Advertisement Competition in a Differentiated Mixed Duopoly: Bertrand vs. Cournot

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Abstract

We examine the Bertrand-Cournot comparisons with advertising competition in differentiated mixed duopoly markets. Comparing with the social optimum, we provide some interesting findings. First, both firms' quantities are higher (lower) in Cournot (Bertrand) than the social optimum, but Bertrand equilibrium is closer to the social optimum. Second, both firms' advertisements are higher (lower) in Cournot (Bertrand) than the social optimum and thus, both firms engage in excessive (insufficient) advertisement in Cournot (Bertrand). Finally, despite lower both firms' prices in Cournot, both firms' profits and social welfare are strictly higher in Bertrand and thus, both firms prefer Bertrand to Counrnot.

Keywords: advertisement; differentiated mixed duopoly; Bertrand-Cournot comparisons
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1. Introduction

Oligopoly models, with Bertrand and Cournot models as leading modes of imperfect competition, are widely used in theoretical economics. Since Singh and Vives(1984) and Vives(1985), it is well-known that Bertrand competition where firms compete in prices yields lower prices and profits and higher quantities and social welfare than Cournot competition where the firms compete in quantities. To date, the literature comparing Bertrand and Cournot outcomes has focused on the various market competitions between profit-maximizing private firms.[2]

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[2] For example, López and Naylor(2004) showed that whether equilibrium profits are higher under Cournot or Bertrand competition would depend upon the upstream agents' bargaining power over the input price. Alipranti *et al.*(2014) demonstrated that the standard results on price and quantity competition can be altered in the context of a vertically related market.

Recently, there are some studies on the Bertrand-Cournot comparisons in mixed market where profit-maximizing private firms coexist with welfare-maximizing public firm.[3] See, for example, Ghosh and Mitra(2010) and Matsumura and Ogawa(2012) found that the public firm's price is lower in Cournot than in Bertrand competition while the private firm's price can be higher or lower in Cournot.[4] Despite the ambiguity in price ordering between Bertrand and Cournot for the private firm's price, comparison of quantities and profits gives unambiguous results. The public firm's quantity is higher in Cournot whereas the private firm's quantity is lower. Thus, contrary to standard findings, both firm's profits and social welfare are lower under Cournot, but consumer surplus is higher under Cournot.

We revisit the classic Bertrand-Cournot comparisons with advertising competition in differentiated mixed duopoly markets, where a welfare-maximizing public firm competes with profit maximizing private firm. Comparing the results between Bertrand and Cournot equilibria, we show that most results in the previous literature on the mixed markets still hold even though we take advertisement into account, e.g., Ghosh and Mitra(2010), Matsumura and Ogawa(2012), Matsumura and Sunada(2013). However, there are some more interesting findings when we compare the results with the social optimum. First, the public firm's quantity is strictly higher in Cournot while private firm's output is strictly lower in Bertrand. Also, both firms' quantities are higher (lower) in Cournot (Bertrand) than the social optimum, but Bertrand equilibrium is closer to the social optimum. Second, the public firm's advertisement is strictly higher in Cournot while private firm's advertisement is strictly lower in Bertrand. However, both firms' advertisements are higher (lower) in Cournot (Bertrand) than the social optimum and thus, both firms engage in excessive (insufficient) advertisement in Cournot (Bertrand). Third, both firm's prices are strictly higher in Bertrand while public firm's price in Cournot is equal to the social optimum. Finally, despite lower both firms' prices in Cournot, both firms' profits and social welfare are strictly higher in Bertrand and thus, both firms prefer Bertrand to Counrnot. This implies that irrespective of the degree of substitutability, Bertrand competition occurs in equilibrium and thus, Bertrand model should be used more in mixed oligopolies.

2. The Basic Model

Using a duopolistic competition model, each firm producing a differentiated good, we consider the case that the representative consumer has the following quadratic and strictly concave utility function:

[3] In reality, public firms in mixed markets are strongly concentrated in a few strategic sectors such as transportation, telecommunications, power generation, electricity, finance, manufacturing, and other energy industries.

[4] Nakamura(2013) examined the comparisons between price and quantity competitions with network effect while Ohori(2014) investigated them with emission tax, and concluded different results.

$$U(q_1, q_2) = aq_1 + aq_2 + z_1q_1 + z_2q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2)$$

where q_i is the amount of consumption of good i, p_i its price, z_i is advertisement level of firm i, and $b \in (0, 1)$ is the degree of substitutability between two products, $i = 1, 2$.

