In this section, we examine the effects of regret on aggregate market fluctuations. As shown in previous sections, to avoid anticipated regret of betting on the wrong side, investors buy when the market price of the asset is high and sell when the price is low. Such behavior will further destabilize the market and cause market turbulence that, in the setting of the present paper, occurs in the form of herds.

A "herd" generally refers to the phenomenon of a large group of people behaving similarly. Herds can occur in various circumstances and for different reasons; for example, clothing fashions, social movements, religious movements, war fever, attitudes toward alcohol, cigarettes and drugs, attitudes toward marriage and sex, choice of political candidates, business strategies, medical practice and so on.[18] In financial markets, herding behavior can be observed in bank runs, bubbles and crashes. Finance researchers are particularly interested in a special form of herd, "informational cascade." According to the definition given by Banerjee (1992) and Bikhchandani et al. (1992), an informational cascade occurs when it is optimal for an investor to imitate the choice of the preceding investor without regard to his own information. The reason for the occurrence of a cascade is informational externality. For example, in a market where the price of an asset is fixed and investors are equally informed, if the first and the second traders both buy the asset, the third trader will buy regardless of his own information; a buy-herd thus occurs because all the other traders will make the same decision.

In the model of the present paper, a buy-herd occurs when the price of the asset is high, whereas a sell-herd occurs when the price is low. However, our model differs from informational cascades models in the following ways. First, herds arise from regret aversion rather than from informational externality. To see this, we only need to note that if investors do not have regrets, that is, $\eta = 0$, then bullish traders always buy and bearish traders always sell. Second, in informational cascade models, investors do not buy overvalued assets or sell undervalued assets; as a result, even during a cascade, an informed trader's expected return is still positive. In contrast, our model shows that an informed trader who joins a herd to avoid anticipated regret may have negative expected returns. Third, depending on the magnitude of regret over inaction, "partial herds" and non-trading can occur in the model of the present paper.[19]

Corollary 3 (Herd) In the case of $z \geq \frac{\sqrt{2}}{2}$, trading and information accumulation in the market take place as follows.

(i) If $p_t \in [\underline{\mu}, \overline{\mu}]$, bullish traders buy and bearish traders sell:

$$x_t^1 = 1 \text{ and } x_t^0 = -1; \tag{33}$$

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[18] See discussions by Bikhchandani et al. (1992) and Bikhchandani et al. (1998).

[19] Informational cascade can provide intuitive explanations for bubbles and crashes; however, as Avery and Zemsky (1998), Chari and Kehoe (2004), and Park and Sabourian (2011) have pointed out, the original model of Bikhchandani et al. (1992) relies on the assumption of fixed prices, which obviously does not apply to financial markets. Furthermore, as noted by Shiller (1995), first movers play a critical role in informational cascade models. The model in the present paper does not have these problems because the market price is flexible and the occurrence of a herd does not rely on fashion leaders.

$$\mu_{t+1} = \begin{cases} \frac{\mu_t \delta}{\mu_t \delta + (1-\mu_t)(1-\delta)} & if \;\; x_t = 1 \\ \frac{\mu_t (1-\delta)}{\mu_t (1-\delta) + (1-\mu_t)\delta} & if \; x_t = -1 \end{cases} \tag{34}$$

where $\delta \equiv \phi q + (1-\phi)/2$.

(ii) If $p_t \in (0, \underline{\mu})$, a "sell-herd" occurs where all informed traders sell:

$$x_t^1 = x_t^0 = -1; \tag{35}$$

$$\mu_{t+1} = \mu_t. \tag{36}$$

(iii) If $p_t \in (\overline{\mu}, 1)$, a "buy-herd" occurs where all informed traders buy:

$$x_t^1 = x_t^0 = 1; \tag{37}$$

$$\mu_{t+1} = \mu_t. \tag{38}$$

Proof for this corollary is abbreviated because it is a direct implication of Proposition 2. According to Corollary 2, informed traders follow their private signals as long as the price of the asset remains inside the "truth-telling" region $[\underline{\mu}, \overline{\mu}]$. If the price drops below $\underline{\mu}$, a sell-herd will occur in the market, where all informed traders submit sell orders regardless of their private information; if the price rises above $\overline{\mu}$, a buy-herd will occur, where all informed traders submit buy orders. These results are illustrated in Figure 4.

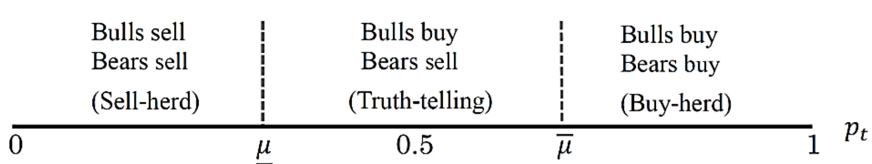

Figure 4

Corollary 4 (Herd and partial herd) In the case of $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \leq z < \frac{\sqrt{2}}{2}$, trading and information accumulation in the market take place as follows.

(i) If $p_t \in [\underline{\kappa}, \overline{\kappa}]$, bullish traders buy and bearish traders sell.

(ii) If $p_t \in (\underline{\nu}, \underline{\kappa})$, a "partial sell-herd" occurs where all bearish traders submit sell orders, while all bullish traders exit the market:

$$x_t^1 = 0 \text{ and } x_t^0 = -1; \tag{39}$$

$$\mu_{t+1} = \begin{cases} \frac{\mu_t \zeta}{\mu_t \zeta + (1-\mu_t)(1-\xi)} & if \;\; x_t = 1 \\ \frac{\mu_t (1-\zeta)}{\mu_t (1-\zeta) + (1-\mu_t)\xi} & if \; x_t = -1 \end{cases} \tag{40}$$

where $\zeta \equiv \frac{(1-\phi)/2}{1-\phi q}$ and $\xi \equiv \frac{\phi q + (1-\phi)/2}{1-\phi(1-q)}$.

(iii) If $p_t \in (\overline{\kappa}, \overline{\nu})$, a "partial buy-herd" occurs where all bullish traders submit buy orders, while all bearish traders exit the market:

$$x_t^1 = 1 \text{ and } x_t^0 = 0; \tag{41}$$

$$\mu_{t+1} = \begin{cases} \frac{\mu_t \xi}{\mu_t \xi + (1-\mu_t)(1-\zeta)} & if \;\; x_t = 1 \\ \frac{\mu_t (1-\xi)}{\mu_t (1-\xi) + (1-\mu_t)\zeta} & if \; x_t = -1. \end{cases} \tag{42}$$

(iv) If $p_t \in (0, \underline{\nu}]$, a sell-herd occurs where all informed traders sell.

(v) If $p_t \in [\overline{\nu}, 1)$, a buy-herd occurs where all informed traders buy.

