Finite mixture of regression modeling for exchange market pressures during the financial crisis: A robust Bayesian approach to variable selection

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Abstract

We propose a Bayesian variable selection approach in a finite mixture regression model with t-errors, which can simultaneously accommodate model uncertainty, population heterogeneity, and outliers. In particular, we adopt a spike and slab prior to deal with highly correlated covariates that are pervasive in large datasets. A Monte Carlo simulation study is conducted to examine the ability of the proposed method to correctly identify the important variables under a number of scenarios for collinear covariates. For the empirical application of exchange market pressures, we identify two clusters of countries that do not match with any country-specific dummies. We find that a number of early warning indicators are robust to heavy-tailed distributions and exert differential impacts on external market pressures across the two groups of countries. In contrast to the earlier 2008-crisis literature, we present optimistic view with regard to the feasibility of an early warning system to predict the likelihood of crises. We also identify outlying countries—most notably Seychelles—in explaining exchange market pressures in a cross-section of countries.

Keywords: Financial crisis, Robust Variable Selection, Heteroscedasticity, Outliers, Finite Mixture Models.

JEL classification: C11, C21, C52, F31, O50.

1 Introduction

The percussion of the global crisis in 2008 has rekindled academic interest in early warning models (see, e.g., [Frankel and Saravelos, 2012;](#page-37-0) [Rose and Spiegel, 2010,](#page-39-0) [2011,](#page-39-1) [2012,](#page-39-2) among others). Researchers have sought to identify risk factors that can indeed predict crisis occurrences. On the other hand, economic theory offers little guidance about the appropriate set of variables included in the underlying true model. Thus, a challenging question is to determine, out of an often large set of candidate variables with a limited number of observations, the variables relevant for crisis events. In contrast to the classical inference, the Bayesian approach provides a natural and general probabilistic framework that simultaneously treats both model and parameter uncertainty [\(Clyde and George, 2004\)](#page-37-1). To address this uncertainty in the context of financial crisis early contributions have applied Bayesian model averaging (BMA) (e.g., [Cuaresma and Slacik, 2009;](#page-37-2) [Dwyer and Tan, 2014;](#page-37-3) [Feldkircher](#page-37-4) [et al., 2014;](#page-37-4) [Ho, 2014\)](#page-38-0).

All the mentioned contributions are, however, plagued by a number of sensitivity issues that determine the relationship between the crisis intensity and the covariates for all considered countries or regions. In particular, the usually considered data sets that comprise very heterogeneous countries or regions make the assumption of a common marginal impact of external shocks, even when controlling for a variety of risk factors, at least worth investigating [\(Doppelhofer and Weeks, 2011;](#page-37-5) [Ho, 2014;](#page-38-0) [Temple, 2000\)](#page-40-0). Despite the wide applicability of the linear regression model powered by the modern variable selection tools, a single regression model can be inadequate if the data come from a heterogenous population that consists of a number of different sub-populations with different characteristics. In this situation, it is possible that a separate linear regression model is needed for each sub-population, moreover, the regression models in different sub-populations may use different subsets of covariates to

explain the response variable. If the memberships of the observations are unobserved, then we naturally have a finite mixture model of linear regressions, where each mixture component is a linear regression model with its own subset of covariates. This gives rise to a variable selection problem that is more complex than that of a single linear regression model.

In this paper, we propose a flexible Bayesian modeling with a finite mixture regression (FMR) model to investigate the robustness of the determinants of the crisis intensity, particularly exchange market pressures during the recent global financial crisis. In FMR models, the characteristics corresponds to the effects of covariates which vary with subpopulations. It implies that the changes of response may be affected by different sets of covariates in the FMR models. Our Bayesian approach is flexible to account for model uncertainty and allow for various forms of heterogeneity. In the Bayesian framework, a popular choice of prior has been Zellner's (1986) g-prior for the regression coefficients which is based on the inverse of empirical covariance matrix of the covariates. However, the difficulties can arise in variable selection when covariates are highly correlated [\(Clyde and George, 2004;](#page-37-1) [Liang et al., 2008\)](#page-38-1). Instead of g-prior, a spike and slab prior [\(George and McCulloch, 1993\)](#page-38-2) is considered here to perform variable selection in the presence of correlated covariates.^{[1](#page-3-0)} In particular, it is straightforward to explicitly incorporate prior information for the relative importance of covariates with a spike and slab prior. Furthermore, to prevent the statistical inferences from being distorted by the presence of outliers, a FMR model with t errors is proposed.

