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The non-linear income-growth relation

A new look at a much analyzed relation

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Abstract:
The paper considers all pairs of initial income and growth in the updated Maddison data from 1950 to 2010. The analysis confirms the well-known fact that the income-growth relation has low explanatory power, but when it is analyzed by kernel regression techniques, a highly significant hump-shaped relation does appear. It gives the long-run dynamics of the incomes of the countries of the world. Further, a moving measure of variation is developed for the growth rate. It is shown that the income-variation relation has a large fall from middle income to high income.

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1. The income-growth relation

Few relations in economics have been researched as much as the income-growth relation. It considers initial income and growth ($y_{i(-)}$, g_i).[3] Income is defined as the logarithm to GDP per capita, in fixed PPP prices; see the appendix. The relation is often illustrated by a diagram with initial income at the horizontal axis and growth at the vertical one. In large data-sets the ($y_{i(-)}$, g_i)-pairs scatter wildly, but the scatter hides an 'underlying' income-growth relation that points to the transition from a traditional to a modern steady state.

The paper studies the relation by kernel regression. This technique has two advantages: (a1) It does not require a model from economic theory. Therefore, it reaches results that all growth theories should be able to replicate. (a2) It works with stacked and sorted data, so that all other country differences than income are randomized.

These advantages come at two disadvantages. (d1) The study merges data across time and countries. This assumes *equivalence*: Long-run and cross-country data tell the same story. (d2) The kernel-technique is a semi-graphical univariate technique. Obviously the world has more dimensions. However, we consider a near-consistent set of N = 8,874 ($y_{i(-)}$, g_i)-pairs of observations. This is enough data so that it could be broken up in many ways, giving some insight in other dimensions.

The kernel has a robust path, which proves highly non-linear. Under the equivalence assumption the path gives the long-run dynamics of the population of countries in the world. Hence, it does answer some much discussed questions, of which we look at two: The first is if a *low level equilibrium trap* exists. This is the case if the relation has a low income interval, where the curve has a negative slope and falls below the y-axis. The second question deals with *convergence*: Do the poor countries catch up with the rich ones? It is the case if the income-growth relation has a negative slope. Convergence may occur throughout or locally at some income interval.

The content of the rest of the paper is: Section 2 looks at the theories. Section 3 describes the data and argues that they are rather representative. Section 4 looks at the pattern in the data. Section 5 considers the development in the variation in the data. Finally, section 6 concludes. This paper is written on the basis of a longer paper, which documents many additional results, see Paldam (2015).

3. The data comes from a (y_{jt-1}, g_{jt})-panel, where j and t are indices for country and time. The data is stacked and sorted to the ($y_{i(-)}$, g_i)-data set where i is an index for the income order. Time only appears as (-) indicating that each data pair is from the same country and the two time periods t and t-1. In the literature g_i is often an average g_{ni}, where $g_{njt} = (g_{jt} + g_{jt+1} + ... + g_{jt+n-1})/n$. See the appendix on definitions.

2. Growth regressions and transitions

The theory and empirics of economic growth is a large field, covered by numerous textbooks, and a massive four volume handbook (Aghion and Durlauf 2005, 2013). Thus, a few notes on a couple of central themes of relevance will suffice. For ease of presentation the World Bank terminology of LICs, MICs, and HICs is used. It refers to Low, Middle, and High Income Countries respectively.

2.1 Growth regressions

Much of the discussion of the ($y_{i(-)}$, $\underline{g}_i$)-relation starts out from the empirical observation that long run time series for income, *y*, often look amazingly linear – perhaps with one or two kinks. This is often taken to represent a Solow model growing along a steady state path.

From this theoretical backbone a large wave of growth regression was started by Barro (1991) and the ensuing textbook Barro and Sala-i-Martin (1995, 2003). It demonstrates how the model leads to a basic estimation model in two versions:

(1) $g_{njt} = \alpha_1 + \beta_1\, y_{jt\text{-}1} + \varepsilon_{1jt}$, the ε's are the noise terms

(2) $g_{njt} = \alpha_2 + \beta_2\, y_{jt\text{-}1} + [\gamma_1\, z_{1jt} + \ldots + \gamma_k\, z_{kjt}] + \varepsilon_{2jt}$, the *z*'s are *k* controls

Indices *j* and *t* are for countries and time, while *n* indicates an average over *n* years.

Equation (1) tests for *absolute* convergence. It occurs if $\beta_1 < 0$. The standard result is that $\beta_1 > 0$ but insignificant, so convergence is rejected.[4] Most data sets used to estimate (1) give quite wild point scatters, so the F-test for the regression (1) is often insignificant, and R^2-scores are low indeed.

Equation (2) tests for *conditional* convergence, which occurs if β_2 turns significantly negative for a reasonable and robust *z*-set of controls for country heterogeneity. It already happens when the *z*-set is fixed effects for countries. Thus, countries would converge, if they were the same. It is almost a tautology, and it is surely not a statement about the real world. However, it is fairly easy to find more interesting control-sets making β_2 negative, as already reported in Barro (1991).

These equations have been estimated in many ways and on many data sets, both in pure cross-country versions (where *t* is fixed) and in panel versions (where both *t* and *j*

4. In our set of N = 8,874 observations β_1 becomes significantly positive. The divergence is 0.15 per logarithmic point, or 0.75 percentage points over the 5 point income range (see Figures 2-4). It will be shown that this is a misleading picture.

varies).[5] About 400 z-variables have been tried in many combinations,[6] and the relation has been adjusted for simultaneity using many sets of instruments. Many n's have been examined, lags have been included, etc.

Since Baumol (1986) it has been known that the HICs converge to the *same* steady state. This is termed club convergence. It is dubious if other clubs than the HIC club exists.

2.2 Two steady states and the Grand Transition

Ever since Rostow (1960) economic historians have pointed out that the most common steady state throughout history has been a traditional one with low income and no growth. This is confirmed by the large efforts of data collections put together by Maddison (2001). It is a steady state in the sense that it is stable, which is due to very slow technical progress. It is a LIC club if a low level equilibrium trap exists, so that the LICs converge to the *same* (low) level.

Thus, it is a basic observation that two steady states exists: A modern and a traditional. The modern started to develop about 200 years ago. The present HICs gradually moved away from their old traditional level and in the last half century converged to much the same income level. There is still a group of LICs, but it is debated if there is convergence at the bottom. If the top of the distribution grows faster than the bottom, the 'bridge' between the two keeps growing. We term the bridge the *Grand Transition* as it consists of many transitions.

Nearly all socio-economic variables have different levels in the traditional and modern steady state and a fairly well-defined path as the country moves between the two states.

(3) $z = F(y)$, with the levels z^T in the traditional and z^M in the modern steady state.

In other work we have shown how the transition paths look for some transitions.[7] Equation (3) models the causal effect $y \Rightarrow z$, while equation (2) models the causal effect $z \Rightarrow g \approx \Delta y$. So that estimates of both (2) and (3) have to be adjusted for simultaneity.

The definition of a steady state is that all ratios in the economy stay constant, and this

5. See Gundlach and Paldam (2015) on the interaction of growth theory and the many estimators tried.
6. It is only possible to include a handful or two of the 400 variables so billions of choices are possible. Each of these gives an estimate of β_2, resulting in a large range of possible estimates. See chapter 12.5 of Barro and Sala-i-Martin (2005).
7. The reader will think of the agricultural and the demographic transitions. We have shown that there is also a democratic transition (see Gundlach and Paldam 2009b and Paldam and Gundlach 2012), a religious transition (see Paldam and Gundlach 2013), a transition of corruption (Gundlach and Paldam 2009a), and a transition in the preferences for socialism (Bjørnskov and Paldam 2010).

is precisely what they fail to do! When the bridge between the LIC and the HIC steady states is a transition, with structural changes in all fields, it becomes rather unclear if this leads to a linear aggregate path. Large-scale structural change creates all kinds of tensions and uncertainties. Political alliances break up and new have to be formed. This may involve changes of the political system. It follows that it is likely that: (i) Growth becomes more variable during the transition. Also, the transition is the period where some MICs catch up with the HICs, i.e., technological catch-up may happen. Once that starts, it is likely that (ii) growth can become particularly fast. So we look for a middle income peak. From this peak to the high end of the income scale, the income-growth curve must have a negative slope resulting in the HIC-club.

2.3 Growth models with two sectors

One way to catch some of the transition is by a two sectors model with a traditional and a modern sector. In the beginning all is traditional, and then the transition is shown as a process where the modern sector gradually replaces the traditional one. This idea goes back to Lewis (1954) and Ranis and Fei (1960), and many formalizations have been made. The model has also been generalized to more sectors. A key feature of these models is that early development leads to large gaps in productivity between the sectors, giving rise to strong tensions in the economies. As development progresses, the gaps close and tensions are reduced.

This model catches a growth process that starts slowly as the modern sector has no weight, but then it becomes gradually more important and able to account for more growth. In the process large differences between the productivity in the sectors emerge. That may lead to large social tensions causing growth to stall in the middle, so that the variability is likely to be much higher in MICs than in the HIC, where a steady state has been reached.

However, even when the two-sector model may catch some of the transition, the Grand Transition is a much more comprehensive process: It includes a large fall in corruption, democratization, a large fall in religiosity, etc. These changes contribute to growth, but they may not be smooth, so they may add political instability that affects investment.

2.4 Two cases: Old west and oil countries

The transition started 200 years ago in a group of ‘old’ western countries, who are the largest group of HICs today. The transition was a gradual process, so it is easier to describe as a log-linear process, where technical progress was randomly distributed throughout the economy. The growth process in these countries will be termed the case of the *old west*. They never had as large tensions between the sectors as the MICs today. This is a qualification to equivalence

All countries earn some resource rent. It is typically 1 to 2% of GDP only, but some countries have abundant resources and resource rent becomes a major part of GDP. Such countries may reach HIC income levels exclusively from resource rent, without going through the Grand Transition. For long they remain LICs in the structure of society and its institutions, but gradually they start to change structure, to conform to income. The most extreme version of this case of development is the oil countries as an oil sector is a small international enclave in the country, employing few people and using a foreign technology. Such sectors are normally carefully fenced and heavily guarded so they are indeed an enclave. Their effect on the rest of the economy is the tax on the resource rent that flows into the treasury. We use OPEC membership as our proxy for the oil-case.

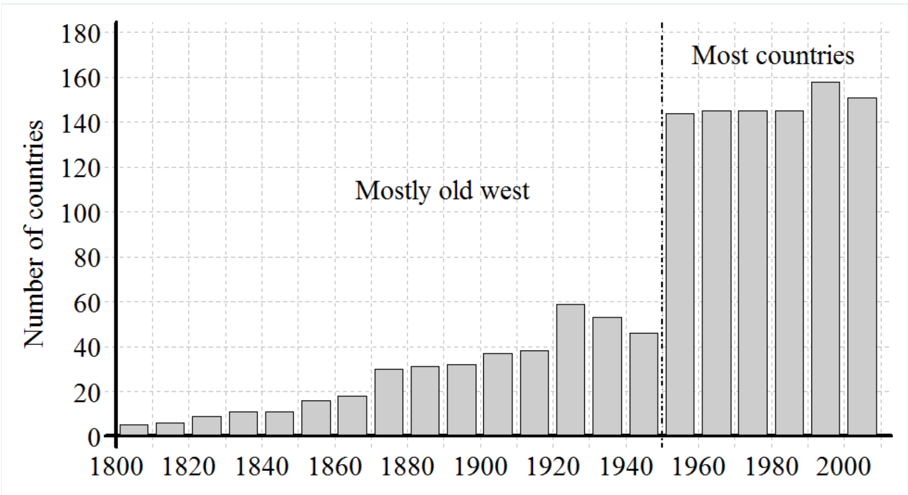
3. The data: ($y_{i(-)}, g_i$) pairs

The data used is from the Maddison-project where the annual data starts in 1800 and (p.t.) ends in 2010. We include data for the present countries as soon as possible. Data for Sub-Saharan African starts in 1950, where all but three counties were colonies. Data for the successor states of Yugoslavia starts in 1952, while data for the successor states of the USSR starts in 1990, and so they does for the successor states of Czechoslovakia. In these cases the data for the 'old' country is stopped, when the new data starts.

The data covers 12,786 data-pairs. Figure 1 gives the average number of countries covered in each decade. During the 19th Century 74% of the observations are from the old west, then some Latin American countries join the countries covered, but before 1950 the data is from a shifting sample of little representativity.[8] From 1950 the sample holds at least 144 countries with more than 95% of the world population. Thus, the 1950-2010 data sample is much more representative. The present study only covers this sample, where N = 8,874.