Corollary 4 follows from Proposition 3. The proof is apparent and is therefore abbreviated. The term "partial" emphasizes the fact that traders only partially ignore their private information. For example, bearish traders trade against their private information in a buy-herd; however, in a partial buy-herd, although they ignore their private information, they do not trade against it. A partial herd in the present paper is similar to a situation addressed by Lee (1998), in whose model informed traders wait and do not trade until someone who has superior information breaks the ice.

Nevertheless, there is an important difference: Lee (1998) uses the trading cost as a "brake" that prevents informed investors from trading. However, in the present paper, the brake is investors' anticipated regret.

Figure 5 illustrates the regions of herds and partial herds. $[\underline{\kappa}, \overline{\kappa}]$ is a truth-telling region where all informed traders act following their signals. If the price drops below $\underline{\kappa}$, a partial sell-herd occurs, where bullish traders exit while bearish traders and noise traders remain in the market. During the partial sell-herd, because buy orders only come from noise traders and all market participants know about this, one tends to presume that the price will remain unchanged after the market maker executes a sell order. However, as shown in equation (42), an interesting result obtained is that the asset price reacts to both buy and sell during partial herds. If the price rebounds and rises above $\underline{\kappa}$, then bullish traders will return to the market to buy and the partial sell-herd will end; if the price keeps dropping and crosses threshold $\underline{\nu}$, then bullish traders will return to the market to sell, and the partial herd will develop into a full herd of sellers.

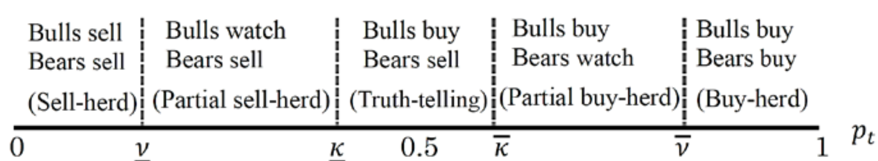

Figure 5

The above results suggest that there are three stages for a sell-herd to form in a market. At first, the price drops but bullish traders still buy; in the next stage, the price drops deeply and bullish traders stop buying; in the final stage, the price drops to such a low level that even bullish traders turn to selling. Buy-herd and partial buy-herd occur in a symmetric manner. It is easy to see that the regions of herds and partial herds depend on z, the magnitude of regret over inaction. If z becomes smaller, the regions of partial herds in Figure 5 will expand, while the truth-telling region will shrink. If regret over inaction becomes so weak that $z < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$, then, as shown in the next corollary, the truth-telling region will entirely disappear and a "non-trading region" will appear.

Corollary 5 (Herd, partial herd, and non-trading) In the case of $\eta > \frac{\lambda_q - 1}{2}$ and $z < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$, trading and information accumulation in the market take place as follows.

(i) If $p_t \in (\overline{\kappa}, \underline{\kappa})$, informed traders do not trade:

$$x_t^1 = x_t^0 = 0; \tag{43}$$

$$\mu_{t+1} = \mu_t. \tag{44}$$

(ii) If $p_t \in (\underline{\nu}, \overline{\kappa}]$, a partial sell-herd occurs.

(iii) If $p_t \in [\underline{\kappa}, \overline{\nu})$, a partial buy-herd occurs.

(vi) If $p_t \in (0, \underline{\nu}]$, a sell-herd occurs.

(vii) If $p_t \in [\overline{\nu}, 1)$, a buy-herd occurs.

Corollary 5 is a direct implication of Proposition 4, so the proof is abbreviated. The main results are illustrated in Figure 6, where $(\overline{\kappa}, \underline{\kappa})$ is the newly appeared non-trading region.

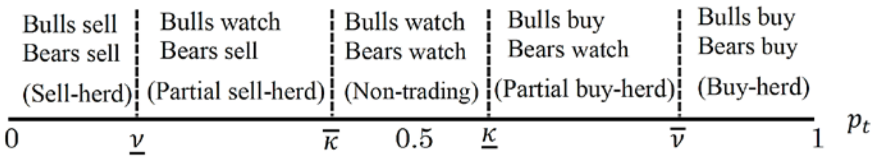

Figure 6

The intuition of non-trading is obvious. When regret over inaction is very weak, investors' regret over action becomes relatively strong. Unless the price is very high or very low, investors are reluctant to take actions. As a result, non-trading can occur in market where all informed traders choose not trading.

Table 3 provides some numerical examples where η and q are fixed while z takes different values. These examples clearly illustrate that investors' strategies are sensitive to z, which controls the magnitude of regret over inaction. When $z = 1$, there is a large truth-telling region (0.1371, 0.8629) where all investors act following their signals. When $z = 0.5$, a region of partial sell-herd (0.0465, 0.3395) and a region of partial buy-herd (0.6605, 0.9535) appear, while the truth-telling region dramatically shrinks to (0.3595, 0.6605). When $z = 0.25$, there is a huge non-trading region (0.2366, 0.7634) between the two partial herd regions, and the truth-telling region totally disappears.

Table 3 Numerical examples with $z = 1, 0.5, 0.25$.

Parameters	Thresholds					
$z = 1,\ \eta = 10,\ q = 0.7$ (The case of Corollary 3)	$\underline{\kappa}$ =0.0275	$\underline{\mu}$ =0.1371	$\underline{\nu}$ =0.4692	$\overline{\nu}$ =0.5309	$\overline{\mu}$ =0.8629	$\overline{\kappa}$ =0.9725
$z = 0.5,\ \eta = 10,\ q = 0.7$ (The case of Corollary 4)	$\underline{\nu}$ =0.0465	$\underline{\mu}$ =0.1371	$\underline{\kappa}$ =0.3395	$\overline{\kappa}$ =0.6605	$\overline{\mu}$ =0.8629	$\overline{\nu}$ =0.9535
$z = 0.25,\ \eta = 10, q = 0.7$ (The case of Corollary 5)	$\underline{\nu}$ =0.0076	$\underline{\mu}$ =0.1371	$\overline{\kappa}$ =0.2366	$\underline{\kappa}$ =0.7634	$\overline{\mu}$ =0.8629	$\overline{\nu}$ =0.9924

Comparing thresholds for buy-herd in Table 3: if $z = 0.25$, a buy-herd occurs when the price rises above 0.9924; if $z = 0.5$, the threshold is 0.9535; and if $z = 1$, the threshold is 0.8629. Thus, the stronger is regret over inaction, the lower is the threshold, and the easier it is for a buy-herd to occur. Symmetrically, Table 3 also illustrates that the stronger is regret over inaction, the easier it is for a sell-herd to occur. Below, we formalize this observation. Let

$$\underline{p} = \begin{cases} \underline{\mu} & for\ z \geq \frac{\sqrt{2}}{2} \\ \underline{\nu} & for\ 0 < z < \frac{\sqrt{2}}{2}; \end{cases} \tag{45}$$

$$\bar{p} = \begin{cases} \bar{\mu} & for\ z \geq \frac{\sqrt{2}}{2} \\ \bar{\nu} & for\ 0 < z < \frac{\sqrt{2}}{2}. \end{cases} \tag{46}$$

It follows from Corollaries 3-5 that given any $z > 0$ and $\eta > 0$, a sell-herd occurs whenever the asset price drops bellow $\underline{p}$, while a buy-herd occurs whenever the asset price rises above $\bar{p}$.

