A simple Trade Model with the Notion of Adaptability

－Endogenous Quality and Productivity－

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Abstract

This paper uses the notion of adaptability which means abilities to understanding, judgement, analyzing and management for production processes. The notion includes sophisticated and complicated task.

By using the notion, we present a new international trade model with endogenous quality and productivity. This paper gives new and wide insights to international trade researches.

We find that relative unit-value added to wage is more accurate index for international competition than relative productivity. We apply our model to intermediate goods and clarify some insights for them

This paper intends to explain recent focused phenomenon, and different productivities and qualities with differentiated goods.

JEL Codes: F10, F12, F23

Key Words: adaptability, endogenous quality, endogenous productivity, intermediate goods, monopolistic competition.

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1. Introduction

The content of international trade has been changing together with the increase in the degree of trade dependence. Advances in information technology and logistics make the economic distance shorter. It promotes the integrated global and it activates the offshore outsourcing. The location of each part of the production process starting from the design and engineering has been possible in the whole world. It is called *trade in tasks* (Grossman and Rossi-Hansberg(2008)).

Baldwin and Ito (2011) classify export goods by quality and price competition. They show that EU major countries have a high proportion of quality-competition goods compared to the other countries. The evaluation of the high price exported goods depends significantly on whether the market competition is price or quality. The lower price goods are superior if the market is the price competition, but if the market is the quality competition, the higher price goods tends to increase value added.

Hallak (2006) finds that rich countries tend to import relatively more from countries that produce high-quality goods. Bastros and Silva (2010) shows that higher-productivity firms tend to ship greater qualities at higher prices by using Portuguese firm-level-data[2]. Feenstra and Romalis (2014) estimate quality and quality-adjusted price by developing a new methodology for quality index. They show that value added and export price in China are still much lower than in developed countries in spite of steep development of China.

On the other hand, demand for skilled workers is growing worldwide in recent years (Dickerson and Green (2004)). There would be correspondence between the increase in

[2] Martin and Mejian (2012) show similar results using firm-level data for France. Schott (2008) confirms that high-wage countries tend to produce better qualities when they compete with low-wage countries' products.

demand for the skilled workers and the fact that developed countries have experienced a specialization in higher quality and sophisticated products. In other words, to produce sophisticated goods, high skilled labors are more necessary.

From these literatures, I interpret that some of recent key words for international trade are quality-competition, trade in tasks, sophisticated products. I provide a new comprehensive model which can explain the above empirical papers and recent phenomenon. Our model considers endogenous decisions for quality and productivity by using new concept "adaptability". In this paper, we define adaptability as the ability to understanding, judgment, adjustment, analyzing for many production process and management[3]. In other words, it means the competence of finding possible solutions and making optimal choices among the solutions, according to each situation. This notion includes both skill in Dickerson and Green (2004)[4] and specific skill for specific jobs and tasks. We treat firm heterogeneity by assuming of adaptability difference for each firm. Hallak(2006) provides empirical framework to estimate the prediction in which quality plays an important role. He confirms some theoretical prediction.

By using this notion, we present a new international trade model with endogenous quality and productivity. The model consists of one type labor with many goods, small country and monopolistic competition. Making quality and productivity to be endogenous, we show that there are price (quality)-productivity lines which reveal

[3] Similar concept is "capability" of the firm to produce a given level of quality in Crozet, Hatte and Zignago (2013).

[4] They show the importance of generic skill which is computing skills, literacy, numeracy, technical know-how, high-level communication skills, planning skills, client communication skills, horizontal communication skills, problem-solving, and checking skills. Most important skill of skilled workers is a high level communication which can persuade colleagues and customers.

production specializations according to wage up.

On the other hand, recent theoretical papers explain the reason why richer countries export higher-quality goods[5]. Das and Donnenfeld (1987) show that a specific tariff leads to higher quality of import goods based on quality control of imports. Murphy and Shleifer (1997) present a model of trade in which high human capital provides the link between tastes and endowments. Fajgelbaum, Grossman and Helpman (2009) show that an income distribution relates to the fractions of consumers who buy higher-quality goods[6]. These theoretical analyses provide a consumer demand-based explanation for the pattern of trade in goods of different quality. Our model is based mainly on a supply side. According to some international trade data, the fraction of consumer goods for international trade is not large[7]. Main of international tarde is intermediate goods and capital goods, and it is the trade between producers.

The paper finds that relative unit value added to wage can be more accurate index for international competitiveness than relative productivity in a Ricardian model for one kind of labor models. In our model, the goods with higher relative unit value added is more competitive and the lower goods is more inferior under some conditions. Smile Curve analyses are concerned for trade in tasks as in Baldwin et.al (2014). Rich countries may worry about the loss of high value added task. On the contrary, low-wage nations worry about getting low value added task. Then what is the source of decision for task allocation among countries? Our paper shows that the key is relative unit value added to each wage.

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[5] The theoretical pioneer literature on quality is Linder (1961).

[6] Their model implies that trade liberalization benefits the poorer households in wealthy countries and the richer households in poor countries.

[7] According to UNIDO (2011), the share of consumer goods is about 30% for world.

Our paper re-evaluates the role of intermediate goods in our globalization world[8]. In our model, intermediate goods are classified to three types. Intermediate goods can promote technological progress and explain quality ladders for one firm. It can provide theoretical analysis for *a rising price-distance link*[9]. Moreover our model can analyzes offshore outsourcing consistently.

We attempt to show the role of human resources and management, which are included in the definition of adaptability, in our global society. We find that our model can theoretically explain recent focused phenomenon which includes vertical-horizontal trade, quality, intermediate goods trade and offshoring. This paper gives new and wide insights to international trade theory and empirical research. It is useful for policy makers who consider not only trade policy but also whole policies of development, education, human recourses and firm competitive.

The rest of the paper is organized as follows, Section 2 presents basic model. Section 3 presents optimal input condition. Section 4 considers some comparative statics. Section 5 classifies intermediate goods and shows the new roles of it. We have seven Propositions.

2. Basic Model

2.1. Preferences and Demand

We use usual CES utility function while quality is added. Preferences for

[8] The empirical literature on intermediate is Amiti and Konings (2007) which pertains to this paper.

[9] If the goods is quality-competitive, the highest priced goods travel the furthest (Baldwin and Ito (2011)). This type of trade goods is characterized by *a rising price-distance link*.

variety i are given by a utility function in h sector for each country m.

$$u^m = U_h^m(x_1^m, x_2^m, x_3^m, \cdots, x_n^m)$$
$$= U_h^m([\sum_{i=1}^{n} (q_i x_i^m)^\rho]^{1/\rho}) \quad (1)$$

where $x_1^m, x_2^m, x_3^m, \cdots, x_n^m$ are differentiated goods in the sector, ρ is love-of variety parameter with $0 < \rho < 1$, n is the number of variety. q_i and x_i^m are the quality and quantity of variety i in h sector. x_i^m is supposed to be produced by a firm which is located in domestic or foreign countries[10]. We assume that the firm of i variety supplies or exports $\sum_{m=1}^{z} x_i^m$ with quality q_i. The variety price p_i is same for all counties from m = 1 to m= z if there is no transportation cost[11]. In subsection 2.2 we explain it in detail.