Our results from the proposed variable selection show that two distinct groups of countries differ in the effects of leading indicators on external market pressures, rendering constant parameter regressions invalid when analyzing the cross-country incidence and severity of global crisis. We also identify a number of important pre-crisis indicators different from the previous studies in rankings and signs. First, for the top ranked variable, our result

¹See also [Korobilis](#page-38-3) [\(2013\)](#page-38-3) in forecasting output and inflation series.

emphasizes the essential role of growth rate prior to the crisis played in explaining exchange market pressures. Overheated economy could have elevated the country vulnerability to external shocks. Second, we find the degrees of globalization affect exchange market pressures across two clusters of countries, and the effect is particularly evident for the second cluster of countries. Third, we do not find supportive evidence for grouping dummies of countryspecific characteristics, implying the FMR model with two clusters is sufficient in uncovering the patterns of exchange market pressures. Finally, a number of countries, including China, Mauritania, Seychelles, Venezuela, and the U.S., are considered as potential outliers. A series of robustness checks suggests that our results are not qualitatively changed by taking into account the effects of outliers and collinearity.

This paper is organized as follows. We briefly review the empirical literature in the early warning models, with a focus of the Bayesian approaches for model uncertainty in Section [2.](#page-4-0) We introduce the FMR models in Section [3.](#page-8-0) The prior distributions and a fully Bayesian approach employed to the problem of variable selection are also discussed. We assess the performance of our proposed variable selection method in the presence of collinear covariates in Section [4.](#page-14-0) We then present the empirical results from applying our proposed method to the data on exchange market pressures in Section [5.](#page-17-0) We conclude this study in Section [6.](#page-26-0) Details of the full conditional distributions and the required MCMC algorithms are given in Appendix [A.](#page-27-0)

2 Model Uncertainty in Cross-Country Crisis Intensity

A growing body of literature has investigated whether pre-crisis conditions and global factors can explain the different impact of the 2008 financial crisis in various countries. [Obstfeld](#page-39-3)

[et al.](#page-39-3) [\(2009,](#page-39-3) [2010\)](#page-39-4) pioneer the study on the global financial crisis in 2008 and suggest that the excessive reserves plays a major role in currency depreciation over 2008. Although the factor is established on a solid theoretical model, its empirical support is weakened by the small sample of countries. In a series of papers, [Rose and Spiegel](#page-39-0) [\(2010,](#page-39-0) [2011,](#page-39-1) [2012\)](#page-39-2) consider a large number of potential explanatory variables for the crisis that have been discussed in the literature, covering such "fundamentals" as: financial system policies and conditions, asset price appreciation in real estate and equity markets, international imbalances and foreign reserve adequacy, macroeconomic policies, and institutional and geographic features. Surprisingly, they find that pre-crisis macroeconomic and financial conditions generally fail to explain the economic performance of countries during the crisis period. There are a few exceptions, however, including run-ups in asset prices and current account deficits prior to the crisis, which were both significantly correlated with the crisis severity. Their general finding of inconclusive relationships presents a pessimistic view with regard to the feasibility of an early warning system to predict the timing of such crises. In contrast, in an extensive review of the early warning indicators literature, [Frankel and Saravelos](#page-37-0) [\(2012\)](#page-37-0) find that the pre-crisis level of reserves and preceding real exchange rate appreciation are consistently useful in predicting exchange market pressures, in particular, [Frankel and Saravelos](#page-37-0) [\(2012\)](#page-37-0) emphasize a more positive role for reserves than other recent studies in reducing vulnerability of developing countries.

Recently, [Aizenman et al.](#page-36-0) [\(2012\)](#page-36-0) investigate the determinants of EMP by focusing on emerging markets (EMs) during the [2](#page-5-0)008–09 crisis.² The authors find that per capita income prior to the financial crisis (as of 2007), inflation and the trade balance appear as useful leading indicators that can explain cross-country difference in EMP.