Corollary 6

(i) Given any $z > 0$, $\underline{p}$ is strictly increasing in η, and $\bar{p}$ is strictly decreasing in η.

(ii) Given any $\eta > 0$, $\underline{p}$ is an increasing function of z that is strictly increasing on $(0, \frac{\sqrt{2}}{2})$, and $\bar{p}$ is a decreasing function of z that is strictly decreasing on $(0, \frac{\sqrt{2}}{2})$.

Statement (i) states that the larger is regret aversion, the easier it is for herds to occur. Statement (ii) claims that the stronger is regret over inaction, the easier it is for herds to occur. See Appendix B for a proof.

6. Discussion

This section discusses the economic implications of the model and provides some testable results. The basic idea of the present paper is that an investor's rational calculation of expected return can be overwhelmed by anticipated regret. Previous sections show that regret aversion can cause the occurrence of herds, partial herds, and non-trading in the market. When a buy-herd occurs, all investors try to buy the asset regardless of their own information about the asset; following previous studies about herd behavior in financial markets, we interpret a buy-herd as a bubble in the asset market. Similarly, a sell-herd can be interpreted as a market crash during which all investors try to sell.[20] Moreover, a partial buy-herd may represent a bull market in which bullish traders play the leading role; in contrast, a partial sell-herd may be interpreted as a bear market where bullish investors have left and only bearish investors and noise traders are active.

To describe the characteristics shown by the market during herds and partial herds, the discussion below will focus on two observable factors. One is the volume of orders, which is the number of orders received by the market maker during one trading round; the other is "order imbalance", which is defined as the ratio between the number of buy orders and the number of sell orders. In real markets, data of these variables

[20] Strictly speaking, the price of the asset does not move during a buy-herd or a sell-herd. This occurs because the model assumes that prices only react to new information, which is in accordance with the efficient market hypothesis, whereas orders in a herd convey no information, which is an important property of herding. To model dramatic price movements during bubbles and crashes, additional assumptions are needed that, however, will make the model far more complex while providing few additional implications about the effects of regret.

exists and has been studied in the literature of market microstructure.[21]

We start with the case in Corollary 4. Recall that when the price of the asset moves from the truth-telling region into the region of the partial buy-herd, bearish traders will exit the market; if the price keeps rising and moves into the region of the buy-herd, then bearish traders will come back to buy the asset. Bearish traders' exiting and reentering will cause changes in both order volume and order imbalance; therefore, Corollary 4 suggests that the market will exhibit the following changes when it is moving toward a bubble.

Remark 1 (Along the path toward a bubble) When a bull market starts after a series of price increases, the market tends to exhibit a high order imbalance but a low volume. If the price keeps rising and a bubble forms in the market, both order imbalance and volume will increase sharply.

In Minsky's model, as described by Kindleberger and Aliber (2005) and Brunnermeier and Oehmke (2013), there are five phases for a bubble to form and to burst: an initial displacement, a boom phase, a phase of euphoria, a phase of profit taking, and a panic phase. Among these, the boom phase is characterized by low volatility, credit expansion, and increases in investment, while the phase of euphoria is associated with high trading volume. It is easy to see that the model in the present paper is consistent with Minsky's model.

Next, consider the case where the asset price moves from the truth-telling region

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[21] For empirical studies about order imbalance and volume, see, for example, Blume et al. (1989) and Chordia et al. (2002).

into the region of partial buy-herd and buy-herd. Corollary 4 implies that, along the path toward a bubble, the following changes will occur in the market.

Remark 2 (Along the path toward a crash) When a bear market starts after a series of price declines, the market tends to exhibit low order imbalance as well as low volume. If the price keeps dropping and triggers a market crash, there will be a sudden increase in volume together with a sharp drop in order imbalance.

When observing the relationship between order imbalance and volume during herds and partial herds, Corollary 4 also implies the following result.

Remark 3 (Order imbalance and Volume) When the market moves from a bear market toward a crash, order imbalance and volume tend to have a negative correlation; when the market moves from a bull market toward a bubble, order imbalance and volume tend to show a positive correlation.

This result helps to understand fluctuations in order imbalance and volume during market turbulences; moreover, the prediction can be empirically tested. In the sense that theoretical models about order imbalance and volume are relatively few, this result may be regarded as an important contribution of this paper.

Below, we discuss an extension to this model. In the present paper, regret over action and regret over inaction are controlled by two parameters, η and z. We can extend this model by allowing the magnitude of regret to evolve dynamically. Specifically, we can use function $\eta(t, p_t)$ instead of parameter η, and function $z(t, p_t)$

instead of parameter z. As discussed in the introduction, psychological studies suggest that, in the short term, people tend to regret actions more, but in the long term, they tend to regret inactions more. Experimental findings also show that regret aversion tends to increase after experiencing regret. Therefore, it is quite likely that people tend to regret missed opportunities more when a bubble is still growing, while they tend to regret incautious investments more after the bubble collapses.[22] In light of psychological evidence, it is realistic to assume that $\eta(t, p_t)$ is decreasing in both p_t and t, whereas $z(t, p_t)$ is increasing in both p_t and t. Most results in this paper will still hold in this generalized setting because the structure of the model remains unchanged. Furthermore, Corollary 5 implies that a market crash will result in the following changes in the market.

Remark 4 (After a crash) If investors' regret over action becomes extremely strong after a crash, the market will enter a non-trading region. In such a case, because order volume is low and buy and sell orders are balanced, price tends to remain at a low level for a long period.

This implication is in accordance with experimental evidence.[23] It is also in line with previous studies on crashes, such as Gennotte and Leland (1990) and Barlevy and Veronesi (2003), who demonstrate that after a crash, asset prices will stay on a path

[22] Empirical studies such as Malmendier and Nagel (2011) and Strahilevitz et al. (2011) suggest that people's willingness to take financial risks is affected by their experiences.

[23] In experimental settings, Ku (2008) shows that the experience of escalation makes people become more cautious in future decision making, and Reb (2008) finds that people make more careful decisions when regret is salient.

lower than the one before the crash.

Conversely, if asset prices grow steadily for a long period, it is likely that investors' regret over action will become weaker whereas their regret over inaction will become very strong. If this is the case, then Corollary 6 implies that a herd can easily occur in such a situation, and Corollary 2 suggests that the market can directly move from the truth-telling region into a herd region. Thus, the model implies that a strong bull market tends to end in a sudden crash.

Remark 5 (After a boom) If investors' regret over inaction becomes stronger after long-continued asset price rising, then the market will become more likely to encounter market turmoil. Moreover, market turmoil in this case will occur in a dramatic manner because a herd can suddenly occur without a partial herd occurring first.