We assume the utility for each sector is maximized for given sector expenditure E_h, each variety price and quality. By using (1), usual optimal condition with quality is given by[12]

$$q_i^\rho (x_i^m)^{\rho-1}/p_i = q_j^\rho (x_j^m)^{\rho-1}/p_j \quad (i, j = 1, 2, n). \quad (2)$$

Since we focus on supply side and one variety firm in the paper, we don't analyze demand system more over. In the following, we consider only one variety i. It is supposed that

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[10] This quality specification is shown as in Hallak(2006).
[11] In the later, this assumption is relaxed.
[12] We obtain from the Lagrangian $F(x_i, E_h, \lambda) = U_h^m([\sum_{i=1}^{n}(q_i x_i^m)^\rho]^{1/\rho} - \lambda(\sum_{i=1}^{n} p_i x_i^m - E_h)$.

$dx_i^m/dq_i > 0$ for constant p_i, (3)

$dx_i^m/dp_i < 0$. (4)

Then we have demand function for x_i^m

$$x_i^m = x_i^m(p_i, q_i) \quad (5)$$

Aggregate demand of i variety for many countries is expressed as

$$X_i^D \equiv \sum_{m=1}^{z} x_i^m(p_i, q_i), \quad (6)$$

2.2. Firms and Production

It is supposed that i variety is produced by i firm in a country. The firm produces only one variety i. Factor of production is one type of labor[13]. L_i denote total employment of a firm which products i variety of h sector[14]. Using L_i, we define a kind of aggregate human resource function, R_i, as

$$R_i = R_i(L_i) \quad \text{with} \quad R_i' > 0, R_i'' < 0. \quad (7)$$

Using this function, we define a nominal adaptability function, H, as

$$H_i = \beta H_i (R_i/L_i, R_i), \quad \text{with } \partial H_i/\partial(R_i/L_i)) > 0, \partial H_i/\partial R_i > 0, \quad (8)$$

[13]Although there are many models with skilled and unskilled labors, we don't consider it in our model for simplicity.

[14] In the following we omit h.

where β is a parameter. The nominal adaptability is interpreted as abilities to understanding, judgment, analyzing and management for production processes. We assume that each personal human resource is same. Then the equilibrium is only equal marginal product of labor for many firms. It is assumed that the nominal adaptability depends on its average value, R_i/L_i, and its absolute value. The term R_i/L_i declines for many employments because of the assumption, $R_i'' < 0$. We assume $R_i'' < 0$ since too many quantity of labor would reduce the marginal adaptability of labor. H_i is raised monotonically for small value of R_i. However as the firms employ labors more, average aggregate human resources, R/L, decreases in the end, so that H_i may declines because of lower average human resources. We rewrite H_i function as

$$H_i = \beta H_i (L_i), \quad \text{with } H_i' > 0 \text{ for small } L_i \text{ and } H_i' < 0 \text{ for large enough } L_i. \qquad (9)$$

We introduce a new concept of difficulty and complexity to accomplish jobs. It is necessary information to understand and accomplish jobs. We denote it as I. It is specific for the sector[15].

Using I, we introduce a key concept of our paper. We call it "adaptability" which is denoted as γ_i. γ_i is defined as $\gamma_i = \beta H_i (R_i)/I = \beta H_i (R(L_i))/I$. γ_i is denoted as

$$\gamma_i = \beta H_i (L_i)/I. \qquad (10)$$

[15]Many goods with different quality are produced in a sector. The difficulty level, "I", is altered according to each product' quality, since high "I" is necessary to produce high quality goods. I_i would be more an accurate notation. However in our paper, we use only I for simplicity.

γ_i is relative not absolute since it is divided by I. Even if a firm has high nominal adaptability, the performance of the job is not successful when complicated management and sophisticated information technology are necessary for high I. One of characteristics in this paper is introducing the notion of sophisticated and complicated knowledge. If the job and the task are more difficult and sophisticated, the performance is lowered compared to easier tasks since the degree of the adaptability is lowered. The adaptability is integrated and aggregate effective skills. The adaptability consists of nominal adaptability and difficulty to conduct a job and it is increasing for nominal adaptability improvement and decreasing for difficulty job. For simplicity, i firm produces only one quality goods among many qualities in the sector.

Noting that $\gamma_i = \partial\gamma_i/\partial L_i$ is not always positive since we find $H_i' \equiv \partial H_i/\partial L_i$ could be negative from (10) for large L_i. It is interpreted as reducing of adaptability on average. More labor input expands production while average aggregate human resource, R_i/L_i, can be reduced.

We assume that each firm is heterogeneous and the production functions for each firm is different. So we express each H function as $H_i(L_i)$ and not H_h. We can interpret that adaptability and management ability differs for each firm. It is assumed that production function is a modified Ricardian model and the factor is labor only[16]. In this paper, each firm's production function is different and γ_i is added to traditional one. Unit labor requirement or productivity is endogenous and it is a function of adaptability. Then production function of the i variety firm and h sector, x_i, is defined as

[16] Later, we introduce intermediate inputs.

$$X_i^S \equiv a(\alpha_i\gamma_i)L_i^{\tau}, \qquad (11)$$

where $a(\alpha_i\gamma_i)$ is labor productivity with $a' > 0$. τ and α are respectively between 0 and 1. α is a weight parameter between productivity and quality improvement. If $\alpha = 1$, all effort is devoted to productivity improvement, and if $\alpha = 0$, it is visa-verse.

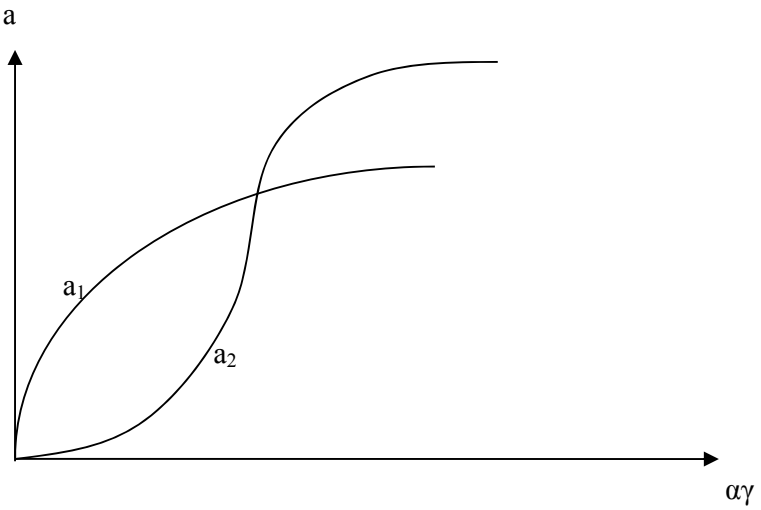


Fig. 1.Productivity functions.