Income is in (natural) logarithms to 1990 international Geary-Khamis dollars. They cover the interval from 5.7 to 10.4. This is 4.7 ***lps*** (logarithmic points) which is a span of 110 times in the *gdp*. To close such a gap in a century means that an excess growth of 4.8% above the one of the rich countries. This is next to impossible.

Figure 1. The coverage of the data



8. Till 1900 only three LICs are included (Indonesia 1815, Sri Lanka 1870, and India 1884). A few more are included before 1950, but the country sample from 1800 to 1950, where N = 3,912, is very skew. Also, national accounting for most countries only started in 1950, so data before that are backward projections. The data from 1800 to 1950 are analyzed in Paldam (2015).

3. The scatter of the ($y_{i(-)}, g_i$)-points analyzed by kernel-curves

The data-pairs are shown as a scatter diagram in Figure 2. The points scatter very much, so it is difficult to see any pattern. The wild scatter means that any simple pattern, such as the one discussed, will explain a modest part of the variation only. Figure 2 is provided with a kernel regression using the Epanechnikov kernel (E-kernel), with a bw (bandwidth) of 0.35. The kernel shows the (smoothed) path of the average. Most of the variation at the high end – notably the negative observations – is due to the oil-rich countries, so the 563 observations for OPEC members are singled out and omitted in the estimate of the kernel.[9]

Figure 2. The scatter of all N = 8,874 from 1950 to 2010

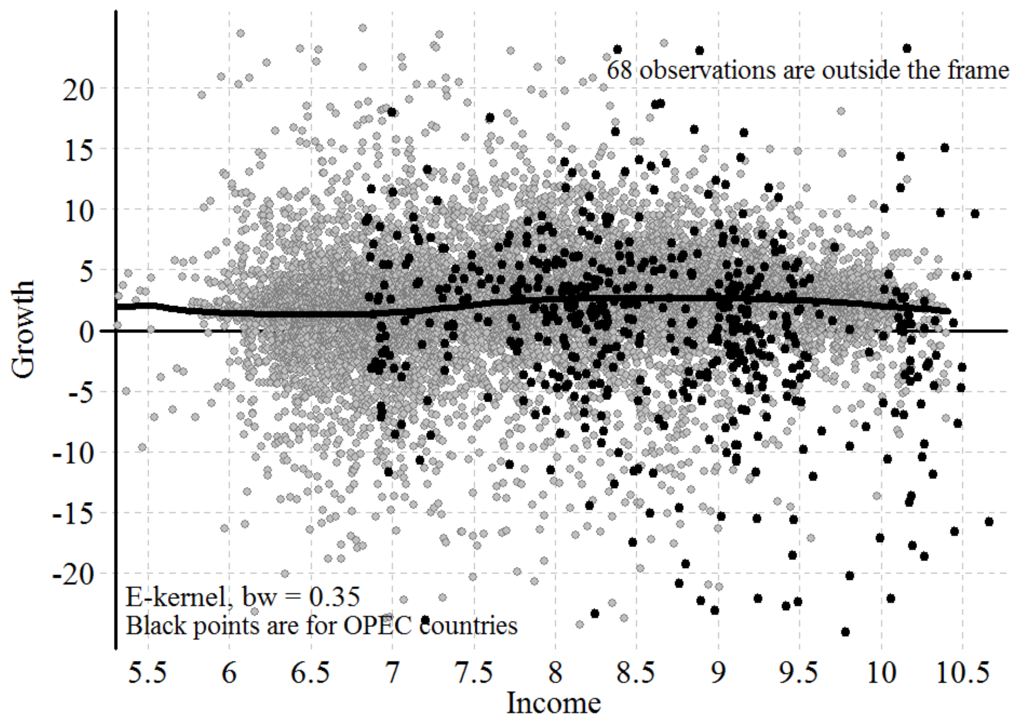


Figure 3 shows how the kernel curve looks when the scatter points are suppressed so that the vertical axis can be enlarged. The 95% confidence intervals are added. Figure 2 shows that the kernel is supported by few points at both ends, but it looks as if the scatter narrows at the high end when the OPEC countries are omitted.

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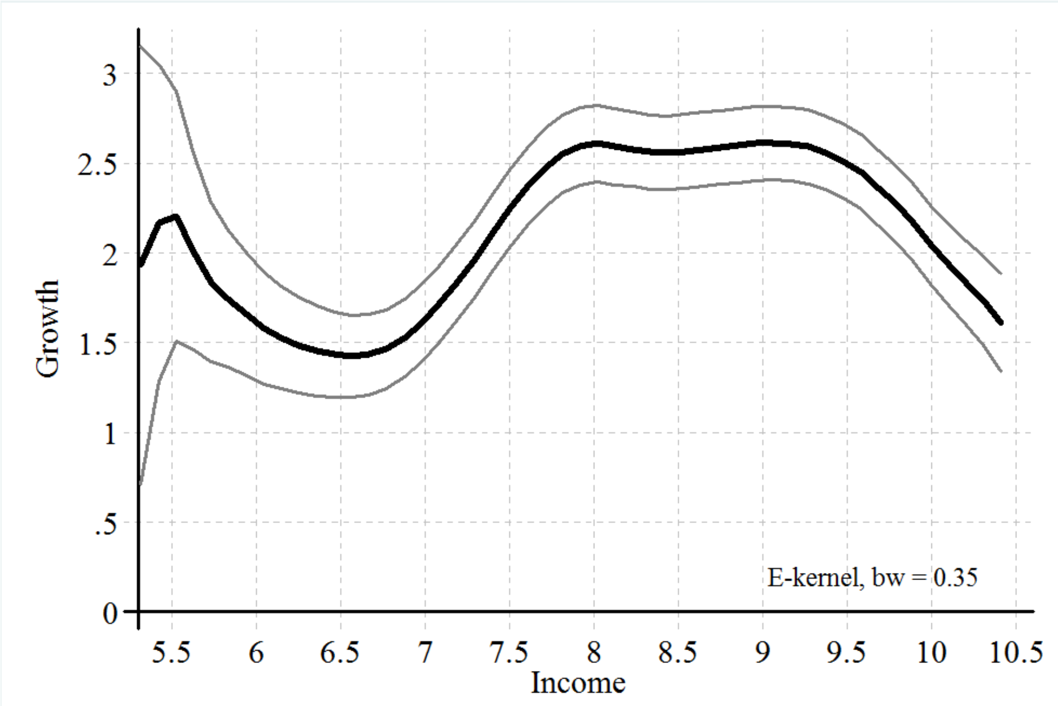
9. The kernel for OPEC observations falls throughout and becomes negative from an income level of about 9. When the OPEC points are included in the estimate of the kernel, it is indistinguishable in most of the range, but it falls more at the high end and becomes negative, see Paldam (2015).

Figure 3. The kernel curve from Figure 2 enlarged



Note: The thick black curve is the kernel. The two gray curves give the 95% confidence interval. This is Figure 10b in Paldam (2015).

The basic form of the kernel-curve is robust to the bandwidth, to a break-up of the time period, and to averaging of the growth variable.[10] The key observation is that the confidence interval is so narrow around the average curve that the non-linearity of the curve is highly significant. There is no way a straight line can be drawn within the confidence interval.

The confidence interval is rather narrow except at the low end, where it widens as expected from Figure 2. This also means that it is possible to draw a horizontal line from y = 5.5 to 7, so that the little top at 5.5 and the negative slope from 5.5 to 6.5 is insignificant. The fact that the confidence interval does not widen at the top suggests that the fall in density of the point is offset by a fall in its variation, as will be confirmed in section 4. The kernel-curve gives an answer to the two questions in the introduction:

A *low level equilibrium trap*: Here that key observation is that no negative section is found. There may be a convergence in the interval from 5.5 to 6.5, but it is a convergence to

10. Paldam (2015) documents the robustness in three ways: (i) The (g_{ni}, $y_{i(-)}$)-kernel is calculated for three values of n = 1, 5 and 10, which looks similar. (ii) bw is varied from 0.2 to 0.7: With a lower bw the curve becomes more wobbly, but it keeps its basic form. For higher bws the 'muddle' at the start disappears and the peak becomes sharper. (iii) The kernel-curve is estimated for all decades separately: It looks much the same for 4 of the 6 decades, but in the 80s the debt crisis of the LICs and MICs gives many countries low growth, and in the 90s the kernel has an extra dip for the MICs due to the large group of countries going through the change from socialism to capitalism.

6.5, where growth becomes faster. It also means that the traditional steady state of a stable income is not visible in the data from 1950 onwards. Countries grow at all income levels.

Convergence or divergence: The kernel-curve has a significant hump in the middle. The countries diverge before the hump and converge after. It looks as if the hump has a rather flat top, which peaks somewhere between 7.8 and 9.5.

It is important that that the curve is positive throughout – the average of all points used to calculate Figure 3 is 2.05% p.a., and as can be seen, it is 1.6% at the two ends and approximately 2.6% at the flat top of the hump. So MICs grow by 1% more than the HICs. This closes the gap by 1 *lp* each century. The gap between the LICs and the HICs is 3.5 to 4 *lp*s. So here the catch-up needs 4-5 centuries in average.

4. A kernel for the standard deviations

To analyze the variation in the growth rate, we have calculated a *moving standard deviation* $s_{11}(g)_i$, which gives a $(y_{i(-1)}, s_{11}(g)_i)$-pair. The s-series is made and analyzed as follows:

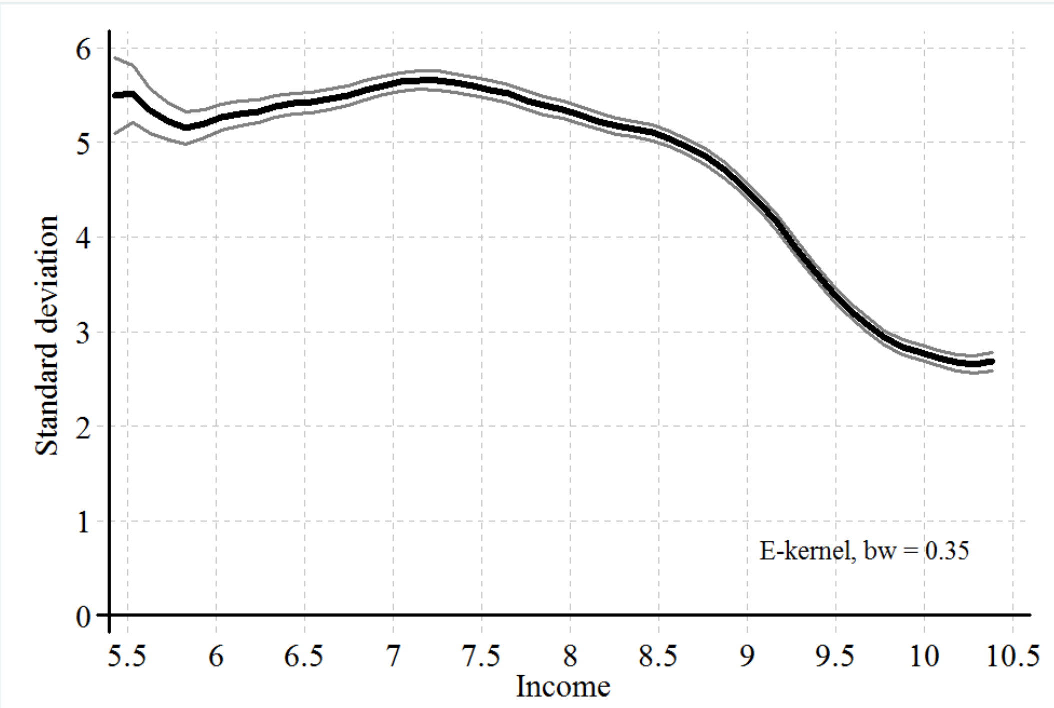
(1) First the $(y_{i(-1)}, g_i)$-series is sorted by $y_{i(-1)}$.

(2) Then the sorted g_i-series is replaced by $s_{11}(g_{i-5}, g_{i-4}, \ldots , g_i, \ldots , g_{i+5})$. Hereby the first 5 and the last 5 observations in the series are lost, so N = 8,864.[11]

(3) Finally, the $(y_{i(-)}, s_{11i})$-series is analyzed by the same kernel-technique as before. The resulting kernel is termed a std$_{11}$-kernel.

Figure 4 shows that the std$_{11}$-kernel rises a little from the start till the peak in y = 7.25. As y increases s_{11} falls. In the end it has fallen to less than half of its peak value. This explains why the confidence intervals on Figure 3 do not fall at the high end, even when the observation density falls. Note also that the confidence interval falls and remains low throughout. The average of the 8,864 s_{11}s calculated is 5.01, as appears rather consistent with Figure 2.