7. Conclusions

Based on Regret Theory, this paper examines the effects of regret on investor behavior and market turbulence in a model where investors not only regret wrong actions, but also regret inaction. We demonstrate that regret aversion can cause investors to ride a bubble, exit and reenter the market, or choose non-trading. As a result, herds and partial herds can occur in the market, and the stronger is regret over inaction, the easier it is for herds to occur. The model predicts that order volume and order imbalance tend to have positive (negative) correlation when a bubble (crash) is forming.

The present paper contributes to the research on asset bubbles by illustrating the role of regret in market fluctuations. It also contributes to the literature in neuroeconomics by showing the effects of regret over inaction on economic decision making. To the knowledge of the author, this is the first study that explicitly incorporates regret over inaction into a decision-making model.

Appendix A. Proofs for the model in Section 3.

Proof of Lemma 1.

Consider a bullish trader's decision making, who observes a signal $s = 1$. By equations (10), (13), and (15),

$$U_t^1(1) - U_t^1(-1) = 2(\mu_t^1 - \mu_t) - \eta(1 - \mu_t^1)\sqrt{2\mu_t} + \eta\mu_t^1\sqrt{2(1 - \mu_t)}. \tag{A.1}$$

Hence,

$$U_t^1(1) - U_t^1(-1) \gtreqless 0 \Leftrightarrow M^1(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta} \tag{A.2}$$

where

$$M^1(\mu_t) = \frac{(1-\mu_t^1)\sqrt{\mu_t} - \mu_t^1\sqrt{(1-\mu_t)}}{\mu_t^1 - \mu_t}. \tag{A.3}$$

Substituting (11) into above equation, we have

$$M^1(\mu_t) = \frac{\sqrt{1+\frac{1}{\lambda_t}} - \lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1} \tag{A.4}$$

where $\lambda_t = \frac{\mu_t}{1-\mu_t}$ and $\lambda_q = \frac{q}{1-q}$. Note that λ_t is strictly increasing in μ_t with $\lambda_t|_{\mu_t=0.5} = 1$. Hence, $M^1(\mu_t)$ is strictly decreasing in μ_t with $M^1(0.5) = -\sqrt{2}$; furthermore, $M^1(\mu_t) \to +\infty$ as $\mu_t \to 0$. As $M^1(\cdot)$ is a continuous function of μ_t, there exists $\underline{\mu} \in \left(0, \frac{1}{2}\right)$ such that $\underline{\mu}$ is the unique solution to $M^1(\mu_t) = \frac{\sqrt{2}}{\eta}$. Obviously, $\underline{\mu}$ depends on q and η, but does not depend on t. Thus, equation (18) holds.

Next, consider a bearish trader's problem who observes a signal $s = 0$. By equations (10), (13), and (15),

$$U_t^0(-1) - U_t^0(1) = 2(\mu_t - \mu_t^0) - \eta\mu_t^0\sqrt{2(1-\mu_t)} + \eta(1-\mu_t^0)\sqrt{2\mu_t}. \tag{A.5}$$

Hence,

$$U_t^0(-1) - U_t^0(1) \gtreqless 0 \Leftrightarrow M^0(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta} \tag{A.6}$$

where

$$M^0(\mu_t) = \frac{\mu_t^0\sqrt{(1-\mu_t)} - (1-\mu_t^0)\sqrt{\mu_t}}{\mu_t - \mu_t^0} = \frac{\sqrt{1+\lambda_t} - \lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}. \tag{A.7}$$

Because $\lambda_t = \frac{\mu_t}{1-\mu_t}$, $M^0(\mu_t)$ is strictly increasing in μ_t with $M^0(0.5) = -\sqrt{2}$; furthermore, $M^0(\mu_t) \to +\infty$ as $\mu_t \to 1$. Therefore, there exists unique $\overline{\mu} \in \left(\frac{1}{2}, 1\right)$ such that $M^0(\overline{\mu}) = \frac{\sqrt{2}}{\eta}$. It is easy to see that $\overline{\mu}$ is the unique solution to $M^0(\mu_t) = \frac{\sqrt{2}}{\eta}$ and it does not depend on t. Thus, equation (19) holds. Q.E.D.

Proof of Lemma 2.

Consider a bullish trader's decision making, who observes a signal $s = 1$. By equations (13)-(15), we have

$$U_t^1(1) - U_t^1(0) = \mu_t^1 - \mu_t - \eta(1-\mu_t^1)\sqrt{2\mu_t} + \eta\mu_t^1\sqrt{1-\mu_t} + \eta(1-\mu_t^1)\sqrt{\mu_t}, \tag{A.8}$$

$$U_t^1(0) - U_t^1(-1) = \mu_t^1 - \mu_t + \eta\mu_t^1\sqrt{2(1-\mu_t)} - \eta\mu_t^1\sqrt{1-\mu_t} - \eta(1-\mu_t^1)\sqrt{\mu_t}. \tag{A.9}$$

Hence,

$$U_t^1(1) - U_t^1(0) \gtreqless 0 \Leftrightarrow K^1(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta}, \tag{A.10}$$

$$U_t^1(0) - U_t^1(-1) \gtreqless 0 \Leftrightarrow G^1(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta} \tag{A.11}$$

where

$$K^1(\mu_t) = \frac{(2-\sqrt{2})(1-\mu_t^1)\sqrt{\mu_t} - \sqrt{2}\mu_t^1\sqrt{1-\mu_t}}{\mu_t^1 - \mu_t} = \frac{(2-\sqrt{2})\sqrt{1+\frac{1}{\lambda_t}} - \sqrt{2}\lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1}, \tag{A.12}$$

$$G^1(\mu_t) = \frac{\sqrt{2}(1-\mu_t^1)\sqrt{\mu_t} - (2-\sqrt{2})\mu_t^1\sqrt{1-\mu_t}}{\mu_t^1 - \mu_t} = \frac{\sqrt{2}\sqrt{1+\frac{1}{\lambda_t}} - (2-\sqrt{2})\lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1}. \tag{A.13}$$

Both $K^1(\mu_t)$ and $G^1(\mu_t)$ are strictly decreasing in μ_t. As $\mu_t \to 1$, $K^1(\mu_t) \to -\infty$ and $G^1(\mu_t) \to -\infty$; as $\mu_t \to 0$, $K^1(\mu_t) \to +\infty$ and $G^1(\mu_t) \to +\infty$. Thus, $K^1(\mu_t) = \frac{\sqrt{2}}{\eta}$ has unique solution $\underline{\kappa}$ and $G^1(\mu_t) = \frac{\sqrt{2}}{\eta}$ has unique solution $\underline{\nu}$. Furthermore, $\forall \mu_t \in (0,1)$, $G^1(\mu_t) > M^1(\mu_t) > K^1(\mu_t)$ holds. Thus, $0 < \underline{\kappa} < \underline{\mu} < \underline{\nu} < 1$. As shown in Figure 1, equation (20) is obtained by comparing $U_t^1(1)$, $U_t^1(0)$, and $U_t^1(-1)$ in each of the intervals $(0, \underline{\kappa})$, $(\underline{\kappa}, \underline{\mu})$, $(\underline{\mu}, \underline{\nu})$, and $(\underline{\nu}, 1)$.