²[Aizenman et al.](#page-36-0) [\(2012,](#page-36-0) p. 600) note that "EMP was a major component of the financial stress in EMs during the 2008–9 crisis, while it played virtually no role in the preceding episodes."

The past studies on the early warning indicators produce mixed evidence about EMP determinants, which may be partly due to the methodological flaws in neglecting model uncertainty and the attendant omitted variable bias. It is common practice for empirical studies to conduct a horse race of linear regressions from some class of early warning models a priori and then make inferences as if the selected were the 'true' model. As [Raftery](#page-39-5) [\(1995,](#page-39-5) p. 113) notes "In this situation, the standard approach of selecting a single model and basing inference on it underestimates uncertainty about quantities of interest because it ignores uncertainty about model form." The early warning models have received much discussion in the literature, the role of model uncertainty, while essential, is only rarely addressed. There are, however, several notable exceptions in which BMA techniques are used to account for model uncertainty in early warning regressions [\(Babecky et al., 2013;](#page-36-1) [Cuaresma and Slacik, 2009;](#page-37-2) [Feldkircher et al., 2014\)](#page-37-4).[3](#page-6-0) [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4) consider an extensive set of pre-crisis leading indicators and explicitly account for the issue of model uncertainty in EMP. Surprisingly, only two leading indicators—inflation and the joint record of domestic savings—stand out as robust determinants of exchange rate pressures. With the updated dataset of [Frankel and Saravelos](#page-37-0) [\(2012\)](#page-37-0), the BMA evidence of [Christofides et al.](#page-36-2) [\(2013\)](#page-36-2) supports a number of early warning signals that are significantly correlated with exchange rate pressure, including real effective exchange rate, remittances, trade deficits, bank liquidity-to-asset ratios and levels of domestic credit.

Nevertheless, all of the studies reviewed above assume constant parameters in their linear regressions, even though the country heterogeneity in the responses to external shocks were well noted by the authors (e.g., [Aizenman et al., 2012;](#page-36-0) [Feldkircher et al., 2014\)](#page-37-4). In particular, [Temple](#page-40-0) [\(2000\)](#page-40-0) forcefully argues that, other than model uncertainty, parameter

³A general overview of BMA refers to [Doppelhofer](#page-37-6) [\(2008\)](#page-37-6); [Hoeting et al.](#page-38-4) [\(1999\)](#page-38-4); [Raftery et al.](#page-39-6) [\(1997\)](#page-39-6). For special emphasis on the applications of BMA to economics refers to [Moral-Benito](#page-39-7) [\(2013\)](#page-39-7).

heterogeneity and outliers have not received adequate attention in the empirical literature. It is of paramount importance to control cross-country heterogeneity in the current empirical literature. [Durlauf](#page-37-7) [\(2000\)](#page-37-7) points out two major drawbacks without proper treatment of heterogeneity. First, ad hoc country groupings may be inconsistent with the true underlying grouping. Second, while fixed effects estimation allows for heterogeneity through the intercept, most studies do not allow for heterogeneity in the slope parameters. In the context of growth models, the effects of covariates such as inflation and investment are assumed to be homogeneous across (groups of) countries. Although the homogeneity simplifies the estimation greatly, it often becomes quite restrictive in an increasingly diverse world economy. Heterogeneity, on the other hand, could be generated by outliers, which is likely to encounter in a large-scale cross-country dataset. Outliers can be seen as the deviations from the typical empirical relationship implied by the regression of dependent variables to independent variables, and they can be caused by fat-tailed or asymmetric error distributions, measurement errors, or model mis-specifications [\(Sturm and de Haan, 2005\)](#page-39-8). As a result, the presence of outliers can adversely affect the statistical inference or even obscure the true relationship. Several recent works have adopted the robust Bayesian estimation to account for potential outliers. To name a few, [Doppelhofer and Weeks](#page-37-5) [\(2011\)](#page-37-5) consider the case of the cross-country economic growth, and [Ho](#page-38-0) [\(2014\)](#page-38-0) investigates the cross-country causes of the 2008-09 crisis. They both highlight the impact of potential outliers on BMA, and to the extent that the major findings can be significantly altered by the robust estimation. To address these issues, we propose a flexible Bayesian modelling to simultaneously account for model uncertainty, population heterogeneity and outliers, while systematically choosing the subset of early warning indicators that are significantly correlated with external market pressures.