Figure 4. The std$_{11}$-kernel on the same data as on Figures 2 and 3



11. The s_{11i}-series uses 11 observations for the calculations of the standard deviation. We have also made the calculations with 51 observations. It had no visible effects on the kernel shown. So the std$_n$-kernel is robust to choice of n as long as N is much larger than n.

8 Conclusions

The above analysis is done using a technique chosen to assume as little theory as possible. We hope the reader will agree that the analysis builds on very few assumptions. This will allow us to draw at least one rather strong conclusion about economic theory: It does appear that there is a transition in the growth rate: From moderate to higher and back again to moderate. Thus the one-sector steady state perspective on economic growth is problematic.

In addition our findings tell a story about the long-run dynamics of income of the world system of countries:

Poor countries (LICs) have a low and unstable growth. However, the growth is still at an average rate of 1.6% per year. The stagnating traditional economy is all but gone today.[12]

Rich countries (HICs) have a growth rate of 1.6% p.a. as well. The instability of this growth is half the one of the LICs.

Middle-income countries (MICs) have, in average, 1 percentage point higher growth than in the LICs and HICs. The peak is rather flat between y = 7.7 and 9.5. During that period the variability of the growth rates falls.

An excess growth of 1 percentage point – as is found for the MICs – accumulates to one logarithmic point over a century. This means that many MICs will actually catch up with the HICs during the next century. It will take considerably longer for the LICs.

Due to the high variability of the growth of the LICs and MICs some countries will catch up much faster than others.

12. The world has about 10 countries with a *gdp* (GDP per capita) that is lower in 2010 than in 1950.

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Appendix on the data definitions and the income scale

Both income and growth are calculated from the *gdp*, which is GDP per capita in real PPP prices. Income $y_{jt} = \ln(gdp_{jt})$, where j is a country and t is time. *Growth*, $g_{jt} = 100(gdp_{jt} - gdp_{j(t-1)})/gdp_{j(t-1)}$. The data used for the calculations is stacked and sorted by income so that the country and time dimensions are scrambled and joined into one index i. We write ($y_{i(-1)}$, g_i) which are for the same country and two adjunct time periods (t and t-1), however ($y_{i+1(-)}$, g_{i+1}) is unlikely to be from the same country as ($y_{i(-1)}$, g_i) or from the next time period. The *gdp* (GDP per capita) is in 1990 Geary-Khamis $. Income is the natural logarithm to *gdp*. The table shows the three closest matching countries to the income scale y = 6, 6.5, ..., 10. Countries with an * miss the observation for 2010, so it has to be assessed. Nearly all countries of the west are between 10 and 10.3.

Table. Income scale: Closest matching countries in 2010

Income	*gdp*	Match	Income	*gdp*	Match	Income	*gdp*	Match	Income	*gdp*	Match
6	400	Burundi* Niger CAR*	7	1100	Mali Korea N* Kenya	8	3000	El Salvador* Libya* Philippines	9	8100	Costa Rica China Turkey
6.5	670	Guinea* Madagascar Haïti*	7.5	1800	Nicaragua* Nigeria Ghana	8.5	5000	Indonesia Bahrain Ecuador	10	22000	Korea S Japan Ireland

Documentation: The non-linear income-growth relation

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This paper is documentation to the following main paper:

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Main:	http://www.martin.paldam.dk/Papers/Growth-trade-debt/Income-growth.pdf
The present:	http://www.martin.paldam.dk/Papers/Growth-trade-debt/Docu-income-growth.pdf

The main paper and the present both study growth as a function of income and show that the relation is non-linear. The premium results are taken to be Figures 10b and 11c. The main paper repeat these graphs and brings most of the theoretical discussion and all references. The technique used is rather space intensive, so the robustness tests and other 'auxiliary' calculations are documented in the present background paper.

Data source: http://www.ggdc.net/maddison/maddison-project/-home.htm
(downloaded 15/11-2014). See also:
Bolt, J., van Zanden, J.L., 2013. The First Update of the Maddison Project; Re-Estimating Growth Before 1820. Maddison Project Working Paper 4.

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Content:

1. The data: From ($y_{jt\text{-}1}$, g_{jt}) to ($y_{i(\text{-})}$, g_i) pairs and Part 1 and Part 2

The paper considers three annual data: $\boldsymbol{gdp_{ji}}$ (in lower case) is GDP per capita in PPP terms where j is the country and t is time. Income $\boldsymbol{y_{jt}} = \ln gdp_{jt}$, and growth is $\boldsymbol{g_{jt}} = 100\Delta gdp_{jt} / gdp_{jt\text{-}1} \approx \Delta y_{jt}$. A data pair is ($y_{jt\text{-}1}$, g_{jt}). Thus, the income scale is in ***lp***s (logarithmic points). Data in the source starts year 1800 and contains 12,786 annual observations.

The analysis stacks these observations and sorts them by income. This produces a series of ($y_{i(\text{-})}$, g_i)-pairs, where the index i is the income order. The '(-)' in the subscript indicates that the lag is for time not for i. When N is large, the stacking-sorting process effectively randomizes the observations, except for the income dimension. That is, it becomes very unlikely that ($y_{i(\text{-})}$, g_i) and ($y_{i+1(\text{-})}$, g_{i+1}) are from the same country and from two adjunct time periods. Thus, it concentrates the analysis on the $g_i = g(y_{i(\text{-})})$ relation.

Table 1 lists the data-pairs available. Data for the present countries is included as soon as possible. The data for the African counties starts in 1950, where nearly all these countries were colonies. The data for the successor states of Yugoslavia starts in 1952, while the data for the successor states of the USSR starts in 1990, and so they does the data for the successor states of Czechoslovakia. In these cases the data for the 'old' countries stops, when the new data starts.

Table 1. Numbers of ($y_{i(\text{-})}$, g_i)-pairs and countries per decade and for Part1 and Part 2

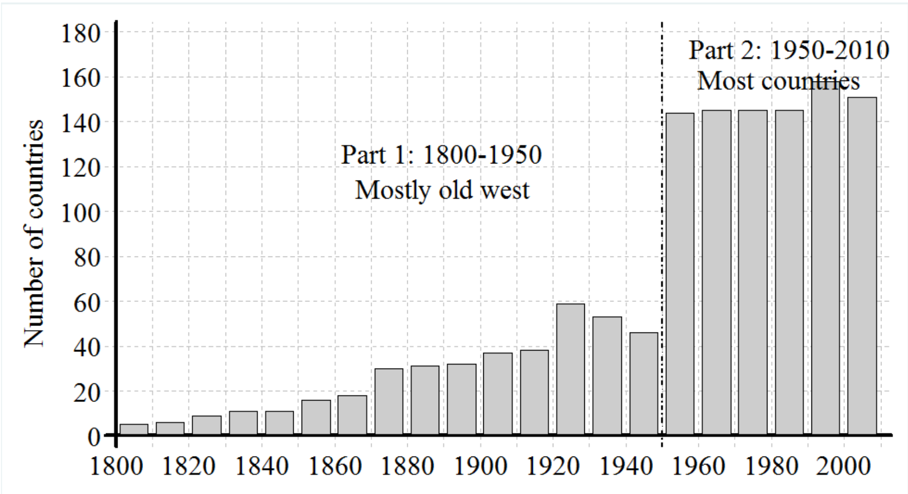
Period	Number of pairs	Descriptive statistics Countries	Mean g	Std g	Period	Number of pairs	Descriptive statistics Countries	Mean g	Std g
Part 1: 1800-1950					1930-40	530	53	1.17	7.68
1800-10	54	5	-0.44	6.27	1940-50	458	46	2.34	10.63
1810-20	60	6	0.03	6.28	Part 1	3,912	26	1.51	7.22
1820-30	92	9	1.08	4.10	**Part 2: 1950-2010**				
1830-40	107	11	1.66	5.47	1950-60	1,438	144	2.70	4.73
1840-50	114	11	0.59	5.23	1960-70	1,450	145	3.21	5.86
1850-60	160	16	1.81	7.79	1970-80	1,450	145	2.21	6.31
1860-70	175	18	1.09	6.33	1980-90	1,450	145	0.18	5.23
1870-80	295	30	1.28	6.09	1990-00	1,580	158	1.00	7.55
1880-90	311	31	1.27	4.95	2000-10	1,506	151	3.01	4.81
1890-00	320	32	1.16	5.63	Part 2	8,874	148	2.04	5.96
1900-10	367	37	1.84	5.36					
1910-20	378	38	0.75	8.69	**All data: 1800-2010**				
1920-30	491	49	2.71	6.75	All	12,786	61	1.88	6.37

Note: Downloaded in November 2014.

To study the ($y_{i(-)}$, g_i)-relation, the data should cover the full income range from LICs over MICs to HICs, which are Low Income, Middle Income, and High Income Countries, respectively. The data from 1800 to 1950 has few LICs.

Figure 1 shows the path of the average number of countries per decade from columns (3) and (8) in Table 1. During the 19th Century 74% of the observations are from 'old western' countries, but from 1870 data from some Latin American countries appears. Till 1900 only three LICs are included. It is Indonesia (from 1815), Sri Lanka (from 1870), and India from (1884). A few more are included before 1950, but the country sample from 1800 to 1950 is very skewed. Also, national accounting for most countries only started in 1950, so data before is backward projections. From 1950 to 2010 the sample holds at least 144 countries with more than 95% of world population. So, the data from 1800 to 1950 is likely to show a much less representative picture than the data after 1950.

Figure 1. The coverage of the data: Part 1 and Part 2



Thus, the data is divided into two parts: **Part 1** 1800-1950 with N = 3,912 and **Part 2** 1950-2010 with N = 8,874.

The income data is in natural logarithms to 1990 international Geary-Khamis dollars. To understand roughly what this means Table 2 gives three closely matching countries in 2010 for selected income levels. When the text refers to some number for income, y, the reader should refer to Table 2 to understand what the number means. The income scale is in *lp* (logarithmic point). As the natural logarithm is used, one lp is e ≈ 2.72 times. Thus, a country that is 1 lp richer than another has a *gdp* that is 2.72 times higher.

Table 2. Income scale: Closest matching countries in 2010

Income	*gdp*	Match	Income	*gdp*	Match	Income	*gdp*	Match	Income	*gdp*	Match
6	400	Burundi* Niger CAR*	7	1100	Mali Korea N* Kenya	8	3000	El Salvador* Libya* Philippines	9	8100	Costa Rica China Turkey
6.5	670	Guinea* Madagascar Haïti*	7.5	1800	Nicaragua* Nigeria Ghana	8.5	5000	Indonesia Bahrain Ecuador	10	22000	Korea S Japan Ireland

Note: The countries with a * missies the observations for 2010, so they are assessments. Nearly all countries of the West are between 9.8 and 10.3.

The most celebrated catch-up story is the one of the Four Asian Tigers. From 1950 to 2010 South Korea and Taiwan both grew by 3.2 lp, which is 25.4 times or 5.5% p.a., while Hong Kong and Singapore both grew by 2.6 lp, which is 13.4 times or by 4.3% p.a. Table 3 contains some simple calculations. The dark gray shaded cells are the ones above 5.5%, which is the growth of South Korea and Taiwan. The cells in light gray show the growth between that of Hong Kong and South Korea.

Table 3. Necessary annual growth to close the gap in 50 or 100 years

	Gap in lp	Times gdp	Over 100 years Growth	Over 100 years Catch up	Over 50 years Growth	Over 50 years Catch up
LIC to DC	4	54.6	4.1	5.6	8.3	10.0
	3.5	33.1	3.6	5.1	7.3	8.9
LIC to DC	2	7.3	2.0	3.6	4.1	5.6
	1.5	4.5	1.5	3.0	3.0	4.6

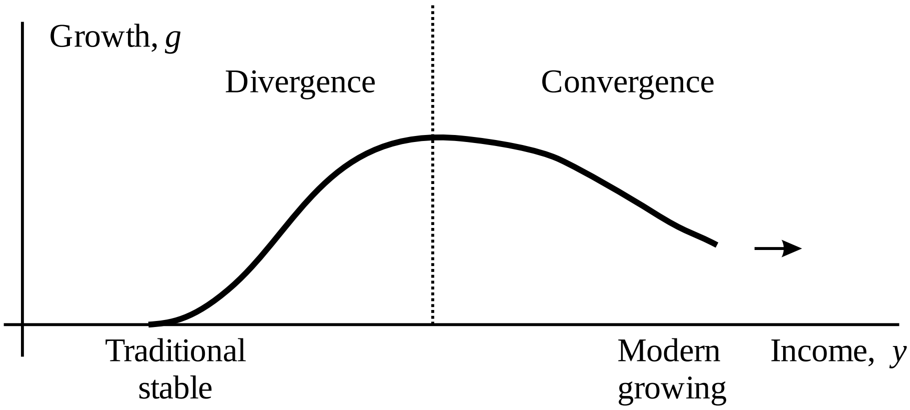
Note: Catch up assumes that the richer country grows at 1.5% per year.