For a bearish trader, equations (13)-(15) imply that

$$U_t^0(-1) - U_t^0(0) = \mu_t - \mu_t^0 - \eta\mu_t^0\sqrt{2(1-\mu_t)} + \eta\mu_t^0\sqrt{1-\mu_t} + \eta(1-\mu_t^0)\sqrt{\mu_t}, \tag{A.14}$$

$$U_t^0(0) - U_t^0(1) = \mu_t - \mu_t^0 + \eta(1-\mu_t^0)\sqrt{2\mu_t} - \eta\mu_t^0\sqrt{1-\mu_t} - \eta(1-\mu_t^0)\sqrt{\mu_t}. \tag{A.15}$$

Hence,

$$U_t^0(-1) - U_t^0(0) \gtreqless 0 \Leftrightarrow K^0(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta}, \tag{A.16}$$

$$U_t^0(0) - U_t^0(1) \gtreqless 0 \Leftrightarrow G^0(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta} \tag{A.17}$$

where

$$K^0(\mu_t) = \frac{(2-\sqrt{2})\mu_t^0\sqrt{1-\mu_t} - \sqrt{2}(1-\mu_t^0)\sqrt{\mu_t}}{\mu_t - \mu_t^0} = \frac{(2-\sqrt{2})\sqrt{1+\lambda_t} - \sqrt{2}\lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}, \tag{A.18}$$

$$G^0(\mu_t) = \frac{\sqrt{2}\mu_t^0\sqrt{1-\mu_t} - (2-\sqrt{2})(1-\mu_t^0)\sqrt{\mu_t}}{\mu_t - \mu_t^0} = \frac{\sqrt{2}\sqrt{1+\lambda_t} - (2-\sqrt{2})\lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}. \tag{A.19}$$

Both $K^0(\mu_t)$ and $G^0(\mu_t)$ are strictly increasing in μ_t. As $\mu_t \to 1$, $K^0(\mu_t) \to +\infty$ and

$G^0(\mu_t) \to +\infty$; as $\mu_t \to 0$, $K^0(\mu_t) \to -\infty$ and $G^0(\mu_t) \to -\infty$. Hence, $K^0(\mu_t) = \frac{\sqrt{2}}{\eta}$ has unique solution $\overline{\kappa}$ and $G^0(\mu_t) = \frac{\sqrt{2}}{\eta}$ has unique solution $\overline{\nu}$. Because $G^0(\mu_t) > M^0(\mu_t) > K^0(\mu_t)$ holds for $\mu_t \in (0,1)$, we have $0 < \overline{\nu} < \overline{\mu} < \overline{\kappa} < 1$. As shown in Fiugre 1, this result ensures the holding of (21). Q.E.D.

Proof of Corollary 1.

First, consider the effect of η. As shown in the proof of Lemma 1, $\underline{\mu}$ is the solution to $M^1(\mu_t) = \frac{\sqrt{2}}{\eta}$, where $M^1(\mu_t) = \frac{\sqrt{1+\frac{1}{\lambda_t}} - \lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1}$ and $\lambda_t = \frac{\mu_t}{1-\mu_t}$. It is easy to see that $\frac{\partial(M^1(\underline{\mu}) - \frac{\sqrt{2}}{\eta})}{\partial\eta} = \frac{\partial M^1}{\partial\mu}\frac{\partial\underline{\mu}}{\partial\eta} + \frac{\sqrt{2}}{\eta^2} = 0$ and $\frac{\partial M^1}{\partial\mu} < 0$; thus, $\frac{\partial\underline{\mu}}{\partial\eta} > 0$. Similarly, $\overline{\mu}$ is the solution to $M^0(\mu_t) = \frac{\sqrt{2}}{\eta}$, where $M^0(\mu_t) = \frac{\sqrt{1+\lambda_t} - \lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}$. Because $\frac{\partial(M^0(\underline{\mu}) - \frac{\sqrt{2}}{\eta})}{\partial\eta} = \frac{\partial M^0}{\partial\mu}\frac{\partial\overline{\mu}}{\partial\eta} + \frac{\sqrt{2}}{\eta^2} = 0$ and $\frac{\partial M^0}{\partial\mu} > 0$, we have $\frac{\partial\overline{\mu}}{\partial\eta} < 0$.

Next, consider the effect of q. From $\frac{\partial(M^1(\underline{\mu}) - \frac{\sqrt{2}}{\eta})}{\partial q} = \frac{\partial M^1}{\partial\mu}\frac{\partial\underline{\mu}}{\partial q} + \frac{\partial M^1}{\partial\lambda_q}\frac{\partial\lambda_q}{\partial q} = 0$, $\frac{\partial M^1}{\partial\mu} < 0$, $\frac{\partial M^1}{\partial\lambda_q} < 0$, and $\frac{\partial\lambda_q}{\partial q} > 0$, we have $\frac{\partial\underline{\mu}}{\partial q} < 0$. Similarly, from $\frac{\partial(M^0(\underline{\mu}) - \frac{\sqrt{2}}{\eta})}{\partial q} = \frac{\partial M^0}{\partial\mu}\frac{\partial\overline{\mu}}{\partial q} + \frac{\partial M^0}{\partial\lambda_q}\frac{\partial\lambda_q}{\partial q} = 0$, $\frac{\partial M^0}{\partial\mu} > 0$, $\frac{\partial M^0}{\partial\lambda_q} < 0$, and $\frac{\partial\lambda_q}{\partial q} > 0$, we have $\frac{\partial\overline{\mu}}{\partial q} > 0$. Q.E.D.

Proof for Corollary 2.

Faced with $p_t^a = \mu_t + \frac{e_t}{2}$ and $p_t^b = \mu_t - \frac{e_t}{2}$, an informed trader's expected utility on position x, denoted by $\breve{U}_t^s(x)$, is as follows.

$$\breve{U}_t^s(1) = \mu_t^s\left(1 - \mu_t - \frac{e_t}{2}\right) + (1 - \mu_t^s)\left(-\mu_t - \frac{e_t}{2} - \eta\sqrt{2\mu_t}\right) = U_t^s(1) - \frac{e_t}{2}, \tag{A.20}$$

$$\breve{U}_t^s(0) = -\eta\mu_t^s\sqrt{1 - \mu_t - \frac{e_t}{2}} - \eta(1 - \mu_t^s)\sqrt{\mu_t - \frac{e_t}{2}}$$

$$= U_t^s(0) + \frac{\eta\mu_t^s e_t}{2\left(\sqrt{1-\mu_t}+\sqrt{1-\mu_t-\frac{e_t}{2}}\right)} + \frac{\eta(1-\mu_t^s)e_t}{2\left(\sqrt{\mu_t}+\sqrt{\mu_t-\frac{e_t}{2}}\right)}, \tag{A.21}$$

$$\breve{U}_t^s(-1) = \mu_t^s\left(\mu_t - \frac{e_t}{2} - 1 - \eta\sqrt{2(1 - \mu_t)}\right) + (1 - \mu_t^s)\left(\mu_t - \frac{e_t}{2}\right) = U_t^s(-1) - \frac{e_t}{2} \tag{A.22}$$

where $U_t^s(x)$ is given by equations (13)-(15) with $p_t = \mu_t$.