3 Finite Mixture Model

FMR models have recently become a popular statistical method for modeling unobserved population heterogeneity, see, e.g., Frühwirth-Schnatter [\(2006\)](#page-38-5); [McLachlan and Peel](#page-38-6) [\(2000\)](#page-38-6), due to the fact that they offer more natural modeling for the population consisting of different subpopulations. These subpopulations may require different parameters to adequately capture their distinct characteristics. In FMR models, the characteristics corresponds to the effects of covariates which vary with subpopulations. It implies that the changes of response may be affected by different sets of covariates in the FMR models. By and large, it becomes a variable selection problem within each subpopulation.

More recently, Bayesian variable selection approach has been extensively developed to identify the important variables, particularly in the regression analysis when the number of available covariates is moderately large, but only a subset of variables are relevant to explain variation in the data, see, e.g., [Khalili](#page-38-7) [\(2011\)](#page-38-7) for review. We apply a Bayesian variable selection to FMR models, where variable selection procedure is implemented to select the important covariates in each subpopulation. In FRM, the regression coefficients may change across subpopulations. Whenever the information is available about the nature of heterogeneity for the problem at hand, it can be incorporated by choosing a specific probabilistic specification for β , which is pre-specified in terms of the density of $\pi(\beta)$ as a prior distribution, imposing some model structure on the individual regression coefficients that may be dominated by the information in the data. Different prior distributions defining different model structures may be compared in a systematic way by Bayesian model comparisons.

To fix notation, let (y_i, x_i) , $i = 1, \ldots, n$, be a data set of n observations that come from a heterogeneous population, where y_i is the response variable of the *i*-th observation, and $x_i = (x_{i1}, x_{i2}, \dots, x_{ip})'$ collects the p covariates of the *i*-th observation. We assume that the heterogeneous population consists of M sub-populations or mixture clusters, and within each sub-population, (y_i, x_i) is fitted by a separate linear regression model. Specifically,

$$
y_i|\beta_m, \sigma_m^2, \rho_m, \omega_i \sim \sum_{m=1}^M \rho_m \cdot N\left(x_i'\beta_m, \omega_i\sigma_m^2\right). \tag{1}
$$

Here $\rho_m = (\rho_1, \ldots, \rho_M)$ describes the proportions of the population distributed among M clusters, or the the mixing proportions, and $\rho_m \geq 0$ and $\sum_{m=1}^{M} \rho_m = 1$, and let $\beta_m =$ $(\beta_{m1},\ldots,\beta_{mp})'$ be the coefficient vector for the mth cluster. We assume that the cluster is normally distributed with a mean $x_i'\beta_m$ and a variance $\omega_i\sigma_m^2$, where ω_i is the variance-inflation factor corresponding to the ith observation and therefore the error variances vary across countries. The model is also flexible enough to place a specific prior on ω_i to accommodate outliers and select relevant covariates simultaneously [\(Geweke, 1993\)](#page-38-8). The main interest is to identify the covariates x_{mp} 's that one believes to have an influence on the response variables y_i in cluster m. To solve this problem within the Bayesian framework, we introduce two set of latent variables. For the first set of latent variables, each observation is associated with an indicator, determining which sub-population or mixture cluster this observation comes from. For the second set of latent variables, within each mixture cluster, each covariate is associated with an indicator, determining whether this variable is included in the regression model of the mixture cluster.

The first latent variable z_i is defined as follows

$$
z_i = m, \text{if } y_i \sim N\left(x_i'\beta_m, \omega_i\sigma_m^2\right), m = 1, \dots, M,
$$

with $P(z_i = m) = \rho_m$ for $i = 1, ..., n$. That is,

$$
z_i \sim \text{Multinomial}(\rho_1, \ldots, \rho_M).
$$

Given $z = (z_1, \ldots, z_n)$, the joint density of (y, z) can be written as follows

$$
f(y, z | \theta) = \prod_{i=1}^{n} \rho_{z_i} N\left(x'_i \beta_m, \omega_i \sigma_m^2\right),
$$

where $\theta = \{\beta_1, \ldots, \beta_M, \sigma_1^2, \ldots, \sigma_M^2, \rho_1, \ldots, \rho_M, \omega_1, \ldots, \omega_n\}$. Conditioning on the latent variable z_i , the cluster to which each observation belongs is known, and therefore, the Bayesian variable selection method is straightforward to carry out for each cluster in the FMR model.