Then, utility maximization yields the linear inverse demand functions:

$p_1 = a + z_1 - q_1 - bq_2$
$p_2 = a + z_2 - bq_1 - q_2$

Also, the direct demand functions are given by:

$q_1 = \frac{a-ab+z_1-bz_2-p_1+bp_2}{1-b^2}$
$q_2 = \frac{a-ab+z_2-bz_1-p_2+bp_1}{1-b^2}$

We assume that firm 1, public firm, maximizes welfare, which is defined as the sum of consumer surplus and firms' profits:

$$\underset{q_1,\ z_1}{\text{Max}}\ W = a(q_1 + q_2) + z_1q_1 + z_2q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - c(q_1 + q_2) - \frac{k}{2}(z_1^2 + z_2^2)$$

while firm 2, private firm, maximizes its own profit:

$$\underset{q_2,\ z_2}{\text{Max}}\ \pi_2 = (p_2 - c)q_2 - kz_2^2/2.$$

We assume that both firms have the same constant marginal production cost, $c > 0$, and quadratic advertisement cost, $\frac{kz_i^2}{2}$, where $k > \frac{1}{1-b^2}$ to ensure the second-order conditions.

The timing of the game is as follows: In the first stage, both firm choose either the price or the quantity contract cooperatively. In the second stage, according to the contract chosen in the first stage, both firms compete with Bertrand fashion where they choose price and advertising simultaneously, or compete with Cournot fashion where they choose quantity and advertising simultaneously.

3. The Analysis

3.1 Social Optimum

As a benchmark, we analyze the social optimum, which maximizes social welfare:

$$\underset{q_1,z_1,q_2,z_2}{\text{Max}}\ W = a(q_1 + q_2) + z_1q_1 + z_2q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - c(q_1 + q_2) - \frac{k}{2}(z_1^2 + z_2^2)$$

From the first-order conditions, the optimal levels of quantity, price and advertisement of each firm are as follows:

$$q_i^{SO} = \frac{k(a-c)}{k+bk-1}\ ,\quad z_i^{SO} = \frac{(a-c)}{k+bk-1}\ ,\quad p_i^{SO} = c$$

Note that marginal cost pricing is obtained at the social optimum. Then, the resulting profits of the firms and social welfare are as follows:

$\pi_i^{SO} = -\frac{\beta^2 k(a-c)^2}{2(k^2-2k-b^2k^2+1)^2} < 0$ and $W^{SO} = \frac{k(a-c)^2}{k+bk-1} > 0$,

where $\alpha = 2k - 1 - bk,\ \beta = k - 1 - bk,\ \gamma = 1 - b^2$. Note also that both firms earn negative profits with marginal cost pricing at the social optimum. However, the social welfare will be maximized at the social optimum.

3.2 Bertrand Competition

Consider the Bertrand competition case where both firm choose price and advertisement simultaneously. The first-order conditions for firms yields the following equilibrium quantity, price and advertisement level of each firm under Bertrand competition:

$p_1^B = \frac{\beta\gamma abk + c(\gamma k(\alpha+b)+1-2k)}{\gamma(2k^2-b^2k^2-k)+1-2k}$, $p_2^B = \frac{ak\beta\gamma + c\left(k^2\gamma\left(1+b-b^2\right)+1-2k\right)}{\gamma(2k^2-b^2k^2-k)+1-2k}$ and

$z_1^B = \frac{(a-c)\left(\alpha-bk\left(1+b-b^2\right)\right)}{\gamma(2k^2-b^2k^2-k)+1-2k}$, $z_2^B = \frac{\beta(a-c)}{\gamma(2k^2-b^2k^2-k)+1-2k}$.

Note that both firms will set higher prices than marginal cost at the Bertrand equilibrium. Then, we have the following equilibrium outputs:

$q_1^B = \frac{k(a-c)\left(\alpha-bk\left(1+b-b^2\right)\right)}{\gamma(2k^2-b^2k^2-k)+1-2k}$ and $q_2^B = \frac{\beta k(a-c)}{\gamma(2k^2-b^2k^2-k)+1-2k}$

The resulting profits of the firms and social welfare are as follows:

$$\pi_1^B = \frac{(a-c)^2 k\,(k(1-b)(2-b^2)-1)\left((1+b)(b^2k+2bk^2(1-b)^2)+1-2k\right)}{2\,(\gamma(2k^2-b^2k^2-k)+1-2k)^2}$$

$$\pi_2^B = \frac{\beta^2 k(a-c)^2\,(2\gamma k-1)}{2\,(\gamma(2k^2-b^2k^2-k)+1-2k)^2}$$

$$W^B = \frac{(a-c)^2 k\left[\alpha - 2\beta k - b^2k + \gamma k\left(\frac{\gamma^2 k(\alpha+k-bk)}{2} + \gamma k(\beta-1) + \beta d - \alpha + bk\right)\right]}{(\gamma(2k^2-b^2k^2-k)+1-2k)^2}$$

Note also that both firms earn positive profit at the Bertrand equilibrium. And the social welfare will be lower than the social optimum.