It is easy to see that $\breve{U}_t^1(1) - \breve{U}_t^1(-1) = U_t^1(1) - U_t^1(-1)$. Recall that $U_t^1(1) - U_t^1(-1) = 0$ when $\mu_t = \underline{\mu}$, and $U_t^s(1) - U_t^s(-1) > 0$ when $\mu_t > \underline{\mu}$. Thus, $\breve{U}_t^1(1) - \breve{U}_t^1(-1) \geq 0$ holds for $\mu_t \in [\underline{\mu}, \overline{\mu}]$. Furthermore, $\breve{U}_t^1(1) - \breve{U}_t^1(0) = U_t^1(1) - U_t^1(0) - \frac{e_t}{2} - \frac{\eta\mu_t^1 e_t}{2\left(\sqrt{1-\mu_t}+\sqrt{1-\mu_t-\frac{e_t}{2}}\right)} - \frac{\eta(1-\mu_t^1)e_t}{2\left(\sqrt{\mu_t}+\sqrt{\mu_t-\frac{e_t}{2}}\right)}$. Obviously, $\breve{U}_t^1(1) - \breve{U}_t^1(0) > U_t^1(1) - U_t^1(0) - \frac{e_t}{2} - \frac{\eta\mu_t^1 e_t}{2\sqrt{1-\mu_t}} - \frac{\eta(1-\mu_t^1)e_t}{2\sqrt{\mu_t}}$. As shown in the proof of Lemma 2, $U_t^1(1) - U_t^1(0)$ is a continuous function of μ_t with $U_t^1(1) - U_t^1(0) > 0$ for $\mu_t \in [\underline{\mu}, \overline{\mu}]$. Obviously, $\exists \check{e} > 0$, such that $\breve{U}_t^1(1) - \breve{U}_t^1(0) \geq 0$ holds for $(\mu_t, e_t) \in [\underline{\mu}, \overline{\mu}] \times [0, \check{e})$.

Symmetrically, for a bearish trader, $\breve{U}_t^0(-1) - \breve{U}_t^0(1) \geq 0$ holds for $\mu_t \in [\underline{\mu}, \overline{\mu}]$; furthermore, $\exists \hat{e} > 0$, such that $\breve{U}_t^0(-1) - \breve{U}_t^0(0) \geq 0$ holds for $(\mu_t, e_t) \in [\underline{\mu}, \overline{\mu}] \times [0, \hat{e})$.

Let $\bar{e} \equiv min\{\check{e}, \hat{e}, 2\underline{\mu}, 2(1 - \overline{\mu})\}$ and consider the following pricing rule.

$$p_t^a = \begin{cases} \mu_t + \frac{e_t}{2} & if \ \ \mu_t \in [\underline{\mu}, \overline{\mu}] \\ \mu_t & otherwise; \end{cases} \tag{A.23}$$

$$p_t^b = \begin{cases} \mu_t - \frac{e_t}{2} & if \ \ \mu_t \in [\underline{\mu}, \overline{\mu}] \\ \mu_t & otherwise \end{cases} \tag{A.24}$$

where $0 \leq e_t < \bar{e}$.

By the analysis above, $\breve{U}_t^1(1) \geq \max\{\breve{U}_t^1(-1), \breve{U}_t^1(0)\}$ and $\breve{U}_t^0(-1) \geq \max\{\breve{U}_t^0(1), \breve{U}_t^0(0)\}$ hold for $(\mu_t, e_t) \in [\underline{\mu}, \overline{\mu}] \times [0, \bar{e})$. Therefore, if the market maker applies pricing rule (A.23)-(A.24), a bullish trader will choose buy whereas a bearish trader will choose sell when $\mu_t \in [\underline{\mu}, \overline{\mu}]$. When $\mu_t \in (0, \underline{\mu})$, because $p_t^a = p_t^b = \mu_t$ by (A.23)-(A.24), all informed traders choose sell as in the case of Proposition 1. Symmetrically, when $\mu_t \in (\overline{\mu}, 1)$, because the price is set at $p_t^a = p_t^b = \mu_t$, all informed traders choose buy. Therefore, if the market maker adopts pricing rule (A.23)-(A.24) with $0 \leq e_t < \bar{e}$, it is optimal for informed traders to act according to strategy (22)-(23).

Conversely, given that informed traders act according to (22)-(23), the pricing rule given by (A.23)-(A.24) satisfies equations (24)-(25) and the requirement that $0 < p_t^b \leq p_t^a < 1$. Thus, the pricing rule given by (A.23)-(A.24) and the trading strategy given by (22)-(23) constitute an equilibrium. This completes the proof. Q.E.D.

Appendix B. Proofs for the model in Section 4

Proof of Proposition 2.

Under Assumption 2, an informed trader's expected utility is as follows.

$$U_t^s(1) = \mu_t^s(1 - p_t) - (1 - \mu_t^s)\left(p_t + \eta\sqrt{2p_t}\right), \tag{B.1}$$

$$U_t^s(0) = -z\eta\mu_t^s\sqrt{1 - p_t} - z\eta(1 - \mu_t^s)\sqrt{p_t}, \tag{B.2}$$

$$U_t^s(-1) = \mu_t^s\left(p_t - 1 - \eta\sqrt{2(1 - p_t)}\right) + (1 - \mu_t^s)p_t\ . \tag{B.3}$$

For a bullish trader with signal $s = 1$, $U_t^1(1) - U_t^1(-1) \gtreqless 0 \Leftrightarrow M_t^1 \lesseqgtr \frac{\sqrt{2}}{\eta}$, $U_t^1(1) -$

$U_t^1(0) \gtreqless 0 \Leftrightarrow K_t^1 \lesseqgtr \frac{\sqrt{2}}{\eta}$, and $U_t^1(0) - U_t^1(-1) \gtreqless 0 \Leftrightarrow G_t^1 \lesseqgtr \frac{\sqrt{2}}{\eta}$, where

$$M^1(\mu_t) = \frac{(1-\mu_t^1)\sqrt{\mu_t} - \mu_t^1\sqrt{1-\mu_t}}{\mu_t^1 - \mu_t} = \frac{\sqrt{1+\frac{1}{\lambda_t}} - \lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1}, \tag{B.4}$$

$$K^1(\mu_t) = \frac{(2-\sqrt{2}z)(1-\mu_t^1)\sqrt{\mu_t} - \sqrt{2}z\mu_t^1\sqrt{1-\mu_t}}{\mu_t^1 - \mu_t} = \frac{(2-\sqrt{2}z)\sqrt{1+\frac{1}{\lambda_t}} - \sqrt{2}z\lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1}, \tag{B.5}$$

$$G^1(\mu_t) = \frac{\sqrt{2}z(1-\mu_t^1)\sqrt{\mu_t} - (2-\sqrt{2}z)\mu_t^1\sqrt{1-\mu_t}}{\mu_t^1 - \mu_t} = \frac{\sqrt{2}z\sqrt{1+\frac{1}{\lambda_t}} - (2-\sqrt{2}z)\lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1}. \tag{B.6}$$