Fig.1. exemplifies productivity functions, $a(\alpha\gamma)$. We can interpret the first goods as low technology goods since lower effort to productivity can achieve enough productivity. The second goods would be vice versa and complicated goods since much effort is necessary to achieve enough productivity.

2. 3 Quality and price function

Quality function is defined as

$$q_i = q_i((1-\alpha_i)\gamma_i) \quad \text{with assumptions of } q_i' > 0 \text{ and } q_i'' < 0. \qquad (12)$$

It represents the index of non-price competition. From (1) we find that q is effective quality. Sometimes even high quality goods can't gain more profit since consumer don't evaluate it. In the case the high quality goods are not effective and q is kept low. From (5), (6) and (12), and market equilibrium condition, $X_i^D = X_i^S \equiv X_i$ for i variety, we obtain

$$X_i = \sum_{m=1}^{z} x_i^m(p_i, q_i) = a(\alpha_i\gamma_i)L_i^{\tau} .^{17} \quad (13)$$

It is assumed that a firm produces only one quality goods and that it maximizes its profit by choice of optimal α and L. We can express p_i from (13) as

$$p_i = p_i\,(q_i\,((1-\alpha_i)\gamma_i)),\ X_i) \quad (14)$$

$$\text{with } \partial p_i/\partial q_i \geq 0,\ \partial p_i/\partial X_i \leq 0,$$

It is assumed that the firm's price with better quality is higher. On the other hand, if consumers don't demand high quality goods, $\partial p/\partial q$ is lower and the firm would hesitate improve quality.

Fig.2. presents examples of the relationship between quality and effort $\{e \equiv (1-\alpha_i)\gamma_i\}$ to quality for variety 1 and 2. Since q is effective, more e doesn't always makes the quality hence. If consumers don't demand higher quality goods and $\partial q/\partial e = 0$ for more than e_1, the q(e) function is like q_1. The $q_2(p)$ curve is the vice versa and consumers would not demand nothing for less than q_2. For $q_2(p)$ curve, lower q is not profitable

[17] In the following h is omitted.

since q is lower compared to much labor to acquire a given quality.

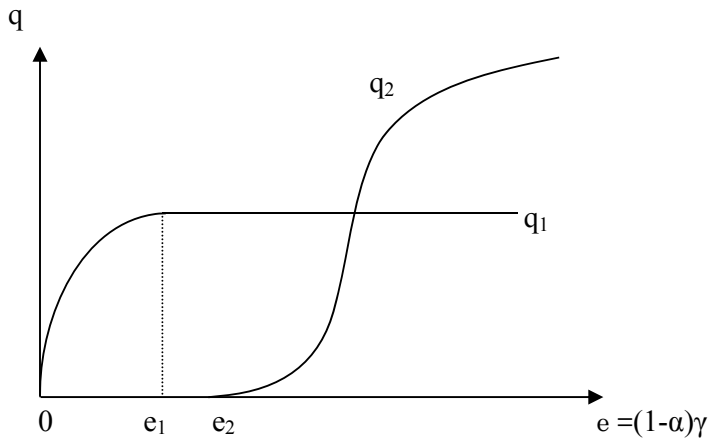


Fig.2.The Effort and quality relationship.

The firm's strategy is choosing quality with optimal labor and α. This is considered in next section. Varity 1 would be lower quality goods than variety 2.

3. Optimal condition

This section considers an optimal decision of α and L for profit maximization. Subsection 3.1. considers optimal labor input. In subsection 3.2., we derive unit revenue isoquant line which represents optimal combination of price (quality) and productivity. In subsection 3.3., we classify trade patterns from the view point of price (quality) and productivity.

3.1 Optimal labor input

In the following we omit i. Profit function is shown as

$$\Pi = p(q((1-\alpha)\gamma), X)X - wL,$$
$$= p(q((1-\alpha)\gamma), a(\alpha\gamma)L^{\tau})\, a(\alpha\gamma)L^{\tau} - wL \quad (15)$$

where Π is profit for i firm in h sector. We assume that the wage, w, is fixed for a country. Using profit function (15), we obtain maximum condition for L of i firm in h sector as

$$d\Pi/dL = (ap_q q'(1-\alpha) + pa'\alpha)\, L^{\tau}(\partial\gamma/\partial L) + p_X\{a'\alpha L^{\tau}(\partial\gamma/\partial L) + \tau a L^{\tau-1}\} + p\tau a L^{\tau-1} - w. \quad (16)$$

There are four effects. First term, $ap_q q'(1-\alpha)\, L^{\tau}(\partial\gamma/\partial L)$, is quality effect. Second term, $pa'\alpha L^{\tau}(\partial\gamma/\partial L)$, is productivity effect. Third term, $p_X\{a'\alpha L^{\tau}(\partial\gamma/\partial L) + \tau a L^{\tau-1}\}$, is price effect and fourth $\tau p a L^{\tau-1}$, is direct revenue effect. Quality, productivity and effects depend on the sign of H' since $(\partial\gamma/\partial L) = \beta H'/I$[18].

From (16), we find that there two types of equilibrium. If the sign of H' is negative, the marginal direct effect would be larger compared to the case of positive H' under some conditions. Since direct effect is related to quantity, we can interpret that firms' marginal expanding production by additional labor is large while average labor quality is lowered from $H' < 0$. On the other hand, for $H' > 0$, both of direct effect and average labor quality are improved.

Then we have Proposition 1.

Proposition 1

[18] $\Pi_{LL} \equiv d(d\Pi/dL)/dL$ is negative from the second order condition of equilibrium although the sign of Π_{LL} is ambiguous.

The optimal conditions for labor input are consist of quality and productivity improvement, and the usual terms, that is, price effect and marginal product of labor.

3.2. Trade-off between quality and efficiency

In this subsection, we analyze the optimal allocation between productivity and quality under given L. From (16), then profit maximization for α under constant L is given by

$$\partial(\Pi)/\partial\alpha = -p_q q' \gamma X + (p_X X + p)\partial X/\partial\alpha, \quad \text{with } \partial X/\partial\alpha = a' \gamma L^{\tau}. \tag{17}$$

In the equation, the first term is quality reduction effect, second is price effect of production and third is productivity improvement effect. If quality effect (q') is small and/or price effect of quality improvement (p_q) is small, α would tend to be larger. On the other hand, if productivity improvement is not expected, α tends to be smaller.