2. The expected pattern

The main paper discusses the theory. For now it needs to be said that:

The theory of the Grand Transition claims that two steady states exist for the economy. A stagnating traditional state and a modern one with fairly low growth as well. Growth is higher, but more variable between the two equilibriums. Thus, the GT-theory predicts that the *g*(*y*) curve has a hump-form like that on Figure 2. There is divergence to the left of the peak and convergence to the right of the peak. It is well-known that there is considerable convergence at the high end. This means that the high end gradually becomes fatter. The hump-formed path is not likely to be clear before the top-end becomes sufficiently fat.

Figure 2. The expected form of the $g = g(y)$ curve



Some countries become rich due to resource rent without going through the Grand Transition. The most extreme case is the wealth created by the exploitation of large oil deposits. An oil sector is a small (basically foreign) enclave in the economy using a highly specialized international technology and labor as well. Its main effect is that a large resource rent accrues to the government, allowing it to pass on large subsidies to its population. This makes the population wealthy, but greatly reduces the competitiveness of all other sectors. This process is known as Dutch Disease. In the short run this is a pleasant disease as it makes everybody rather wealthy, but in the longer run the growth rate becomes small – even negative, as demonstrated in section 8.

Figure 10b below on page 20 is our best estimate as it looks at the most representative data set, when the most resource rich counties are deleted.

3. Some polynomial regressions

One way to study the non-linearity in the income-growth relation is to run sets of polynomial regressions as done in Table 4. The set starts by the linear regression, and then terms of higher power are gradually added:

(1) $g_i = \alpha + \beta_1 y_{i(-)} + u_{1i}$

(2) $g_i = \alpha + \beta_1 y_{i(-)} + \beta_2 y_{i(-)}^2 + u_{2i}$

(3) $g_i = \alpha + \beta_1 y_{i(-)} + \beta_2 y_{i(-)}^2 + \beta_3 y_{i(-)}^3 + u_{3i}$

(4) $g_i = \alpha + \beta_1 y_{i(-)} + \beta_2 y_{i(-)}^2 + \beta_3 y_{i(-)}^3 + \beta_4 y_{i(-)}^4 + u_{4i}$

...

These regressions have been run up to (8). What happens is that after a certain number the power-terms become too collinear and start to eat the significance of each other. Once it happens, it does not work to add additional power-terms.

The first observation is that the R^2-score is very low throughout as the reader will expect from the literature. However, significant coefficients are still found. They are quite different in Part 1 and Part 2 of the data. When all data is merged Part 2 dominates, but the fit decreases.

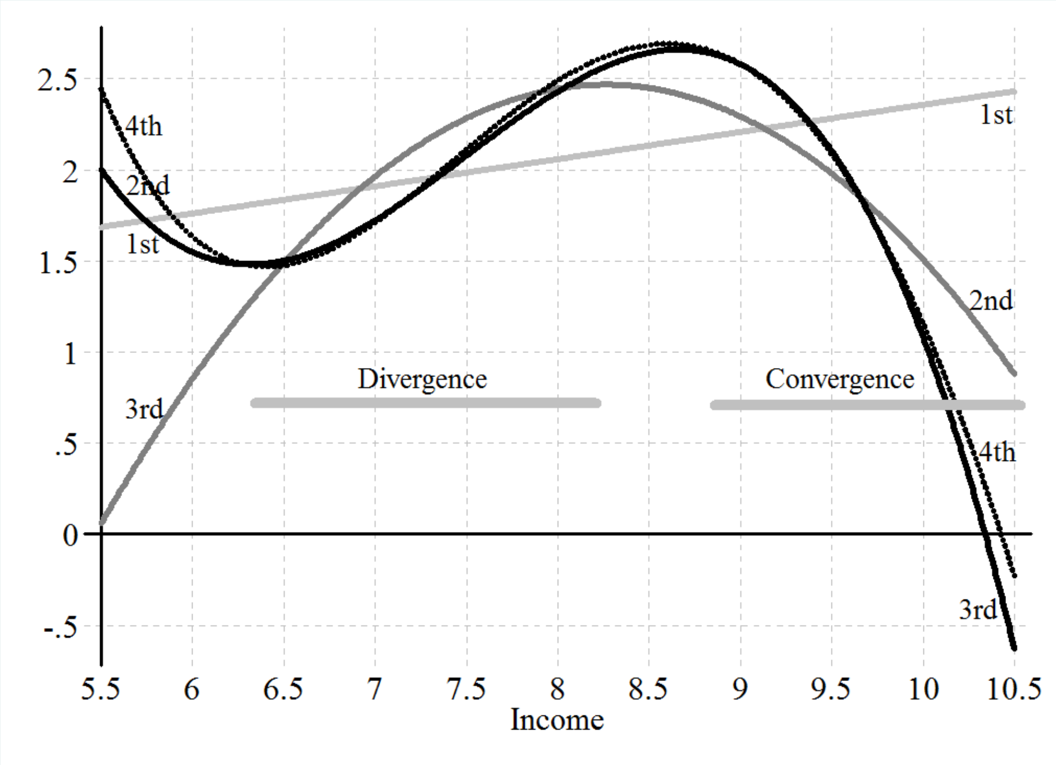
The regressions for Part 1 are not improved by adding power-terms to (1). Thus, it is clear that the regressions show a linear relation with a negative coefficient. There is convergence throughout.

Table 4. Polynomial regressions: Growth, g, explained by initial income, $y_{(-)}$, $y_{(-)}^2$, $y_{(-)}^3$

Nr	Constant	$y_{(-)}$	$y_{(-)}^2$	$y_{(-)}^3$	$y_{(-)}^4$	R^2
Part 1: Data shown on Figure 4a, N = 3,912						
(1)	**4.902 (3.6)**	**-0.451 (-2.5)**				0.002
(2)	5.163 (0.4)	-0.520 (-0.1)	0.005 (0.0)			0.002
Part 2: Data shown on Figure 4b, N = 8,874. The estimates are depicted of Figure 3						
(1)	**0.865 (1.9)**	**0.149 (2.6)**				0.001
(2)	**-19.09 (-5.8)**	**5.219 (6.3)**	**-0.316 (-6.1)**			0.005
(3)	**72.79 (3.3)**	**-29.84 (-3.5)**	**4.085 (3.9)**	**-0.182 (-4.2)**		0.007
(4)	164.54 (1.2)	-76.95 (1.1)	13.06 (1.0)	-0.935 (1.1)	0.023 (0.7)	0.007
All data: N = 12,786						
(1)	**1.001 (2.3)**	**0.113 (2.0)**				0.000
(2)	**-9.093 (3.2)**	**2.682 (3.4)**	**-0.161 (-3.2)**			0.001
(3)	**69.73 (3.2)**	**-27.42 (3.3)**	**3.625 (3.5)**	**-0.157 (-3.7)**		0.002
(4)	-86.57 (-0.7)	52.76 (0.8)	-11.65 (-0.9)	1.124 (1.1)	-0.040 (1.2)	0.002

Note: Bold coefficient estimates are statistical significant at the 5% level.

Figure 3. The paths of the four estimated relations from Part 2 of Table 4



The convergence in Part 1 is misleading, as it was the period where the West developed into wealthy countries, while the rest of the world hardly developed. So a full data-sample should show divergence. Thus, the convergence found must be due to the narrow county selection in the sample concentrating on the West.

The regressions for Part 2 improve when the squared and cubed term is added, as these terms are significant. The data is clearly pointing to a non-linear relation. The relations are shown in Figure 3. The curves fitted disagree for low incomes between 5.5 and 6.5. But they agree that there is divergence from about 6.3 to 8.3 and convergence from 8.7 onwards.

4. The scatter of the ($y_{i(-)}$, g_i)-points analyzed by kernel-curves

All data-points for the two parts are shown as the scatter diagrams of Figure 4a and b. The points scatter very much, so it is difficult to see any pattern.

Figure 4a. Part 1: The point scatter, N = 3,912

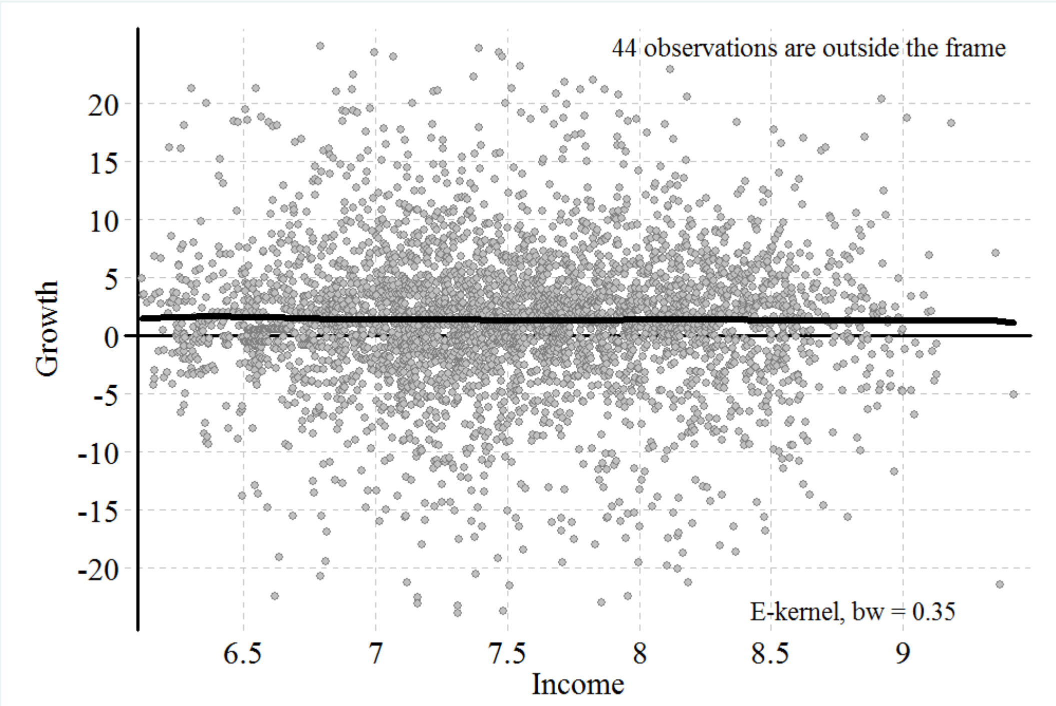
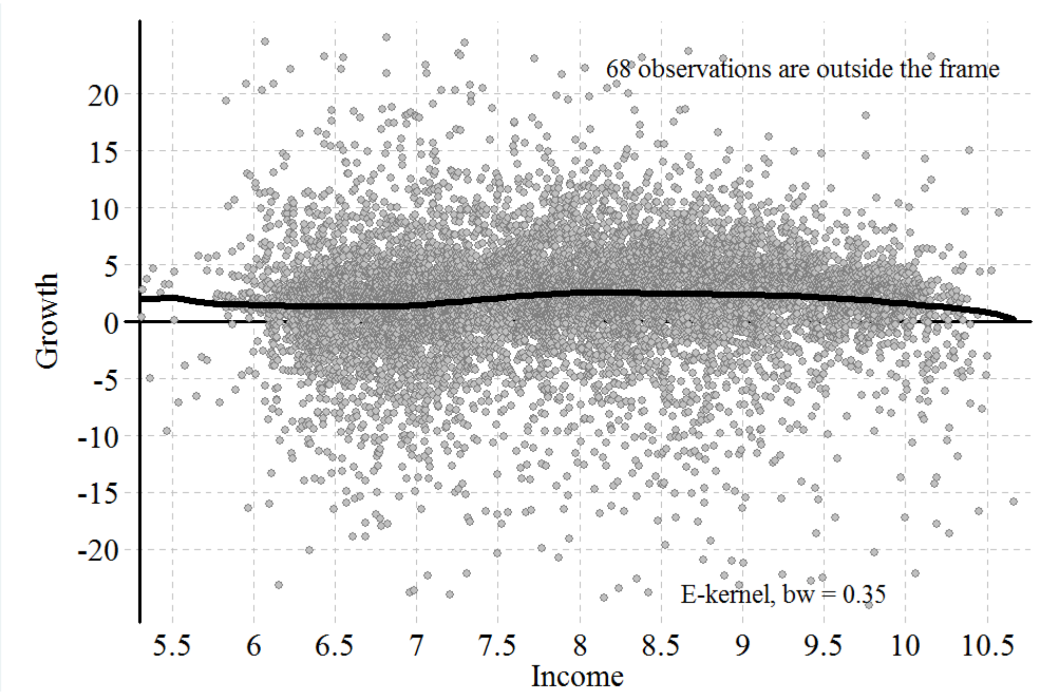


Figure 4b. Part 2: The point scatter, N = 8,874



The wild scatter means that any simple pattern, such as the one discussed, will explain a modest part of the variation only. This was amply confirmed by the 10 polynomial regressions in Table 4, which certainly obtain very low R^2 scores.