For a bearish trader with signal $s = 0$, $U_t^0(-1) - U_t^0(1) \gtreqless 0 \Leftrightarrow M^0(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta}$, $U_t^0(-1) - U_t^0(0) \gtreqless 0 \Leftrightarrow K^0(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta}$, and $U_t^0(0) - U_t^0(1) \gtreqless 0 \Leftrightarrow G^0(\mu_t) \lesseqgtr \frac{\sqrt{2}}{\eta}$, where

$$M^0(\mu_t) = \frac{\mu_t^0\sqrt{1-\mu_t} - (1-\mu_t^0)\sqrt{\mu_t}}{\mu_t - \mu_t^0} = \frac{\sqrt{1+\lambda_t} - \lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}, \tag{B.7}$$

$$K^0(\mu_t) = \frac{(2-\sqrt{2}z)\mu_t^0\sqrt{1-\mu_t} - \sqrt{2}z(1-\mu_t^0)\sqrt{\mu_t}}{\mu_t - \mu_t^0} = \frac{(2-\sqrt{2}z)\sqrt{1+\lambda_t} - \sqrt{2}z\lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}, \tag{B.8}$$

$$G^0(\mu_t) = \frac{\sqrt{2}z\mu_t^0\sqrt{1-\mu_t} - (2-\sqrt{2}z)(1-\mu_t^0)\sqrt{\mu_t}}{\mu_t - \mu_t^0} = \frac{\sqrt{2}z\sqrt{1+\lambda_t} - (2-\sqrt{2}z)\lambda_q\sqrt{1+\frac{1}{\lambda_t}}}{\lambda_q - 1}. \tag{B.9}$$

Obviously, equation (B.4) is the same as (A.4), and (B.7) is the same as (A.7). Thus, as in the case of Lemma 1, M^1 is a monotonically decreasing function while M^0 is a monotonically increasing function; furthermore, $M^1(\underline{\mu}) = \frac{\sqrt{2}}{\eta}$, $M^0(\overline{\mu}) = \frac{\sqrt{2}}{\eta}$, and $\underline{\mu} < \frac{1}{2} < \overline{\mu}$.

When $z \geq \sqrt{2}$, it is easy to see that $K^1(\mu_t) < M^1(\mu_t) < G^1(\mu_t)$ and $K^0(\mu_t) < M^0(\mu_t) < G^0(\mu_t)$. Thus, for $\mu_t \in (0, \underline{\mu})$, we have $G^1(\mu_t) > M^1(\mu_t) > \frac{\sqrt{2}}{\eta}$, which implies $U_t^1(-1) > U_t^1(0)$ and $U_t^1(-1) > U_t^1(1)$; for $\mu_t \in [\underline{\mu}, 1)$, we have $K^1(\mu_t) < M^1(\mu_t) \leq \frac{\sqrt{2}}{\eta}$, which implies $U_t^1(1) > U_t^1(0)$ and $U_t^1(1) \geq U_t^1(-1)$. Therefore, equation (27) holds. Equation (28) can be obtained by an analogous argument.

When $\frac{\sqrt{2}}{2} < z < \sqrt{2}$, $K^1(\mu_t)$ and $G^1(\mu_t)$ are strictly decreasing functions of μ_t. Analogous to the proof of Lemma 2, it is easy to see that $\underline{\kappa}$ and $\underline{\nu}$ exist such that $K^1(\underline{\kappa}) = \frac{\sqrt{2}}{\eta}$ and $G^1(\underline{\nu}) = \frac{\sqrt{2}}{\eta}$. Symmetrically, $K^0(\mu_t)$ and $G^0(\mu_t)$ are strictly increasing in μ_t with $\overline{\kappa}$ and $\overline{\nu}$ existing such that $K^0(\overline{\kappa}) = \frac{\sqrt{2}}{\eta}$ and $G^0(\overline{\nu}) = \frac{\sqrt{2}}{\eta}$. Because $K^1(\mu_t) < M^1(\mu_t) < G^1(\mu_t)$ and $K^0(\mu_t) < M^0(\mu_t) < G^0(\mu_t)$ hold when $z > \frac{\sqrt{2}}{2}$, we have $0 < \underline{\kappa} < \underline{\mu} < \underline{\nu} < 1$ and $0 < \overline{\nu} < \overline{\mu} < \overline{\kappa} < 1$. As shown in Figure 1, we can compare $U_t^s(x)$ resulting from different positions in each interval of μ_t. For a bullish trader, it is easy to see that $U_t^1(1) \geq U_t^1(-1)$ and $U_t^1(1) > U_t^1(0)$ hold for $\mu \in [\underline{\mu}, 1)$, while $U_t^1(-1) > U_t^1(1)$ and $U_t^1(-1) > U_t^1(0)$ hold for $\mu \in (0, \underline{\mu})$; thus, equation (27) holds. For a bearish trader, equation (28) can be obtained similarly.

When $z = \frac{\sqrt{2}}{2}$, we have $0 < \underline{\kappa} = \underline{\mu} = \underline{\nu} < 0.5 < \overline{\nu} = \overline{\mu} = \overline{\kappa} < 1$. It is easy to see that (27) and (28) also hold in this case. Q.E.D.

Proof of Proposition 3.