Eq.(17) is rewritten as

$$\begin{aligned}\partial(\Pi)/\partial\alpha &= a\gamma L^{\tau}(-p_q q' + p_X X a'/a + pa'/a) \\ &= apL^{\tau}(-p_q \gamma q'/p + p_X X \gamma a'/ap + \gamma a'/a) \\ &= apL^{\tau}\{-\eta_{pq}\eta_{q\gamma} + (-\eta_{pX} + 1)\,\eta_{a\gamma}\} = 0.\end{aligned} \tag{18}$$

where $\eta_{pq}(\equiv qp_q/p > 0)$ is quality elasticity of price, $\eta_{q\gamma}(\equiv \gamma q'/q > 0)$ is quality elasticity of adaption and $\eta_{a\gamma}(\equiv \gamma a'/a > 0)$ is productivity elasticity of adaption, and η_{pX} ($\equiv p_X X/p$) is usual demand elasticity of price.

Then we have Proposition 2.

Proposition 2

There is trade-off between quality and productivity. The optimal allocation conditions between quality and productivity depend on elasticity of price-quality, quality-adaptability, demand and productivity-adaptability.

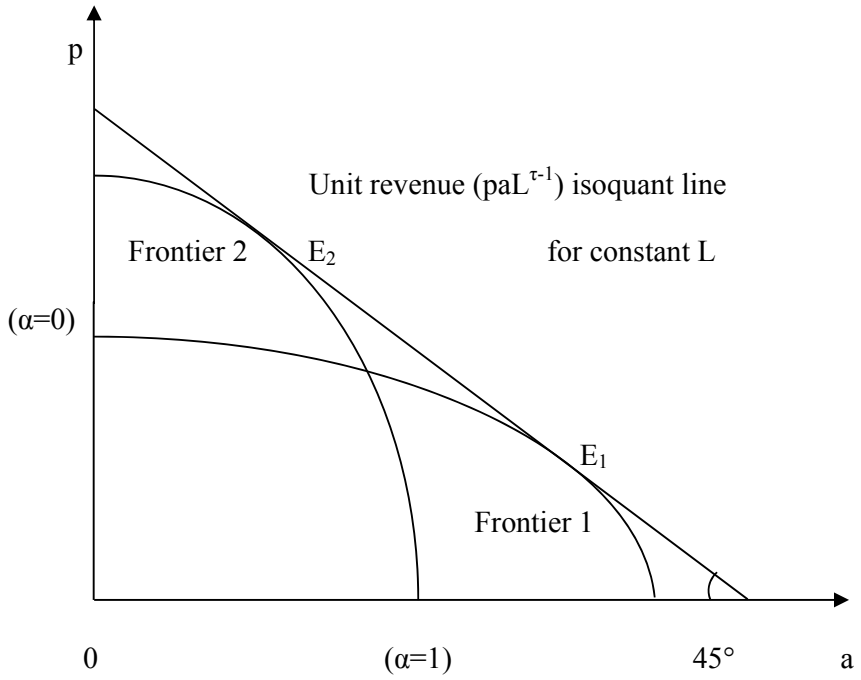


Fig. 3.Price - productivity (quality-efficiency) frontier for given L.

Fig. 3. shows that the quality of production depends on price (quality) – productivity (efficiency) frontier. Frontier 1 refers to 1^{th} firm in h sector. Frontier 1 has more productivity advantage compared to Frontier 2. As a result, firms which have characteristics of Frontier 1 produce high productivity goods and less quality goods. If α is slightly apart from optimal value, profit is reduced since marginal revenue of α is decreased from the second optimal condition, $\partial(\partial(\Pi/L)/\partial\alpha)/\partial\alpha < 0$. So the shape of

frontier would be convex. Unit revenue is defined as $paL^{\tau-1}$ which is revenue per labor. Since the cost, wL, is fixed for constant L, we consider revenue maximization only. For simplicity, unit revenue line is the same tangent for two frontiers[19].

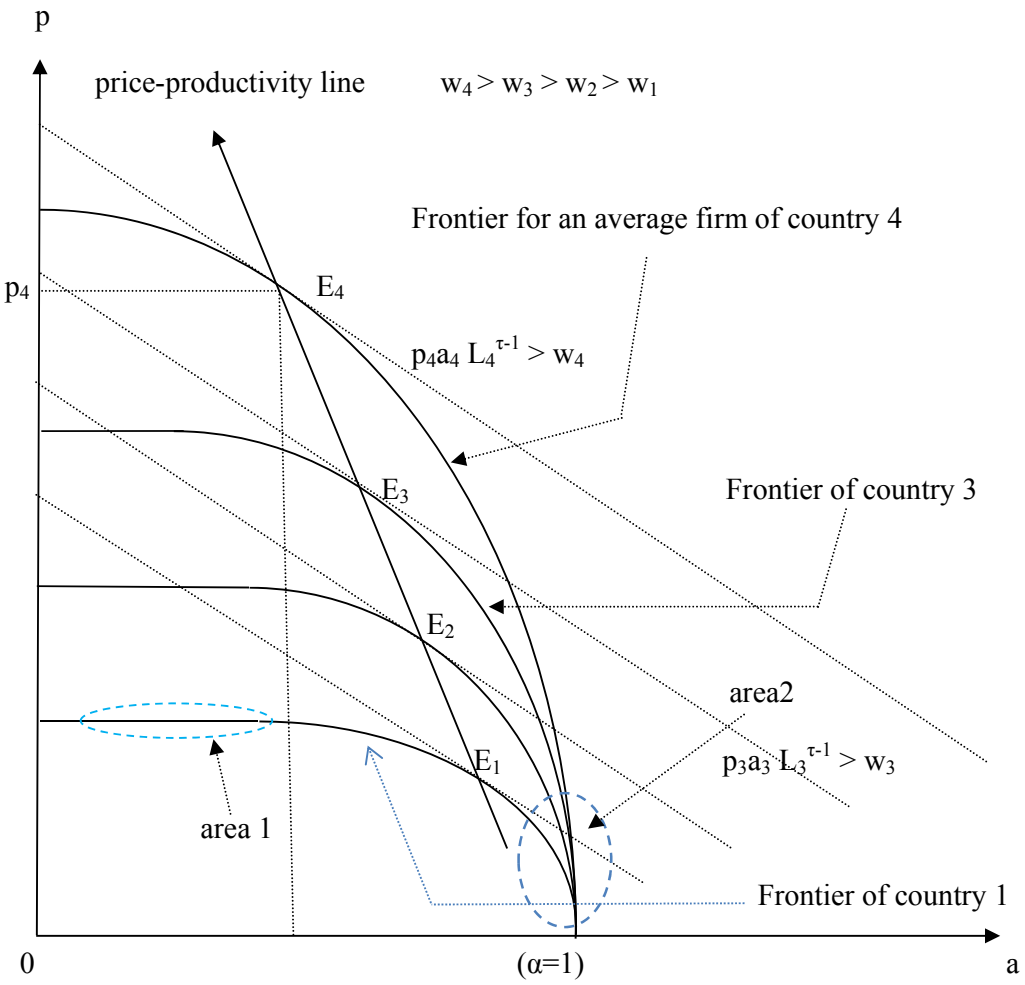


Fig. 4. Price-productivity line for different γ and wage.