The point scatters on Figures 4a and b are provided with a kernel regression using the Epanechnikov kernel (E-kernel), with a bandwidth of 0.35 (bw = 0.35). TYhis is the bold black curve. Other bandwidths are analyzed in section 7. The kernel shows the (smoothed) path of the average – using a fixed bandwidth. In spite of the wild scatter the graphs confirm the regressions in Table 4 and add some points for further investigation:

To see these points more clearly, Figure 5a and b are presented. They delete the points of the scatter and concentrate on the kernel. This allows a great enlargement of the vertical axis, and the 95% confidence interval is added. It is the gray lines around the kernel.

For Part 1 the kernel shows a rather straight line with a negative slope. However, the slope is barely significant as it is almost possible to draw a horizontal line within the 95% confidence lines. This is precisely the same as found by the regression analysis.

However Figure 5b is another matter. Thanks to the large data-sample the 95% confidence interval around the curve is rather narrow. There is no way to draw a straight line within the confidence interval. The non-linear curve has four features:

Figure 5a. Part 1: The kernel-curve from Figure 4a, *N* = 3,912

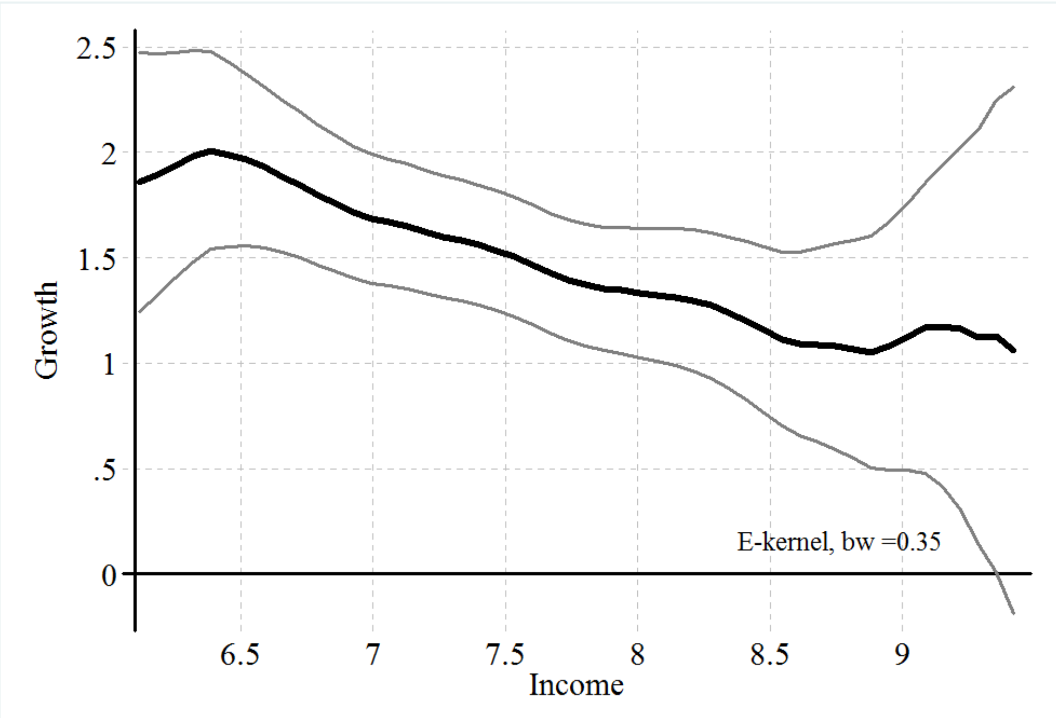
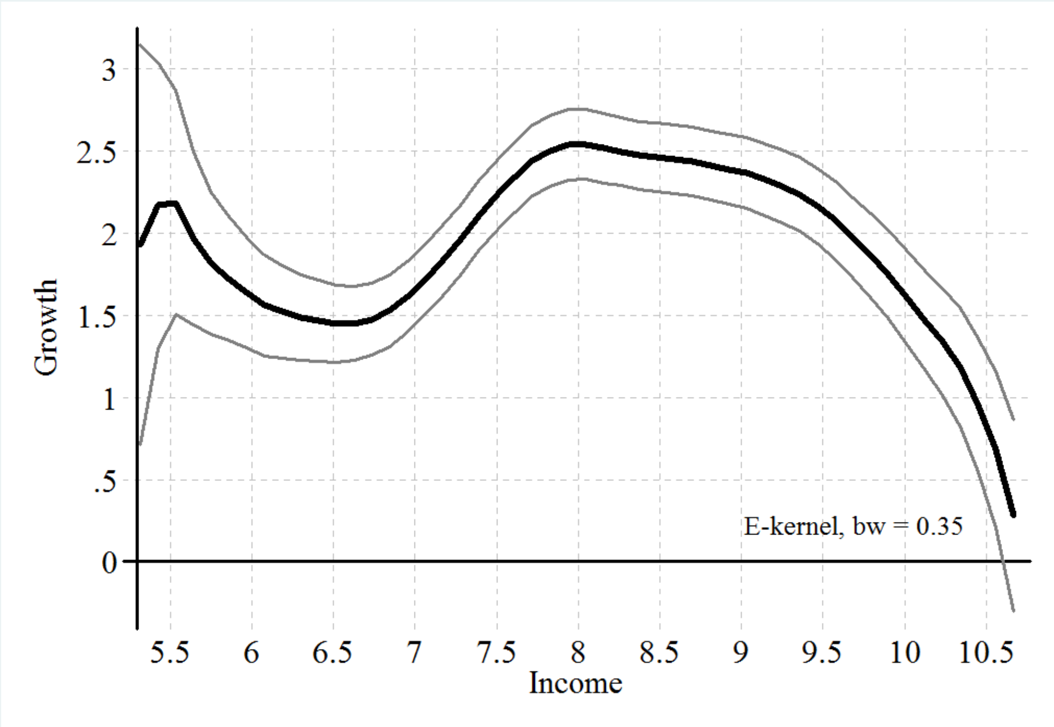


Figure 5b. Part 2: The kernel-curve from Figure 4b, N = 8,874



(f1) At the low end, where y is within [5.3, 6.5], the slope is largely negative, but here the confidence interval is rather wide. So it is consistent with a horizontal line indicating a constant growth of a little more than 1.5%.

Thus, it is possible that the underlying 'true' curve is flat at about 1.5%. However, the lower rim of the confidence interval is well above zero, so there is no sign of a low level equilibrium trap.[14] This indicates that the traditional stagnating steady state has vanished. . The wide confidence interval corresponds to the great difference between the 4 curves at Figure 3. Also, it is clear from Figure 4b that the data is thin for income levels below 6 lp.

(f2) For y within [6.5, 10.7] a significant hump-form appears. The hump has a rather flat top somewhere between 8 and 9.

(f3) From y = 6.5 to the top the figure shows a strong divergence.

(f4) From the top to the high end at 10.7 the kernel has a significant negative slope, which ends almost at zero. This is the well-known convergence to the modern steady state.

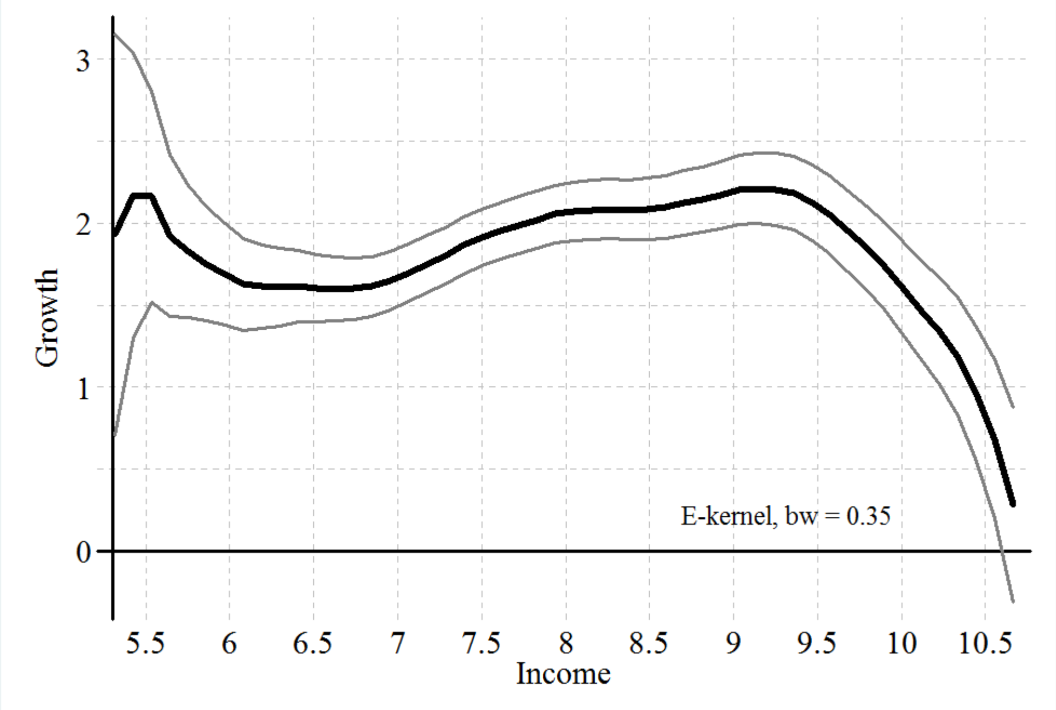
These four features will be further examined below. It is clear that the kernel explains more of the variation than the polynomial regressions in Table 4, but it is hard to imagine that they

[14]. A low level equilibrium trap means that LICs that starts growing reaches an income level with negative growth so that they fall back to a previous low level.

have a R^2-score above 0.03, so we are still dealing with a small level of explanatory power.

Figure 5c shows that the pattern from Period 2 dominates in all the data as they did in section 3. But the curve on Figure 5c is less clear than the curves from the two parts.

Figure 5c. Part 1 and 2 together: The kernel curve, *N* = 12,786



5. Robustness 1: Averages over *n* growth rates

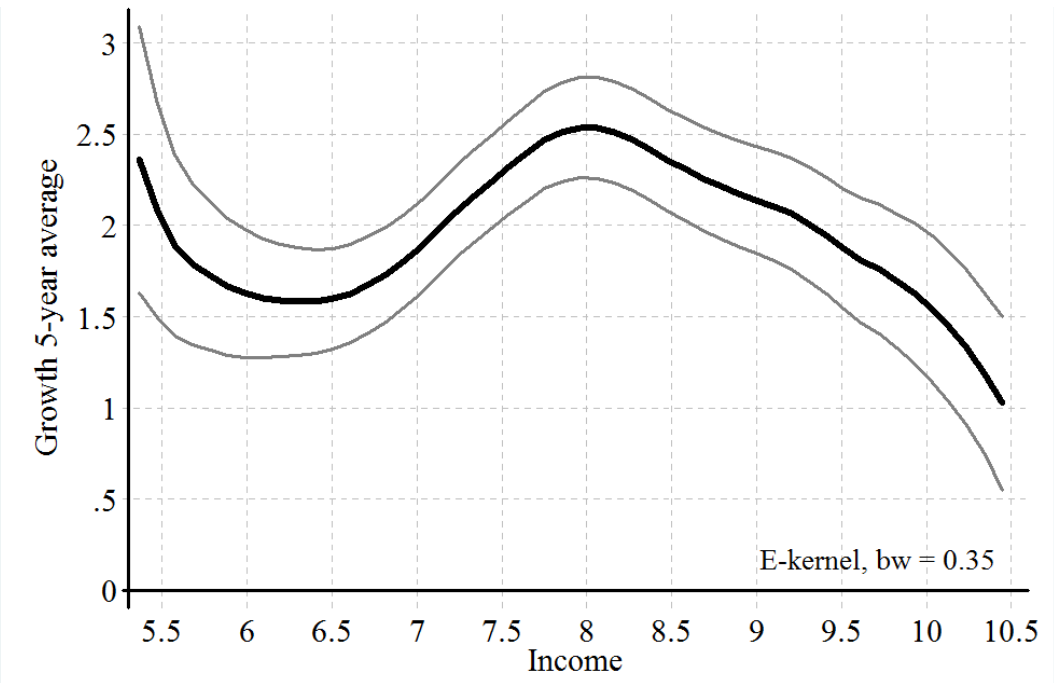
Much of the literature on growth regressions uses longer run averages of the growth rate instead of the annual one. Thus it is important that the analysis above is robust to averaging. Define the n-year average as:

(1) $g_{njt} = (g_{jt} + g_{jt+1} + \ldots + g_{jt+n})/n$, where $g_{1jt} = g_{jt}$

The $(y_{jt\text{-}1}, g_{njt})$ can be stacked and sorted into a $(y_{i(\text{-})}, g_{ni})$ data set as before. The analysis in the other sections of the paper is made for $n = 1$. It is important that the analysis is robust to $n = 1, \ldots, 10$. This is done by recalculating Figure 5b, where $n = 1$, for $n = 5$ and 10. This is the figure for all observations of Part 2. By increasing n, the number of points in the scatter decreases. For $n = 1$ $N = 8{,}874$, for $n = 5$ and 10 N becomes 1,787 and 892 respectively.[15]

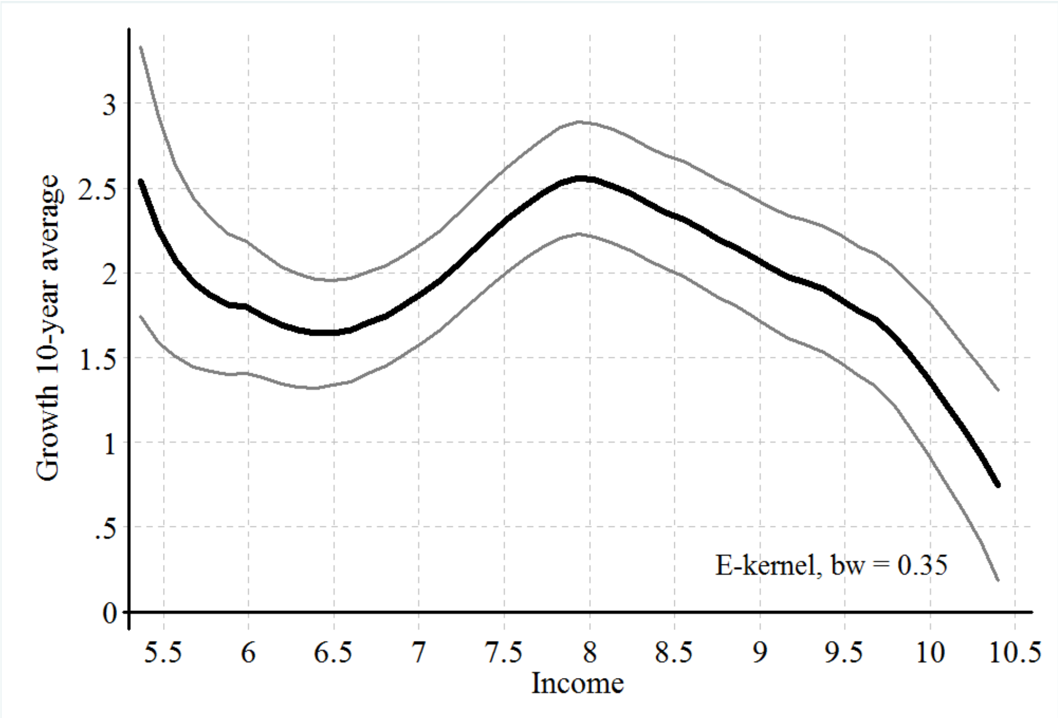
Figure 6a and b show the two kernels. They look remarkably alike each other and also much like Figure 5b. The small hump at 5.5 on Figure 5b disappears, but it was not significant anyhow. The fall from $y = 5.3$ to 6.4 still appears, but it remains insignificant. Also, the hump looks much the same with a top for growth 3.5% at $y = 8$.

Figure 6a. Part 2: Kernel curve for $n = 5$, $N = 1{,}787$, cf. Figure 5b



[15]. To use as much data as possible, values of g_{5i} are included even in a few cases where n is 3 or 4. And in the same way g_{10i} is accepted even if $n = 7$, 8 or 9.

Figure 6b. Part 2: Kernel curve for n = 10, N = 892, cf. Figure 5b



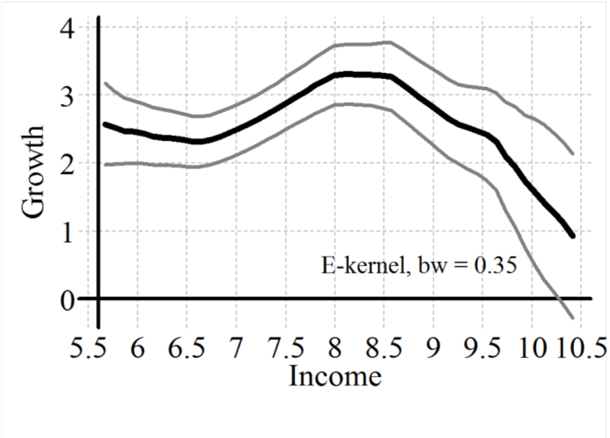
The three figures 5b and 6a and b show that the non-linear form is significant in the data throughout the range of averages used in the growth literature.

6. Robustness 2: The 6 decades 1950 to 2010

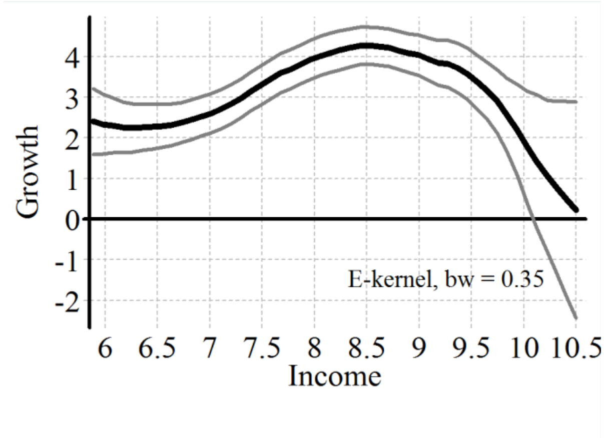
The pattern on figure 5b is broken up in the 6 decades on the 6 small graphs of Figure 7. The hump-form is much as in Figure 5b and Figures 7a, b, c and f. The four curves support the idea that the average is flat for low *y*-values. The last two decades are special.

Figure 7. Part 2: For each of the six decades, cf. Figure 5b

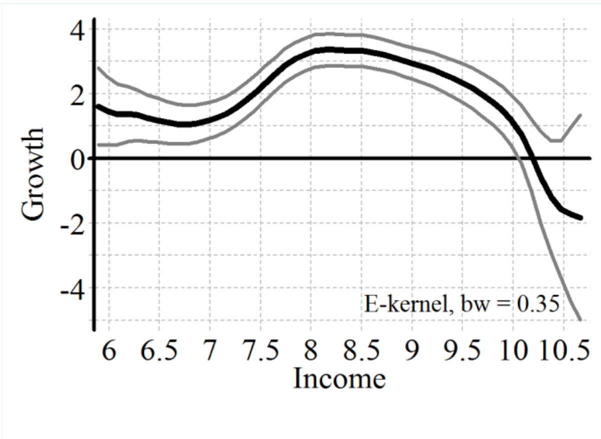
7a. Decade 1950-60



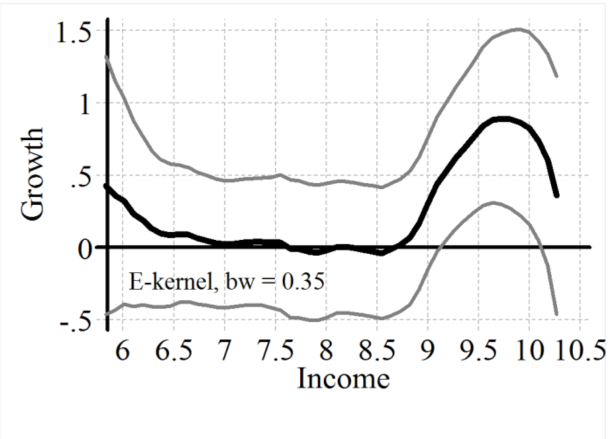
7b. Decade 1960-70



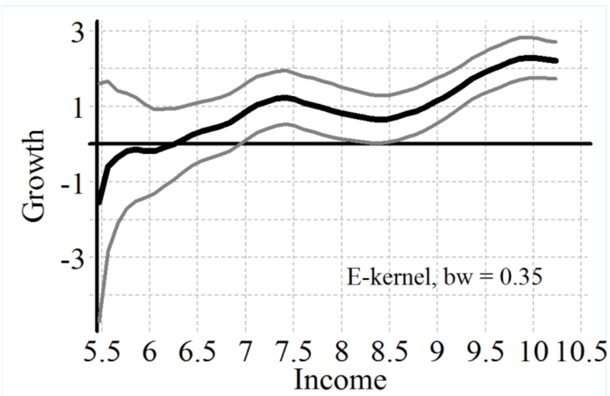
7c. Decade 1970-80



7d. Decade 1980-90



7e. Decade 1990-2000



7f. Decade 2000-10

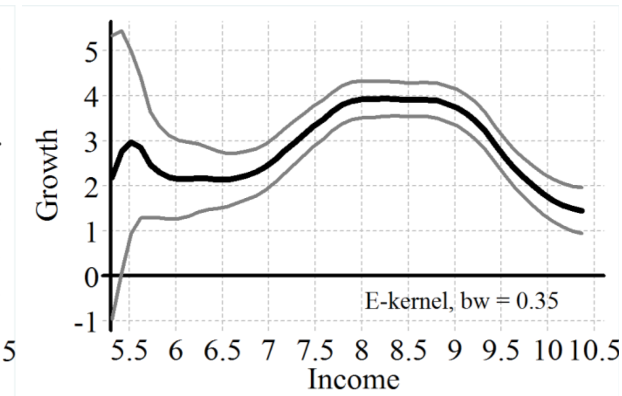


Figure 7d is the ‘lost’ decade of the debt crisis in the LDC world. Here many middle income countries had zero economic growth, as shown.

Table 5. The post-communist vs. other countries: 1990-2000

Country group	*Countries*	*N*	Income		Growth	
			Mean	Std	Mean	Std
Initial communist [a)]	34	340	8.18	0.67	-0.79	11.07
Other	124	1240	8.05	1.20	1.48	6.21

Note a: The countries include Cambodia, China, Cuba, Laos, North Korea, Vietnam.

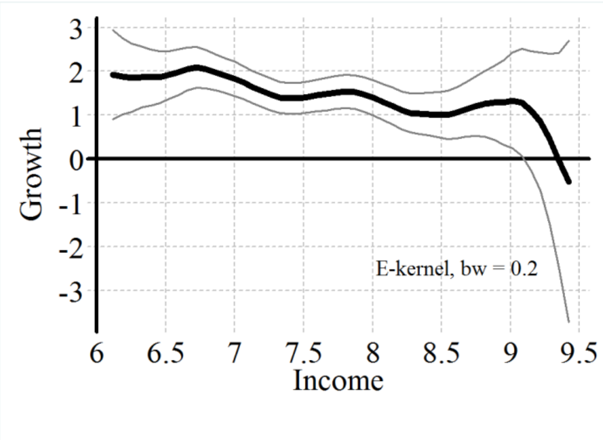
Figure 7e is the period where 15 comuniust countries became 35 countries, where most went through a rather painful transition to capitalism. This is a major group of middle income countries with *y* between 7 and 9 as shown in Table 5. Nearly all of these countries saw a large initial fall in GDP. This appears to explain the strange zig-zag path of the kernel on Figure 7e.

7. Robustness 3: Varying the bandwidth

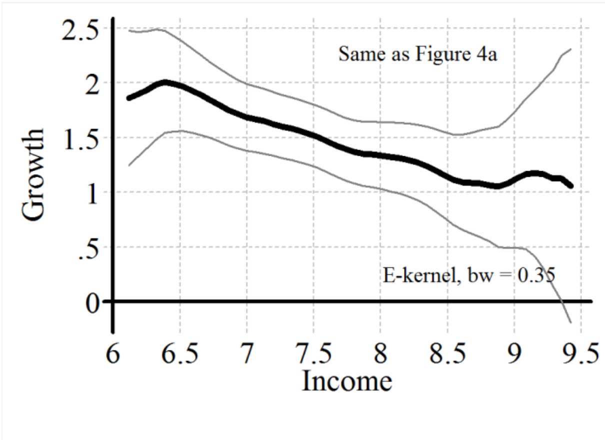
The stata-program chooses a kernel bandwidth close to 0.35 for both parts of the data. The experiments reported look at bw = 0.2, 0.35, 0.5, and 0.7.