First, note that $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} < \frac{\sqrt{2}}{2}$. Hence, the condition $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \leq z < \frac{\sqrt{2}}{2}$ is well defined, where $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \gtreqless 0 \Leftrightarrow \eta \gtreqless \frac{\lambda_q - 1}{2}$. Because $z > 0$ by assumption, $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \leq z < \frac{\sqrt{2}}{2}$ holds in two subcases: $\eta \leq \frac{\lambda_q - 1}{2}$ and $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \leq 0 < z < \frac{\sqrt{2}}{2}$; $\eta > \frac{\lambda_q - 1}{2}$ and $0 < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} \leq z < \frac{\sqrt{2}}{2}$. In either subcase, $K^1(\mu_t)$, $M^1(\mu_t)$, and $G^1(\mu_t)$ in equations (B.4)-(B.6) are decreasing functions of μ_t, with $K^1(\mu_t) > M^1(\mu_t) > G^1(\mu_t)$. It is easy to see that there are $\underline{\nu} < \underline{\mu} < \underline{\kappa}$ such that $K^1(\underline{\kappa}) = \frac{\sqrt{2}}{\eta}$, $M^1(\underline{\mu}) = \frac{\sqrt{2}}{\eta}$, and $G^1(\underline{\nu}) = \frac{\sqrt{2}}{\eta}$. Symmetrically, $K^0(\mu_t)$, $M^0(\mu_t)$, and $G^0(\mu_t)$ are strictly increasing functions of μ_t with $K^0(\mu_t) > M^0(\mu_t) > G^0(\mu_t)$; moreover, it is easy to see that there are $\bar{\kappa} < \bar{\mu} < \bar{\nu}$ such that $K^0(\bar{\kappa}) = \frac{\sqrt{2}}{\eta}$, $M^0(\bar{\mu}) = \frac{\sqrt{2}}{\eta}$, and $G^0(\bar{\mu}) = \frac{\sqrt{2}}{\eta}$. Furthermore, it follows from (B.5) and (B.8) that $K^1(0.5) = K^0(0.5) \le \frac{\sqrt{2}}{\eta}$ holds when $z \ge \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$, which implies the holding of $\underline{\kappa} \le 0.5 \le \bar{\kappa}$. Thus, as shown in Figure 2, we have $0 < \underline{\nu} < \underline{\mu} < \underline{\kappa} \le 0.5 \le \bar{\kappa} < \bar{\mu} < \bar{\nu} < 1$, which ensures the holding of (29) and (30). Q.E.D.

Proof of Proposition 4.

It is easy to see that $\frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} > 0$ when $\eta > \frac{\lambda_q - 1}{2}$; thus, the condition of Proposition 4 is well defined. Analogous to the proof of Proposition 3, it is easy to show that $\underline{\nu} < \underline{\mu} < \underline{\kappa}$ and $\bar{\kappa} < \bar{\mu} < \bar{\nu}$ hold when $z < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)} < \frac{\sqrt{2}}{2}$. Thus, to prove Proposition 4, we only need to prove $\underline{\nu} < \bar{\kappa} \le 0.5 \le \underline{\kappa} < \bar{\nu}$.

By (B.5) and (B.8), $K^1(0.5) = K^0(0.5) > \frac{\sqrt{2}}{\eta}$ if $z < \frac{\sqrt{2}}{1+\lambda_q} - \frac{\lambda_q - 1}{\sqrt{2}\eta(1+\lambda_q)}$; thus, $\bar{\kappa} < 0.5 < \underline{\kappa}$.

By (B.6) and (B.8), $G^1(\mu_t) - K^0(\mu_t) = \frac{(1+\lambda_q)\left[\sqrt{2}z\sqrt{1+\frac{1}{\lambda_t}} - (2-\sqrt{2}z)\sqrt{1+\lambda_t}\right]}{\lambda_q - 1}$. Note that $G^1(\underline{\nu}) = \frac{\sqrt{2}}{\eta}$ and $G^1(\mu_t) = \frac{\sqrt{2}}{\eta} \Leftrightarrow \frac{\sqrt{2}z\sqrt{1+\frac{1}{\lambda_t}} - (2-\sqrt{2}z)\lambda_q\sqrt{1+\lambda_t}}{\lambda_q - 1} = \frac{\sqrt{2}}{\eta} \Rightarrow \sqrt{2}z\sqrt{1+\frac{1}{\lambda_t}} - \left(2-\sqrt{2}z\right)\sqrt{1+\lambda_t} > 0$. Therefore, $G^1(\underline{\nu}) > K^0(\underline{\nu})$ holds, which implies $\underline{\nu} < \bar{\kappa}$. By an analogous argument, $\underline{\kappa} < \bar{\nu}$ also holds. Thus, $\underline{\nu} < \bar{\kappa} < 0.5 < \underline{\kappa} < \bar{\nu}$. As shown in Figure 3, by comparing the utilities of buy, sell, and inaction in each interval of μ_t, equations (31) and (32) are obtained. Q.E.D.

Proof of Corollary 6.

We first prove statement (i). It follows from $M^0(\overline{\mu}) = \frac{\sqrt{2}}{\eta}$ that $\frac{\partial M^0}{\partial \mu}\frac{\partial \overline{\mu}}{\partial \eta} < 0$. Because $\frac{\partial M^0}{\partial \mu} > 0$ by equation (B.7), we have $\frac{\partial \overline{\mu}}{\partial \eta} < 0$. Furthermore, it follows from $G^0(\overline{v}) = \frac{\sqrt{2}}{\eta}$ that $\frac{\partial G^0}{\partial \mu}\frac{\partial \overline{v}}{\partial \eta} < 0$, and it follows from equation (B.9) that $\frac{\partial G^0}{\partial \mu} > 0$; thus, we have $\frac{\partial \overline{v}}{\partial \eta} < 0$. By equation (46), $\overline{p} = \overline{\mu}$ for $z > \frac{\sqrt{2}}{2}$ and $\overline{p} = \overline{v}$ for $z < \frac{\sqrt{2}}{2}$. Obviously, given any $z > 0$, $\overline{p}$ is strictly decreasing with respect to η. By analogous argument, we can obtain $\frac{\partial \underline{\mu}}{\partial \eta} > 0$ and $\frac{\partial \underline{v}}{\partial \eta} > 0$; therefore, given any $z > 0$, $\underline{p}$ is strictly increasing with respect to η.

Next, we prove statement (ii). For $z < \frac{\sqrt{2}}{2}$, it follows from $G^0(\overline{v}) = \frac{\sqrt{2}}{\eta}$ that $\frac{\partial G^0}{\partial \mu}\frac{\partial \overline{v}}{\partial z} + \frac{\partial G^0}{\partial z} = 0$, and it follows from (B.9) that $\frac{\partial G^0}{\partial \mu} > 0$ and $\frac{\partial G^0}{\partial z} > 0$; therefore, $\frac{\partial \overline{v}}{\partial z} < 0$ holds. Because $\overline{p} = \overline{v}$ for $z < \frac{\sqrt{2}}{2}$, it is clear that $\overline{p}$ is strictly decreasing in z on $(0, \frac{\sqrt{2}}{2})$. For $z \geq \frac{\sqrt{2}}{2}$, $\overline{p} = \overline{\mu}$ by definition; furthermore, from (B.7) and $M^0(\overline{\mu}) = \frac{\sqrt{2}}{\eta}$, it is clear that $\overline{\mu}$ does not depends on z. Recall that it follows from Propositions 4 and 5 that $\overline{v} \leq \overline{\mu}$ holds for $z < \frac{\sqrt{2}}{2}$. Therefore, $\overline{p}$ is a decreasing function of z that is strictly decreasing on $(0, \frac{\sqrt{2}}{2})$. By an analogous argument, we can prove $\underline{p}$ is an increasing function of z that is strictly increasing on $(0, \frac{\sqrt{2}}{2})$. Q.E.D.

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