For $paL^{\tau-1} > w$, the firm has positive profit. In our paper, this can be equilibrium even for free entry assumption. Since production function and adaptability of each firm are

[19] In the Fig., we assume that $p = 0$ for $\alpha = 1$, and $a = 0$ for $\alpha = 0$.

different. The superior firms can achieve positive profit even for free entry. If a firm has negative profit, that is, $paL^{\tau-1} < w$, the firms exit from the market or change the labor input. If many firms over the world have negative profit and intend to exit from the market, world price would be raised. In the case, price-productivity frontier for given L is expanded outward.

In the Fig.4., there are four countries and one firm in each country for the sector. The wage for country 4 is highest and that of country 1 is lowest. It is supposed that $da/d\alpha$ is close to zero for near to $\alpha = 1$, and $dp/d\alpha$ is close to zero for near to $\alpha = 0$. In other words, it is assumed that more effort on quality makes no return for larger $(1 - \alpha)\gamma$ compared to a small effort with small value of quality. It is depicted in "area 1" in Fig. 4. In the close to $\alpha = 1$, it is "area 2" in the Figure. Then quality-efficiency frontiers for many countries which have different γ are shown as in the Figure. The firm in the country 1 has smallest γ on average, and the firm in the country 4 has largest.

According to recent papers, developed countries export high-quality and more sophisticated goods (Scott 2008). Country 4 is supposed to be a developed country and country 1 to be developing. If higher wage countries produce goods with more quality with less productivity, price-productivity line for different γ would be depicted as in the Fig. 4. When the line is upward to the right, price and productivity are not alternative. Sometimes the line is downward to the right. These are implicated to trade pattern and we considered it later.

Suppose that there is an additional fixed unit cost, C_F, for export. It is consist of transportation cost, maintaining foreign sales network, and other costs about barriers of national borders. If the unit revenue of the firm is in the region of $p_4a_4L^{\tau-1} \geq w_4 \geq (p_4a_4L_4^{\tau-1} - C_F)$, the firm can exist only in domestic market since exporting is deficit.

We can interpret equilibrium E_4 as that of average firm if there are many kinds of firms which have different varieties in country 4. In Fig. 5., we show the image of p_4 for i firm in country 4 under the assumptions of constant productivity. The region in which firm i can't exist both in domestic and foreign market are "A". The firm of region in "A" exit from the market because of its deficit. The region of "B" shows that it can exist only in domestic market, and "C" in both markets since $pL_4^{\tau-1} > (w+C_F)/a$. The region of "C" is decreased as C_F is larger.

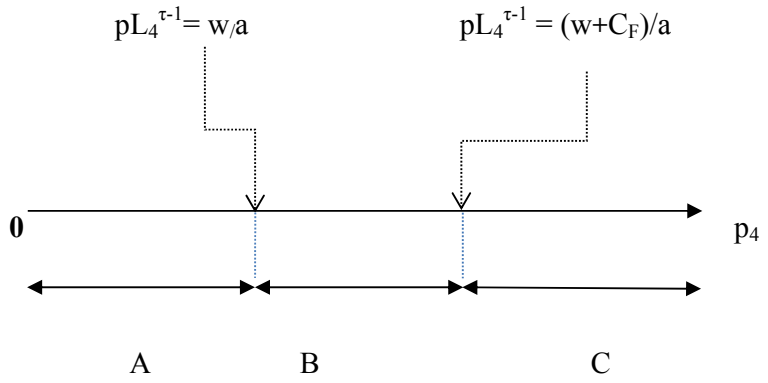


Fig.5. The image of firm exit and exist.

Before ending this subsection, we have Proposition 3. The proposition is derived from the above discussion. Since price is endogenous and p is increased as the quality is higher, the firms in high wage country must increase unit value added ($paL^{\tau-1}$) by raising p and/or productivity. In the Fig. 5, we find that firms with higher unit-value to wage can export and have competitive. Then we obtain Proposition 3.

Proposition 3

In our model with endogenous quality and productivity, and with heterogeneous firms, relative unit value added to wage, $paL^{\tau-1}/w$, can be international competitive index.

3.3 The trade pattern

Fig.6. is derived from Fig.4. From Fig.6., we find that there are three types of price-productivity lines. This line is according to wage rising from developing countries to developed. In type A, the quantity of production would be small while the price is high in developed countries. Developed countries export high price with high quality goods. On the contrary, in type B, Developing countries export high quality goods, which are in opposition to our intuition.

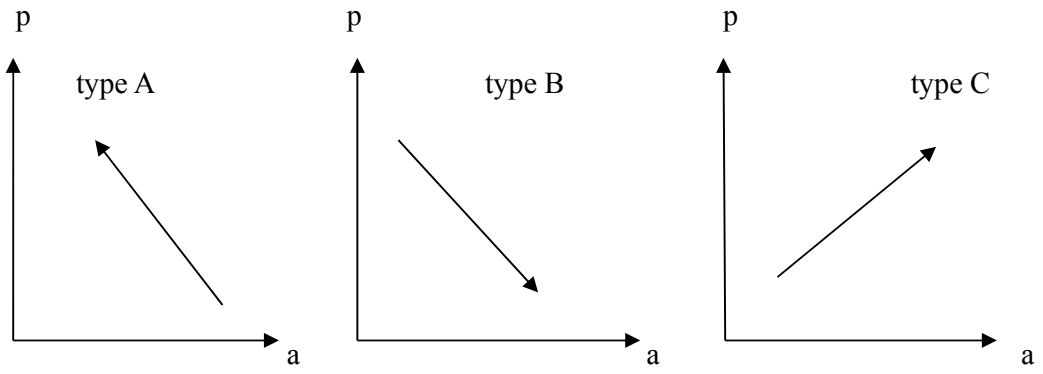


Fig. 6. Three types of price-productivity lines and trade pattern.

Using Portuguese firm-level data on exports, Bastos and Silva (2010) find that higher-productivity firms tend to ship greater quantities at higher prices to a given market. In this case, quality-productivity frontier is a balanced shape and type C. We have Proposition 4.