Another latent vector r_m is used to identify active variables for each regression model in each cluster of the mixture model. It is equivalent to identify the non-zero elements in β_m for each m. In order to perform the variable selection, for the mth cluster, we define a $p \times 1$ vector $r_m = (r_{m1}, \ldots, r_{mp})'$ so that for covariate x_j in cluster $m, \beta_{mj} = 0$ if $r_{mj} = 0$ and $\beta_{mj} \neq 0$ if $r_{mj} = 1$. Therefore, given r_m , let $\beta_m(r_m)$ consist of all nonzero elements of β_m and let $x(r_m)$ be the active elements of x corresponding to those elements of r_m that are equal to 1. Thus, the FMR model in equation [\(1\)](#page-9-0) can be re-written as

$$
y_i|\beta_m, \sigma_m^2, \rho_m, r_m, \omega_i \sim \sum_{m=1}^M \rho_m \cdot N\left(x_i(r_m)'\beta_m(r_m), \omega_i \sigma_m^2\right).
$$

Based on the augmentation of these two sets of indicators, it allows one to transform the complex structure of mixture model into a set of simple structures, so that in the Bayesian analysis the Gibbs sampler can be easily implemented to draw the sample from the posterior distribution. In the following subsections, we first introduce the prior specifications, and then describe the implementation details of our proposed Bayesian approach.

3.1 Priors

We first consider the mixing proportion vector ρ . Similar to [Viele and Tong](#page-40-1) [\(2002\)](#page-40-1), we assume a conjugate Dirichlet prior distribution for ρ

$$
\rho \sim \text{Dirichlet}(\alpha_1, \ldots, \alpha_M).
$$

In each component of mixture regression model, the prior of the indicator variable r_{mj} is independently Bernoulli (d_{mj}) for $j = 1, \ldots, p$. As a result, the joint density of $r_m =$ $(r_{m1},\ldots,r_{mp})'$ is

$$
\pi(r_m) = \prod_{j=1}^p d_{mj}^{r_{mj}} (1 - d_{mj})^{1 - r_{mj}}.
$$

Consider the spike and slab prior for the coefficient vector β_m . That is, given r_m , the prior of the regression coefficient vector, β_{mj} for all j and m is assumed to be

$$
\beta_{mj}|r_{mj} \sim (1-r_{mj})\delta_0 + r_{mj}N(0, \tau_{mj}^2),
$$

where δ_0 is a point mass at 0.

To eliminate the selection bias on τ_{mj} , we further assume τ_{mj}^2 independently distributed $IG\left(\frac{a_{\tau_{m j_0}}}{2}, \frac{b_{\tau_{m j_0}}}{2}\right)$ $\left(\frac{m_{j_0}}{2}\right)$. To address the effect of outliers on the estimation and statistical inference, we place a specific prior on ω_i which follows a inverse Gamma distribution, $IG(v/2, v/2)$. Under this setting, the linear model is equivalent to a model whose errors have independent and identical Student-t distributions with the degree of freedom equal to v , as in [Geweke](#page-38-8) [\(1993\)](#page-38-8). Note that lower values of v correspond to heavy-tailed distributions and hence more

Figure 1: The figure illustrates the hierarchical structure of the priors on the parameter of the proposed model. A green pentagon indicates the observed data, a red circle indicates the latent variables or parameters to be estimated, and a gray square indicates a hyperparameter, which is considered to be a constant for the corresponding prior distribution. The arrows indicate the conditional dependence structure of the model.

accommodating of outliers, and also imply relatively larger variances in the inverse Gamma distribution.