3.3 Cournot Competition

Consider the Cournot competition case where both firm choose quantity and advertisement simultaneously. The first-order conditions yield the following equilibrium quantity, price and advertisement level of each firm under Cournot competition:

$q_1^C = \frac{\alpha k(a-c)}{2k^2-3k-b^2k^2+1}$, $q_2^C = \frac{\beta k(a-c)}{2k^2-3k-b^2k^2+1}$ and

$$z_1^C = \frac{\alpha(a-c)}{2k^2-3k-b^2k^2+1}, \quad z_2^C = \frac{\beta(a-c)}{2k^2-3k-b^2k^2+1}.$$

Then, we have the following equilibrium prices.

$$p_1^C = c \quad \text{and} \quad p_2^C = \frac{\beta ak - c\left(2k-k^2-bk^2+b^2k^2-1\right)}{2k^2-3k-b^2k^2+1},$$

where $\alpha = 2k-1-bk,\ \beta = k-1-bk,\ \gamma = 1-b^2$. Note that public firm will set the marginal cost pricing while private firm will set higher price than marginal cost at the Cournot equilibrium.

The resulting profits of the firms and social welfare are as follows:

$$\pi_1^C = -\frac{\alpha^2 k(a-c)^2}{2(2k^2-3k-b^2k^2+1)^2}, \quad \pi_2^C = \frac{\beta^2 k(2k-1)(a-c)^2}{2(2k^2-3k-b^2k^2+1)^2}$$

$$W^C = \frac{k(a-c)^2\left(\alpha(2+k^2-6k)+2\beta\gamma k^2+k^2(3k-3bk+2b)\right)}{2(2k^2-3k-b^2k^2+1)^2}$$

Note also that public firm earns negative profits while private firm earns positive profit at the Cournot equilibrium. And the social welfare will be lower than the social optimum.

4. Comparisons

We now compare the results under Bertrand and Cournot competitions with the social optimum. We will show that most results in the previous literature on the mixed markets still hold even though we take advertisement into account, e.g., Ghosh and Mitra(2010), Matsumura and Ogawa(2012), Matsumura and Sunada(2013). However, there are some more interesting findings when we compare the results with the social optimum.

Proposition 1. *Public firm's output is strictly higher in Cournot whereas private firm's output is strictly lower in Bertrand. However, both firms' outputs are higher (lower) under Cournot (Bertrand) competition than the social optimum. Thus, the outputs of both firms are closer to the social optimum in Bertrand.*

Proof. (i) $q_1^B - q_1^C = \frac{bk^2(a-c)\left(k^2(1-b)\left(b^2-2\right)+k\left(3-b-2b^2+b^3\right)-1+b^2\right)}{(2k^2-b^2k^2-3k+1)(1-2k+\gamma(2k^2-b^2k^2-k))} < 0$ *and*

$$q_1^B - q^{SO} = \frac{\gamma bk^2(a-c)}{(k+bk-1)\left(1-2d+\gamma(2k^2-b^2k^2-k)\right)} > 0 \ \Rightarrow\ q_1^C > q_1^B > q^{SO}.$$

(ii) $q_2^B - q_2^C = \frac{\beta b^2k^2(a-c)\left(2k-b^2k-1\right)}{(2k^2-b^2k^2-3k+1)\left(1-2k+\gamma(2k^2-b^2k^2-k)\right)} > 0$ *and*

$$q_2^B - q^{SO} = \frac{\gamma(a-c)k^2\left(1-k+b^2k\right)}{(k+bk-1)\left(1-2k+\gamma(2k^2-b^2k^2-k)\right)} < 0 \ \Rightarrow\ q^{SO} > q_2^B > q_2^C.$$

Proposition 2. *Public firm's advertisement is strictly higher in Cournot whereas private firm's advertisement is strictly lower in Bertrand. But, both firms' advertisements are higher (lower) under Cournot (Bertrand) than the social optimum. Thus, both firms engage in excessive advertising in Cournot whereas insufficient advertising in Bertrand.*

Proof. (i) $z_1^B - z_1^C = \frac{bk(a-c)\,(k^2(1-b)(b^2-2)+k(3-b-2b^2+b^3)-1+b^2)}{(2k^2-b^2k^2-3k+1)(1-2k+\gamma(2k^2-b^2k^2-k))} < 0$ *and*

$$z_1^B - z^{SO} = \frac{\gamma bk(a-c)}{(k+bk-1)(1-2d+\gamma(2k^2-b^2k^2-k))} > 0 \quad \Rightarrow z_1^C > z_1^B > z^{SO}.$$