Figure 8. Part 1: The kernel with four bandwidths, cf. Figure 5a

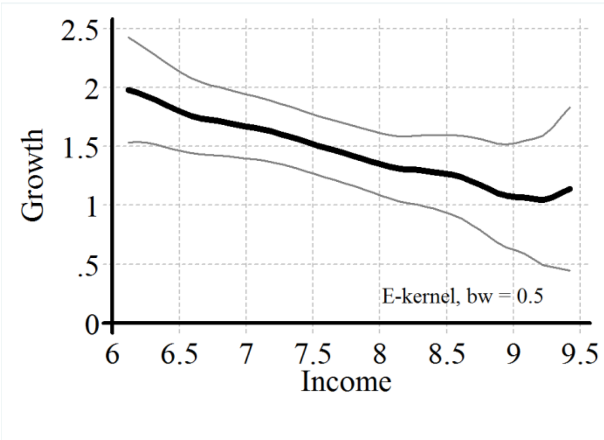
8a. For bw = 0.2



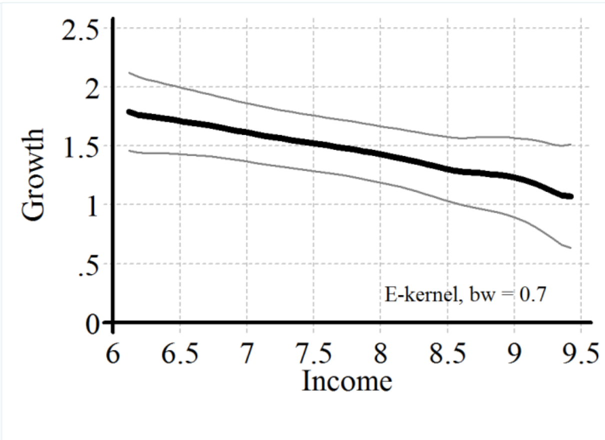
8b. For bw = 0.35 same as Figure 5a



8c. For bw = 0.5



8d. For bw = 0.7

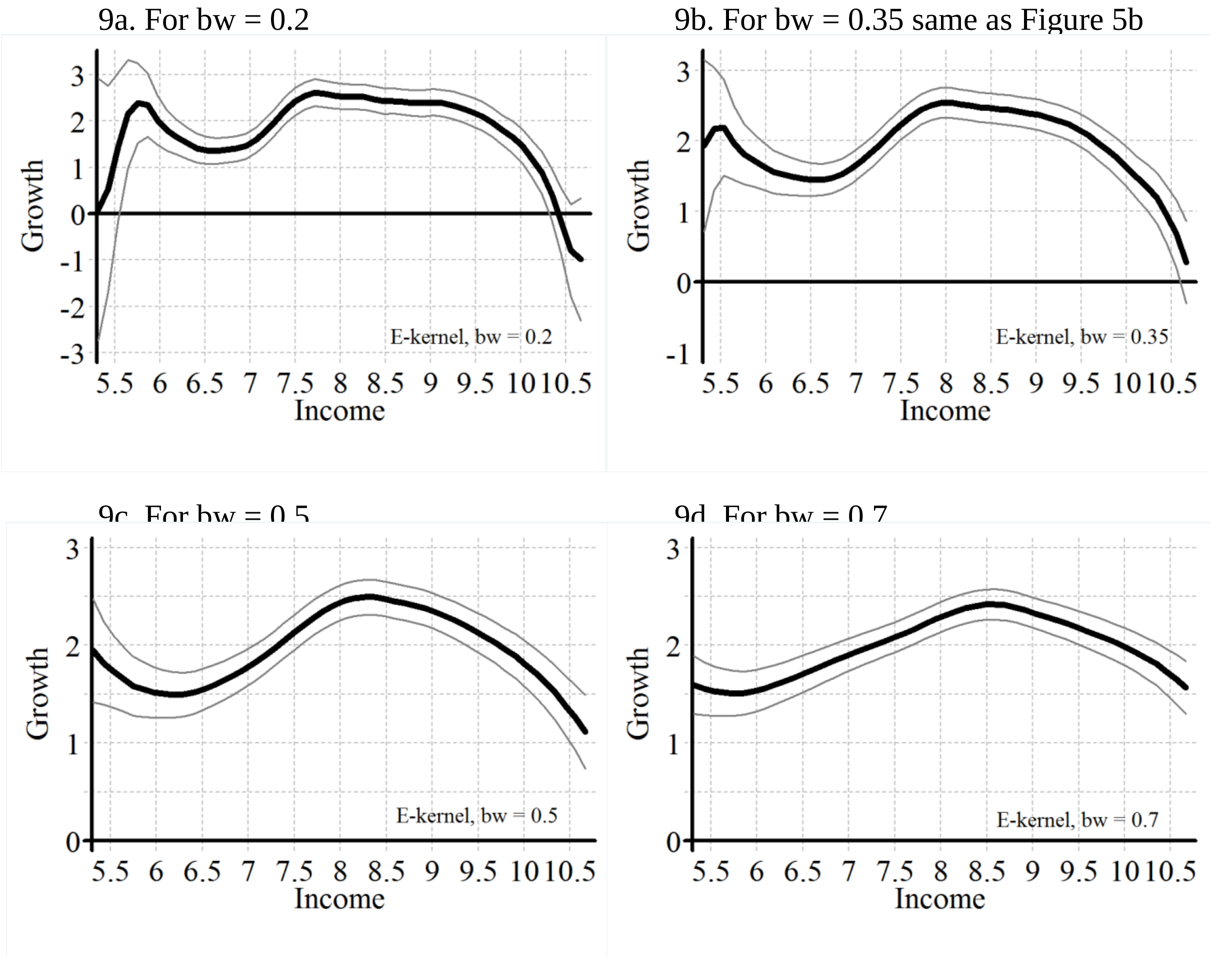


For easy comparison Figure 5a is repeated as Figure 8b. The other three kernels look rather like the original kernel with bw = 0.35.

In all cases the kernel starts around g = 2 and falls throughout the range. For bw = 0.2 the kernel becomes more wobbly and takes a dive at the end, but here the confidence interval widens. So basically all four kernels end at g = 1. Thus, the graphs confirm the impression till now that the $g = g(y_{t-1})$ relation is a straight line with a negative slope.

Figure 9 is the same exercise repeated for Figure 5b. All kernels have a basic hump-form, but the curves differ a bit at the low end, where the confidence intervals are rather wide.

Figure 9. Part 2: The kernel with four bandwidths, cf. Figure 5b



In all cases the top of the kernel is around 8. The convergence of the rich countries is rather strong throughout, but the divergence to the left of the top is a bit less clear. However, for the two largest bandwidths the wobble for the poorest countries goes away.

As Stata chooses 0.34 it is worth to consider this choice. Clearly a higher choice such as 0.5 makes the kernel simpler in form and thus easier to read, while lower bws make the kernel more wobbly. It is interesting if the extra wobbles makes sense. However, the key observation is that the basic picture is almost the same from 0.30 to o.0.5 and fairly similar all the way from 0.2 to 0.7.

8. Getting wealthy from resource rent

As mentioned in section 2, some countries become rich from resource rent without a Grand Transition. Above we have considered the univariate relation $g_i = g(y_{i(-1)})$. Now one more variable enters. It is, r_i, the share of y generated by resource rent. It enters in a complex way, as resource rent typically enters through the treasury as a resource tax. It is partly passed on to people in the form of transfers, and partly used to finance some parts of the transition, i.e., it accumulates in the form of physical capital, such as buildings and infrastructure, and gradually also in the form of human capital. However, the full accumulation of skills and the transition in all fields that constitutes the Grand Transition is a very complex process. Thus, it requires detailed data to explain how the resource rent enters into the relation analyzed.

To stay within the simple framework used, we have simply used OPEC membership to sort the data. Table 6 lists the present and past members of OPEC. 563 observations of the 8,878 from Part 2 are for OPEC countries.

The OPEC observations are used for Figure 10a. 563 observations are too few for a full randomization of countries and time, so it is less reliable than the other kernel-curves reported. The OPEC kernel shows a rather clear case of convergence: It has a negative slope throughout, and it even intersects the horizontal axis a little above the middle, so the convergence is to y = 9.3. Clearly, oil-countries have a growth problem. It explains the difference between Figure 5b and Figure 10b.

When the OPEC observations are deleted from the data, Figure 10b results. It is almost the same as Figure 5b, except at the high end. The non-oil kernel does not go down as much as when data includes all countries. As the high end is where the most resource rich countries are concentrated, this is precisely where the difference should appear. In the rest of the range the oil-countries are just a few observations among many.

Table 6. OPECs list of present and past members

Country	From	To	Country	From	To
Algeria	1969	Now	Kuwait	1960	Now
Angola	2007	Now	Libya	1962	Now
Ecuador	1973	1992	Nigeria	1971	Now
Ecuador	2007	Now	Qatar	1961	Now
Gabon	1975	1995	Saudi Arabia	1960	Now
Indonesia	1962	2009	UAE	1967	Now
Iran	1960	Now	Venezuela	1960	Now
Iraq	1960	Now	15 countries	563 observations	

Source: OPEC home page at URL: http://www.opec.org/opec_web/en/about_us/24.htm.

Figure 10a. Part 2: The kernel for the OPEC observations, *N* = 563

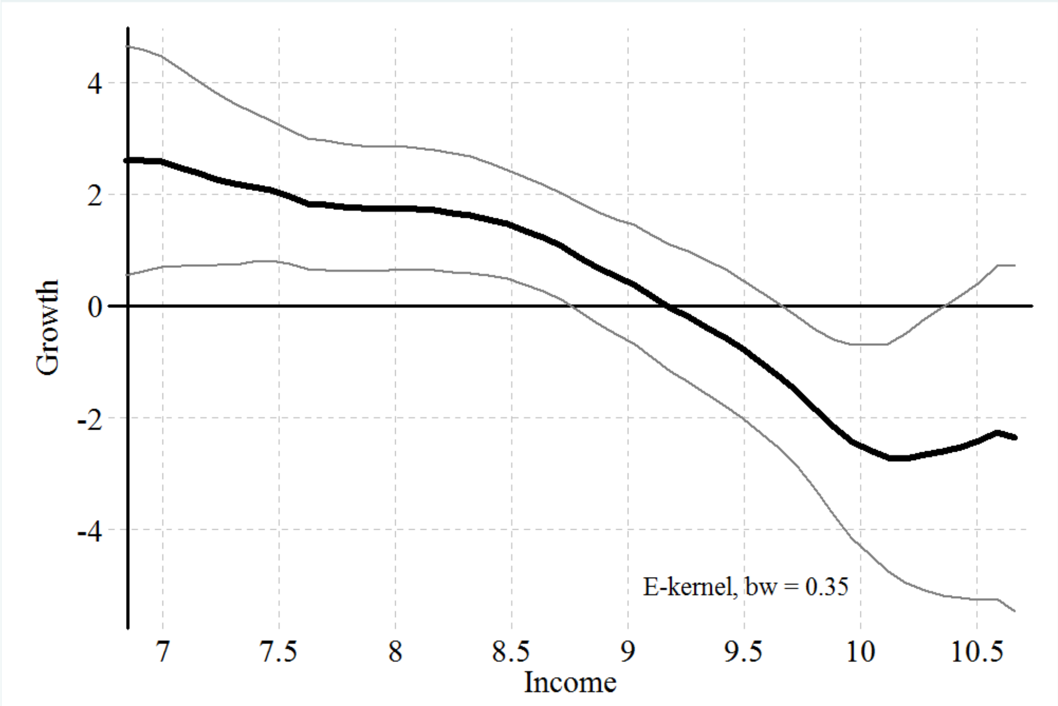
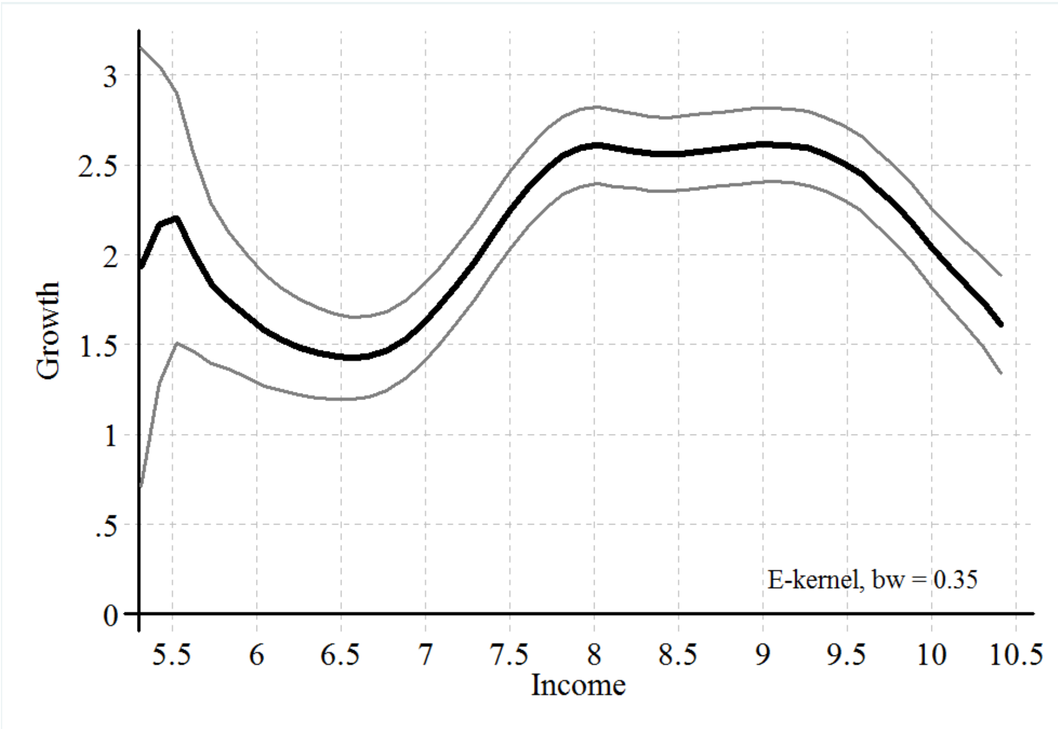