Proposition 4

Price-productivity relationship is classified to three patterns. Two types, A and C are

coincident with our intuition. For type B, developing countries can export high quality goods with low productivity.

4. Comparative statics

Usually we solve the following simultaneous equations.

$\Pi_{LI}dI + \Pi_{L\beta}d\beta = dL$

$\Pi\alpha_I dI + \Pi\alpha_\beta d\beta = d\alpha$

Since the calculation is complicated and we can't obtain economic intuitions, we solve each equation.

4.1. dL/dI under constant α

From (15), we have:

$$\Pi_{LI}dI + \Pi_{LL}dL = 0, \tag{19}$$

where $\Pi_{LI} \equiv d[(ap_q q'(1-\alpha) + pa'\alpha)\, L^{\tau}(\partial\gamma/\partial L) + p_X\{a'\alpha L^{\tau}(\partial\gamma/\partial L) + \tau a L^{\tau-1}\} + p\tau a L^{\tau-1}]/dI$.

The term, $d\{(ap_q q'(1-\alpha) + pa'\alpha)\, L^{\tau}\beta H'/I + \tau pa L^{\tau-1}\}/dI$ is rewritten as

$$(ap_q q_\gamma(1-\alpha) + pa_\gamma\alpha)\, L^{\tau}\beta H'/(-I^2) + \partial\{(ap_q q_\gamma(1-\alpha) + pa_\gamma\alpha)\, L^{\tau}\beta H'/I + \tau pa L^{\tau-1}\}/\partial\gamma \cdot \partial\gamma/\partial I, \tag{20}$$

where $\partial\gamma/\partial I \equiv \beta H/(-I^2) < 0$. In (22), $(ap_q q_\gamma(1-\alpha) + pa_\gamma\alpha)L^{\tau}\beta H'/(-I^2) < 0$ for $H' > 0$, $\partial(\tau pa L^{\tau-1})/\partial\gamma \cdot \partial\gamma/\partial I < 0$ and the sign of $\partial\{(ap_q q_\gamma(1-\alpha) + pa_\gamma\alpha)\, L^{\tau}\beta H'/I\}/\partial\gamma \cdot \partial\gamma/\partial I$ is

ambiguous. The sign of $d[p_X\{a'\alpha L^{\tau}(\partial\gamma/\partial L) + \tau a L^{\tau-1}\}]/dI$ is also ambiguous. We call $\partial(\tau p a L^{\tau-1})/\partial\gamma \cdot \partial\gamma/\partial I$ direct effect. Then if direct effect overwhelms the others, we have $\Pi_{LI} < 0$ and we obtain

$$dL/dI < 0. \tag{21}$$

This would be coincident with our intuition. The increase in I makes the job more difficult. Then it makes adaption γ lower, which decreases marginal labor revenue, $paL^{\tau-1}$. From $\Pi_{LI} < 0$, labor input is deceased so that marginal revenue is increased from the second order condition.

Lemma 1: *If the degree of difficulty to perform a job in a sector, I, is raised, the firms of the sector lowers its employment under plausible conditions.*

4.2. dL /dβ under constant α

The effect of β is the reveres of I from (10), $\gamma = \beta H(L)/I$. If direct effect, $\partial(pa)/\partial\gamma \cdot \partial\gamma/\partial\beta$, overwhelms the others, $\Pi_{L\beta} \equiv d[(ap_q q'(1 - \alpha)+pa'\alpha) L^{\tau}(\partial\gamma/\partial L) + p_X\{a'\alpha L^{\tau}(\partial\gamma/\partial L) + \tau a L^{\tau-1}\} + p\tau a L^{\tau-1}]/d\beta > 0$, we obtain for $H' > 0$:

$$dL/d\beta > 0. \tag{22}$$

Then we have Lemma 2.

Lemma 2: *If the human resource parameter β, is increased, the firms raises its employment under plausible conditions.*

4.3. dα/dI and dα/dβ under constant L

From (19), we have:

$$\partial(\partial\Pi/\partial\alpha)/\partial I \cdot dI + \partial(\partial\Pi/\partial\alpha)/\partial\alpha \cdot d\alpha = 0, \qquad (23)$$

where, $\partial(\partial\Pi/\partial\alpha)/\partial I = (\partial(\partial\Pi/\partial\alpha)/\partial\gamma)(\partial\gamma/\partial I)$. From (17), $(\partial(\partial\Pi/\partial\alpha)/\partial\gamma) = \{\partial[-p_q q' \gamma X + (p_X X + p)\, a' \gamma L^{\tau}]/\partial\gamma\}$ is ambiguous and we can't derive economic some intuitions.

It is same for $d\alpha/d\beta$. Then we have Lemma 3.

Lemma 3: *The effects of dI and dβ on dα are ambiguous. When $\partial(\partial\Pi/\partial\alpha)/\partial\gamma) > 0$, that is, the partial effect of γ on $\partial\Pi/\partial\alpha$ is positive, $d\alpha/dI > 0$ and $d\alpha/d\beta < 0$.*

Although the result is ambiguous, the Lemma is related to the problem how degree of effect of productivity is altered by the difficulty of the job.

5. Intermediate input

According to some international data, the ratio of final consumption goods in trade is very low. The main is intermediate and its recent share in world trade without fuel is about 55%[20]. Subsection 5.1 treats simple intermediate model, and subsection 5.2 decisions for offshore outsourcing and direct investment (foreign subsidiary).

[20] See UNIDO (2011).

5.1. Simple model with intermediate input

5.1. 1. Three types of intermediate goods

In this sub-section, I introduce intermediate input explicitly in our model. Intermediate input is classified to three types, A, B and C. Type A is relatively close to traditional type, and (13) is rewritten as

$$X = X(a(\alpha\gamma), M^A(\gamma, M^A_0), L), \tag{24}$$

with $M^A_\gamma(\gamma, M^A_0) \equiv \partial M^A(\gamma, M^A_0)/\partial\gamma > 0$ and $M^A_{K0}(\gamma, M^A_0) \equiv \partial M^A(\gamma, M^A_0)/\partial M^A_0 > 0$, and M^A_0 is intermediate goods. M^A_0 is usual definition. $M^A(\gamma, M^A_0)$ is defined as effective intermediate and it depends on also γ. For example, when γ is high, labors can treat machines well and efficiently. Conversely it is un-effective if the machines are difficult for labors to operate for inferior γ.

The second type B, γ function of (10) is rewritten as

$$\gamma = \beta H(L)/I(M^B), \quad \text{with} \quad I'(M^B) < 0. \tag{25}$$

The effect of M^B on γ is positive from the assumption of $I'(M^B) < 0$. By using machines and software, even unskilled labors can achieve complicated jobs, which were only accomplished by skilled labors before. Introducing such intermediate input makes many difficult jobs easier. Then the assumption of $I'(M^B) < 0$ would be plausible. It is interpreted as a kind of technical progress.