(ii) $z_2^B - z_2^C = \frac{\beta b^2k(a-c)(2k-b^2k-1)}{(2k^2-b^2k^2-3k+1)(1-2k+\gamma(2k^2-b^2k^2-k))} > 0$ *and*

$$z_2^B - z^{SO} = \frac{\gamma(a-c)k^2\,(1-k+b^2k)}{(k+bk-1)(1-2k+\gamma(2k^2-b^2k^2-k))} < 0 \quad \Rightarrow z^{SO} > z_2^B > z_2^C.$$

Proposition 3. *Both firm's prices are strictly higher in Bertrand. Under Cournot competition, public firm's price is equal to the social optimum while private firm's price is greater than the social optimum.*

Proof. (i) $p_1^B - p_1^C = \frac{\beta\gamma bk(a-c)}{(1-2k+\gamma(2k^2-b^2k^2-k))} > 0 \quad \Rightarrow \; p_1^B > p_1^C = p^{SO} = c.$

(ii) $p_2^B - p_2^C = \frac{\beta b^2k\,(a-c)\,(2k-1)}{(2k^2-3k-b^2k^2+1)(1-2k+\gamma(2k^2-b^2k^2-k))} > 0$ *and* $p_2^C - p^{SO} = \frac{\beta k(a-c)}{(2k^2-3k-b^2k^2+1)} > 0$

$$\Rightarrow \; p_2^B > p_2^C > p^{SO} = c.$$

Proposition 4. *Both firms' profits are strictly higher in Bertrand. But, the profit of public firm in Cournot is lower than the social optimum whereas the profit of private firm in Cournot is higher than the social optimum.*

Proof. (i) $\pi_1^C - \pi^{SO} = \frac{bk^3(a-c)^2\left(6k-2-k^2\left(4+b-2b^2\right)\right)}{2(k+bk-1)^2(2k^2-b^2k^2-3k+1)^2} < 0, \;\; \pi_1^B > 0, \;\; \pi^{SO} < 0, \;\; \pi_1^C < 0$

$$\Rightarrow \; \pi_1^B > \pi^{SO} > \pi_1^C$$

(ii) $\pi_2^B - \pi_2^C = \frac{k^2(\gamma b^2(k^3(8\gamma+2b^4)-k^2\left(b^2-4\right)^2+k(10-2b^2)-2)-b^6k^2-2b^4k+b^4)}{\left((2k^2-b^2k^2-3k+1)(1-2k+\gamma(2k^2-b^2k^2-k))\right)^2} > 0 \;\Rightarrow\; \pi_2^B > \pi_2^C > \pi^{SO}.$

Proposition 5. *Welfare is strictly higher under Bertrand.*

Proof. $W^B - W^C = \frac{\beta^2b^2k^3(a-c)^2\left(\gamma k^2\left(b^4-6+k\left(4\gamma+b^4\right)\right)-2k^2-1-b^2+b^4+k\left(5+b^2-3b^4\right)\right)}{2\,(1-3k+b^2k+(2-3b^2+b^4)k^2)^2} > 0.$

Note that proposition 4 and 5 yields that both public and private firms prefer Bertrand competition to Cournot competition. Thus, irrespective of the degree of substitutability, Bertrand competition occurs in equilibrium in the first stage when both firms choose price or quantity contract. It implies that Bertrand model should be used more in differentiated mixed duopoly. Furthermore, this result is in stark contrast to the result in a private duopoly case where Cournot competition occurs in equilibrium. As discussed by Singh and Vives(1984) and Matsumura and Ogawa(2012), it indicates that privatization of a public firm changes the

competition structure from Bertrand to Cournot. Therefore, if Cournot competition takes place after privatization, the welfare loss of privatization will be higher.

Remark 1: We can consider the sequential choice game in which advertisement is determined before firms decide quantities or prices. In such a game, the social optimum is the same, but the strategic effect between advertisement and quantity (or prices) can affect the equilibrium. However, we can find that the main results in the simultaneous choice game are robust.

Remark 2: We can consider the pure private market where each firm maximizes its own profit, and compare the results of privatization policy. We can show that as far as the cost efficiency is the same between public firm and private firm, privatization will reduce the social welfare.

5. Conclusion

We have examined the Bertrand-Cournot comparisons with advertising competition in differentiated mixed duopoly markets and compare them with the social optimum. We have provided some interesting findings. First, both firms' quantities are higher (lower) in Cournot (Bertrand) than the social optimum, but Bertrand equilibrium is closer to the social optimum. Second, both firms' advertisements are higher (lower) in Cournot (Bertrand) than the social optimum and thus, both firms engage in excessive (insufficient) advertisement in Cournot (Bertrand). Finally, despite lower both firms' prices in Cournot, both firms' profits and social welfare are strictly higher in Bertrand and thus, both firms prefer Bertrand to Counrnot. Thus, Bertrand competition occurs in equilibrium and thus, Bertrand model should be used more in differentiated mixed duopoly.

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