Figure 10b. Part 2: The kernel from data without the OPEC observations, cf. Figure 5a

Replicated as Figure 3 in main paper



9. The standard deviation of the growth rate as a function of income

To analyze the variation in the growth rate, we have calculated a *moving standard deviation* $(y_{i(-)}, g_i)$, where n is the number of observations used in the calculation. Below we use $n = 11$. The (y_i, s_{ni})-series is made and analyzed as follows:

(1) The $(y_{i(-)}, g_i)$-series is sorted by y as before.

(2) Then the g_i-series is replaced by the standard deviation $s_{11}(g_{i\text{-}5}, g_{i\text{-}4}, \ldots, g_i, \ldots, g_{i+5})$. Hereby the first 5 and the last 5 observations in the series are lost.

(3) Finally, the (y_i, s_{11i})-series is analyzed by the same kernel-technique as before.

Note that the (y_i, s_{11i})-series uses 11 observations for the calculations of the std. We have also made the calculations with 51 observations, but it had no visible effects on the std-kernels shown. We first look at the std-kernel corresponding to Figure 5a.

Here the pattern is rather dull. Though it is not a fully flat and horizontal curve, it does not deviate much. However, when the same exercise is repeated for Part 2 of the data, an interesting pattern appears. The std-kernel has a clear path. It increase till the peak that occurs at $y = 7.2$, well before the growth peak, and then it falls.

Figure 11a. Part 1: The std-kernel, cf. Figure 5a

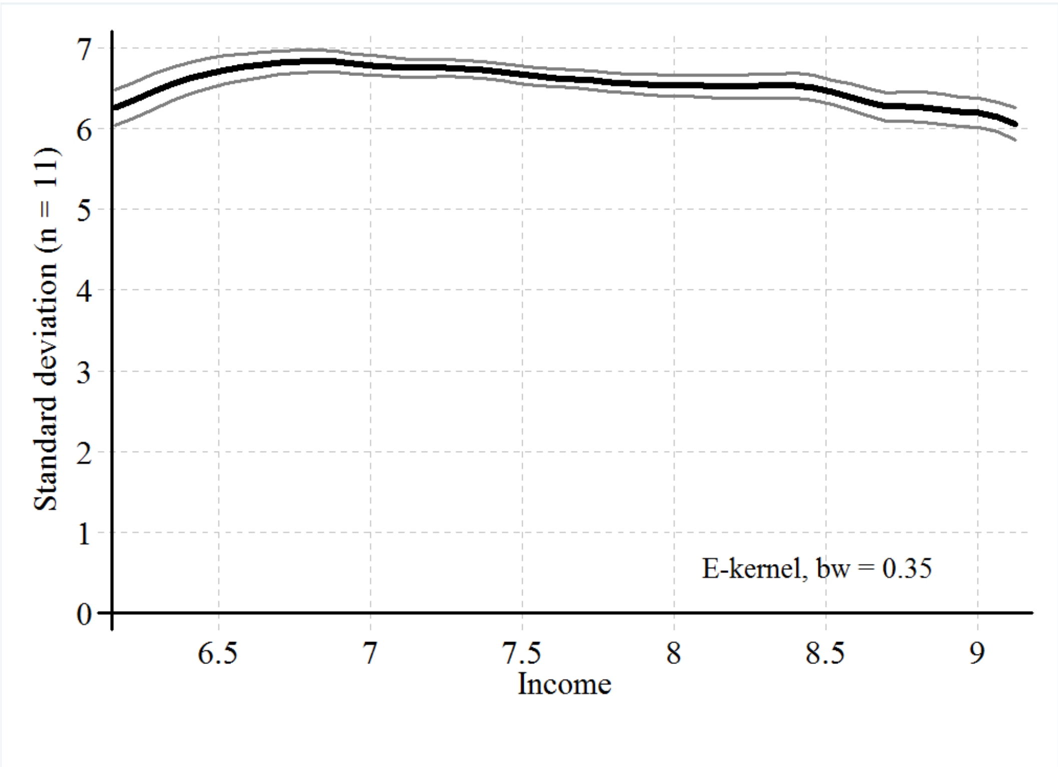


Figure 11b. Part 2: The std-kernel, cf. Figure 5b

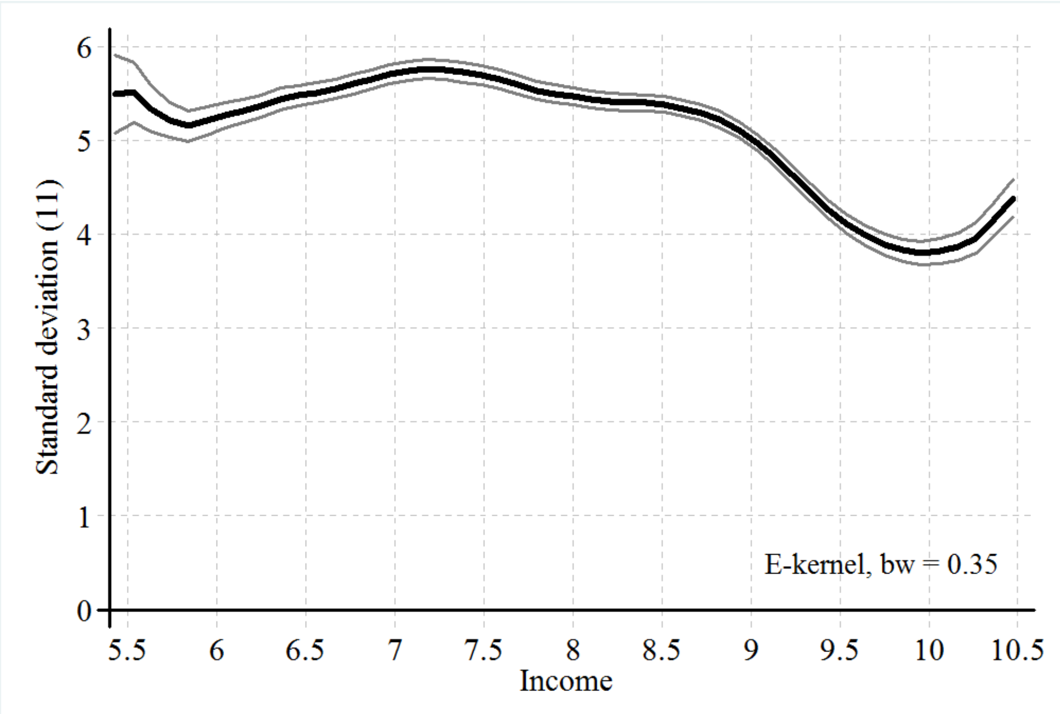
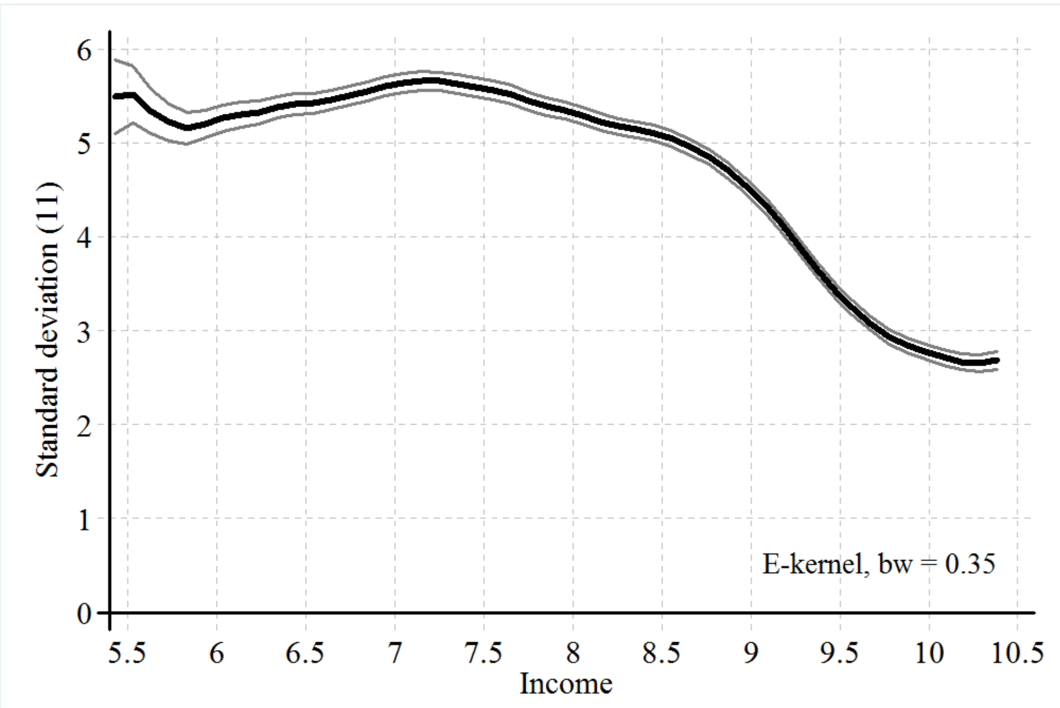


Figure 11c. Part 2: The std-kernel without OPEC, cf. Figures 10b and 11b

Replicated as Figure 4 in the main paper



When the OPEC data is deleted in Figure 11c, the pattern becomes even stronger. Now the fall in the std is two times, and the confidence intervals become very narrow at the end.

10 Conclusions

The above analysis is done using a technique chosen to be as a-theoretical as possible. We hope the reader will agree that the analysis builds on few assumptions. This allows us to draw rather strong conclusions about economic theory.

Part 1 of the data, before 1950, is a sample of countries that is heavily concentrated on Western countries. It is therefore unrepresentative as shown.

Part 2 of the data, after 1950, is a rather full sample covering countries with more than 95% of the world population. Here the data shows a rather strong transition that is especially clear when the OPEC countries are deleted: this gives Figures 10b and 11c that are the 'premium' figures in the paper.

Poor countries (LICs) have a moderate and unstable growth. However, the growth is still at an average rate of 1.6% per year. So the stagnating traditional economy is all but gone in the world of today.[16]

Rich countries (HICs) have a growth rate of much the same 1.6 as well. The variation around that growth is much smaller than the one for the LICs and the MICs.

Middle-income countries (MICs) have higher growth with on average 1 percentage point. The peak is rather flat, and on Figure 9a and Figure 10b it appears that it is somewhere between y = 8 to 9. Here growth is rather variable, but rapidly falling as income increases.

An average excess growth rate of 1% reduces the gap to the top by 2.7 times over a century, which is 1 logarithmic point. The difference from the poorest to the richest countries is 4 logarithmic points. So the full process will take 400 years. However, the variation is large, so some countries do it much faster and others not at all.

With all said, our analysis shows why the group of high income countries keeps increasing.

[16]. However, the world still has about 10 countries where gdp is lower in 2010 than in 1950.