The third type C is introduced in revenue function as

$$p(q((1-\alpha)\gamma, p_M^C, X)X(a(\alpha\gamma), L, M^C) \quad (26)$$

In intermediate type C, the quantity M^C is fixed and the price, p_M^C, is variable. Higher p_M^C is interpreted as enhancing quality and not useful for productivity improvement. For example, in watch and automobile industries, using luxury parts make the quality enhance while it keeps the productivity[21].

Using (15), (24), (25) and (26), profit function is respectively given by

$$\Pi = p(q((1-\alpha)\gamma, X)X(a(\alpha\gamma), M^A(\gamma, M^A{}_0), L) - wL - p_M M^A. \quad (27)$$

$$\Pi = p(q((1-\alpha)\gamma, X)a(\alpha\gamma)L - wL - p_M M^B, \quad \text{with } \gamma = \beta H(L)/I(M^B). \quad (28)$$

$$\Pi = p(q((1-\alpha)\gamma, p_M, X)a(\alpha\gamma)L - wL - p_M M^C. \quad (29)$$

We don't analyze moreover in detail since our purpose presents endogenous quality and productivity model and our main concern is not intermediate input.

Amiti and Konings (2007) estimate that the reduction of intermediate input tariffs leads to be more productivity gain. The sources of productivity gain are tougher import competition, cheaper imported inputs and the effects of learning, variety and quality. At least in my knowledge, there are few theoretical models analyzing these effects.

Explicitly, our model presents some mechanism in which intermediate input causes the productivity and quality gain. The effects of trade liberalization for intermediate

[21] Jewel is sometimes used in high grade watches, and expensive aluminum wheel and sheets are used in luxury automobiles.

input are different from final consumption goods. For type B, this effect would be larger since introducing intermediate input cause adaptability large and can lead to terms-of-trade improvement through higher export price, and welfare gain.

Then, we have proposition 5:

Proposition 5

We clarify the roles of intermediate goods. First, firms with high adaptability can operate intermediate goods well. Second, it makes unit value to rise. Intermediate goods promote a kind of technological progress through making difficult jobs to be easier and improvement in real adaptability. Third, using expensive intermediate goods even for constant productivity, it increases its quality.

5.1. 2. A rising price-distance link

Using type C of intermediate goods, we can theoretically explain *a rising price-distance link* which means that the highest priced goods travel the furthest (Baldwin and Ito (2011))[22]. The firm exports to a foreign country if the profit for a f country is positive. We assume that there is an additional fixed unit cost for f country export, and that the firm change its price according to the. It is supposed that each price for each f country is different because of additional each fixed cost. Then the variety goods profit for a f foreign country for the firm is written as

$$\Pi_f = p_f(q_f((1-\alpha)\gamma, p_{Mf})a(\alpha\gamma)L^{\tau}{}_f - wL_f - p_M M - C_{Ff}, \quad \text{with } \partial p_f/\partial p_M > 0, \qquad (30)$$

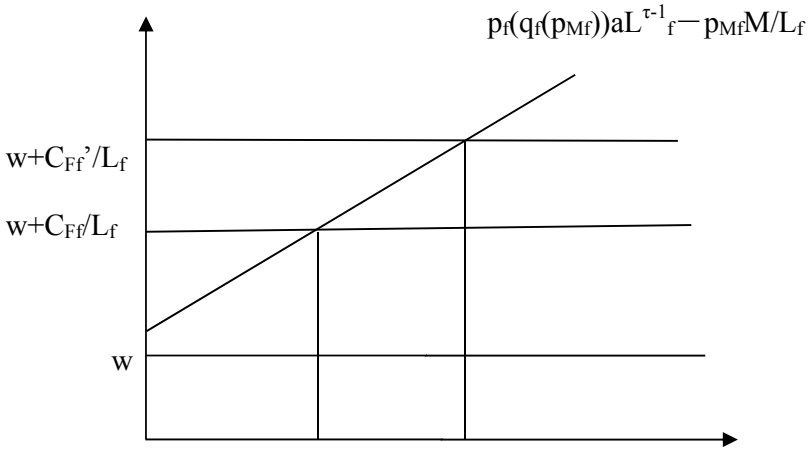
[22] Although we can explain by type A and B, we use type C since it is easiest to analyze *a rising price-distance link.*

where C_{Ff} is an additional fixed unit cost for f country export, and it is basically the same as C_F in Fig.5. We assume that C_{Ff} is increasing function of distance from home country to the export country. If the firms use luxury parts in the differentiated goods, the firm can compete in higher price range.

In the Fig. 7, a rising price-distance link is depicted. Vertical axis is shown as unit revenue minus unit intermediate cost, $p_f(q_f(p_{Mf}))aL_f^{\tau-1} - p_{Mf}M/L_f$. The Figure shows the relationship between intermediate price p_{Mf} and profit. It is assumed that a, L_f, M, and C_{Ff} are given and the firm doesn't export if export is deficit. Then we find that higher C_{Ff}' ($C_{Ff}' > C_{Ff}$) cause cheaper variety goods (AB) to be deficit. Since unit cost (C_{Ff}/L_f) is increased when the distance from home country to export is longer, the firm can't export lower p_{Mf} or cheap varieties (OB)[23]. So *a rising price-distance link* can occur.

If the line, $p_f(q_f(p_{Mf}))aL^{\tau-1}{}_f - p_{Mf}M/L_f$, is more flat, the firm profit is indifferent for higher p_M. The profit rate for luxury and high price goods is lowered. Then there is no distance link.

$p_f(q_f(p_{Mf}))aL^{\tau-1}{}_f - p_{Mf}M/L_f$



[23] There could be unit cost (C_{Ff}/L_f) rising when scale merit is low because sales are small and unit transportation, maintaining sales network is high.

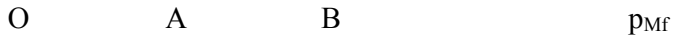


Fig.7. A rising price-distance link.

From type C, we have proposition 6

Proposition 6

Using our intermediate goods of type C, we can analyze a rising price-distance link theoretically. Under the assumption of profit increasing for higher price, the firms intend to export luxury goods for longer distance country.

The model in the subsection is directly relevant not to adaptability. But since p function is denoted as $p_f(q_f((1-\alpha)\gamma, p_{Mf})$, the line, $p_f(q_f(p_{Mf}))aL^{\tau-1}{}_f - p_{Mf}M/L_{f,}$ may be more flat and brand is low if adaptability is low.

5.2. Vertical international trade -offshore outsourcing or subsidiary-

The increase in intermediate goods trade in parts and components is due to decreasing of transaction costs including logistics and so called service link costs[24]. The trade in intermediate goods includes both inter-firm and intra-firm international trade. This sub-section treats decisions for offshore outsourcing and direct investment (foreign subsidiary). Offshore outsourcing is categorized to inter-firm trade, and foreign subsidiary to intra-firm trade. We analyze the decision by using our model and we show some new perspectives for this.

We may interpret Fig. 4. as a kind of vertical and/or horizontal trade. However the

[24] E.g., Jones et.al (2005)

Figure can't explain inter and intra firm trade, and outsourcing explicitly. We focus on the decision of inter and intra firm trade. In other words, we consider why offshore outsourcing occurs and why firms don't produce some goods by themselves. As "The terms outsourcing and fragmentation refer to moving parts to different locations, not necessarily to different firms" in Jones (2008), outsourcing and fragmentation treats the production location. This subsection analyzes inter-firm and intra-firm transactions focusing on contract for inner or outside supplier.

Suppose that there are two companies initially, one is headquarter which is located in home country, and the other is its subsidiary in a foreign country. It is assumed that the subsidiary provides intermediate goods, X*, which is used in the headquarters plant in home country. In the case, the profit function in its subsidiary and adaptability function are respectively shown as:

$$\Pi_S^* = p^* (q^* ((1-\alpha^*)\gamma_S^*, X^*) X^*(a^* (\alpha^*\gamma_S^*), L^*) - w^*L^* - C_S, \quad (31)$$

$$\gamma_S^* = \beta^* H^* (L^*)/I^*(\lambda_S^*), \quad (32)$$

In the following, we assume that $H^{*\prime} > 0$. In the equation, λ_S^* is synergy effect which drives from the fact that the headquarter transfers its knowledge and experience to subsidiary, and $\partial I^*/\partial\lambda_S^* < 0$. The synergy effect includes licensing technology and improvements on the production process. The s synergy effects make the tasks easier by teaching to its subsidiary. Using data on U.S. imports, Nunn and Trefler (2008) provide evidence of the positive relationship between intra-firm trade and two measures of headquarter intensity, namely capital intensity and skill intensity. "λ*" is interpreted to be higher when headquarter intensity is high level.

C_S is service link cost which includes communication, organization cost, trade barrier and shipment[25]. It drives from the fact that the subsidiary must be managed by foreign headquarters, and its management cost is necessary.

If λ^* is large and there is an enough synergy effect, I^* is lowered and a^* could be higher, so that productivity is high under constant quality and subsidiary is profitable. It is profitable for the firm to produce the goods. Then the firm keeps the subsidiary since adaptability is high due to high λ^*. Intra-firm trade is more attractive for headquarter.

On the other hand, outsider supplier profit and adaptability functions are shown as:

$$\Pi_F^* = p^* (q^* ((1-\alpha^*)\gamma_F^*, X^*) X^*(a^* (\alpha^*\gamma^*), L^*) - w^*L^*, \quad (33)$$

$$\gamma_F^* = \beta^* H^* (L^*)/I^*_F, \quad (34)$$

where I^*_F is that of foreign supplier. It is supposed that headquarters buy the foreign firm if $\Pi_S^* > \Pi_F^*$.

If headquarter is not used to manage the subsidiary in foreign country or unfamiliar to produce goods, λ^* and C_S would be respectively small and large so that the headquarter may let go of the subsidiary. Even if the firm know the production process and λ_S^* is large, headquarter would sell the subsidiary when a firm in the foreign country can produce it easily or $I^*(\lambda_S^*)$ is not enough small compared with I^*_F.

Considering the headquarter profit together, offshore outsourcing can occur even if $\Pi_S^* < \Pi_F^*$. Let λ denote synergy effect for the headquarters. Then "I" function of the headquarters is written as $I(\lambda)$. When the firm closes the subsidiary, λ is decreased. If $I(\lambda)$ is increased for closing the subsidiary, the firm would intend to maintain the

[25] In the paper, we don't consider C_S explicitly.

subsidiary since the performance of the firm is improved to make profit enough to offset the difference of profit .

For the decision of subsidiary or outsider, the optimal allocation of human resources is related. If the firm can gain from the concentration of human resources in the headquarters, the headquarters would sell the subsidiary. Moreover the distance from the home country to foreign is important. In the paper we don't treat these also. Then we have proposition 7.

Proposition 7

In our model, synergy effect and service link cost are key factors when headquarter has subsidiary in foreign country (FDI, foreign direct investment) or consign its production to outsider supplier (offshore outsourcing). Sometimes offshore outsourcing can occur even if $\Pi_S^* < \Pi_F^*$ *because of synergy effects to headquarter.*

7. Conclusion

We try to explain recent phenomenon of international trade comprehensively by using a new notion, "adaptability". While this concept is near to human resources and human capital, it includes the difficultness of task, so that it reflect our sophisticated society. This paper interprets the source of productivity and quality as adaptability. Although this notion is not observable directly, we find that it has much power to analyze different phenomenon. Using a new model with endogenous quality-productivity, we obtain some Propositions.

In our model, price is correspondence to quality through consumers, and quality

depends on allocated efforts on quality. In our model, firms allocate their resources optimally between quality and productivity. There is trade-off between quality and productivity, and we can depict price-productivity frontier. The shape of the frontier depends on the production and demand characteristics of each differentiated goods. We find each firm decides optimal price-productivity under price-productivity frontier.

Unit-value is obtained by price multiplying productivity. We find that unit-value added ratio to wage could be more accurate index for competitiveness than comparative advantage. The firms which can produce high price goods are more competitive for quality competition. We show wage and price-productivity lines. The lines depict production structure for wage rising from developing to developed countries. We can ensure some empirical findings in which developed countries tend to export high quality goods. For optimal labor input, we consider quality-productivity improvement as well as marginal product since quality-productivity is function of labor.

Our applications of our model refer to intermediate goods. We find new insights of intermediate goods. We show that there are three types of intermediate goods. One is effective intermediate goods and it's near to traditional. It reflects the fact that efficient operation and using with high adaptability make the value of intermediate goods to raise. Second, we can accomplish sophisticated job easily by using some machines and software. It makes difficulty of jobs easier and contributes improvement on quality and productivity for labors. Third, higher expensive intermediate goods raises the quality and the price. Firms with superior brand use expensive parts as diamond in watch, and they supply with higher price for rich.

Our model has further possible researches and many kinds of applications. We can analyze a model with skilled and unskilled labors. This model show theoretically that

demand for skilled labor become higher in our society as in some papers. In our paper, we don't analyze welfare economics and trade gain. Since there is no distortion in the paper, it is not enough effective for our simple model. We could have interesting results for another modified model. International trade and educational policies are also not considered as well as free trade gain. Theses analysis would be more fruitful.

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