As usual, an independent inverse Gamma distribution, $IG\left(\frac{a_{m_0}}{2}\right)$ $\frac{m_0}{2}, \frac{b_{m_0}}{2}$ $\left(\frac{n_0}{2}\right)$, is placed on σ_m^2 , that is,

$$
p(\sigma_m^2|a_{m_0}, b_{m_0}) \propto (\sigma_m^2)^{-\frac{a_{m_0}}{2}-1} \exp\left\{-\frac{b_{m_0}}{2\sigma_m^2}\right\}.
$$

We assume $r, \sigma, \rho, \tau, \omega$ are a priori independent, β conditionally independent. A hierarchical representation of our Bayesian model is shown in Figure [1.](#page-12-0)

3.2 Posterior estimation

Under the model and prior specifications laid out in the above section, the joint posterior distribution can be derived. The posterior distribution is not available in explicit form so we use the MCMC method, specifically Gibbs sampling [\(Brooks et al., 2011\)](#page-36-3) to simulate the parameters from the posterior distribution. To implement the Gibbs sampler, the full conditionals of all parameters must be determined. A derivation of the full conditional distributions is provided in Appendix [A.](#page-27-0) With the conditional probability of each parameter, the parameters in each cluster are then updated individually using a Gibbs sampler (where available), or a Metropolis-Hastings sampling algorithm.

Under mild regularity conditions and for sufficient iterations, the sample simulated from the above Gibbs sampler can be used to approximate the joint posterior distribution. We collect a sequence of MCMC samples and then approximate the posterior probability of covariate x_j within subpopulation (cluster) m by

$$
\hat{p}(r_{mj} = 1|y) \approx \frac{1}{K_j} \sum_{k=1}^{K} I\{r_{mj}^{(k)} = 1\}.
$$
\n(2)

This gives an estimate of the posterior inclusion probability (PIP) as a measure of the relative importance of the *j*th covariate within cluster m . Higher posterior inclusion probabilities indicate the covariate is important in explaining the response variable for the mth cluster.

A researcher may also be interested in drawing inference about the economic importance of a variable in terms of posterior estimates. Both can be approximated in a straightforward manner from the corresponding PIP. The posterior mean for the regression coefficient β_j associated with covariate x_j , for cluster $m = 1, \ldots, M$.

$$
E(\beta_{mj}|\mathbf{y}) = \sum_{r_{mj}} E\left[\beta_{mj}|r_{mj},\mathbf{y}\right] p\left(r_{mj}|\mathbf{y}\right) \approx \frac{1}{K_j} \sum_{k=1}^K \beta_{mj}^{[k]},
$$

where K is the number of samples generated from the posterior distribution using the MCMC procedure. Moreover, $r_{mj}^{[k]}$ and $\beta_{mj}^{[k]}$ is the MCMC sample in the kth iteration, and $K_j =$ $\sum_{k=1}^{K} r_{mj}^{[k]}$.

4 Simulation study

In this section, we conduct several simulation experiments to validate our proposed model and the estimation procedure described above. This is done to examine the degree of accuracy on selecting the important variables. In particular, we investigate the potential impacts of severe and complicated collinearity in the design matrix.

4.1 Simulation Settings

We consider a simple situation to illustrate the important properties of the model and the crucial aspects of the simulation procedure. For this simulation, we assume there are two clusters and the 150×20 design matrix X follows a multivariate normal distribution with a mean of 0. We consider four pair correlation structures with $\text{corr}(x_i, x_j) = 0, 0.5, 0.75, 0.95$ for covariates i and j, $i \neq j$. The parameters of the model are randomly generated from the

following distributions:

$$
\rho \sim \text{Dirchlet}(3, 3)
$$

\n
$$
r_{mj} \stackrel{iid}{\sim} \text{Bernoulli}(0.5)
$$

\n
$$
\tau_{mj}^2 \stackrel{iid}{\sim} IG(2, 2)
$$

\n
$$
\beta_{mj}|r_{mj}, \tau_{mj} \stackrel{iid}{\sim} (1 - r_{mj})\delta_0 + r_{mj}N(0, \tau_{mj}^2)
$$

\n
$$
\sigma_m^2 \stackrel{iid}{\sim} IG(2, 2)
$$

\n
$$
\omega_i \stackrel{iid}{\sim} IG(2, 2).
$$

The main point we want to address is the accuracy of our proposed model in this simulation. In addition, we would expect the proposed method to provide a sufficiently parsimonious model that contains as many active variables and as fewer noise variables as possible. In this simulation study, the median probability criterion [\(Barbieri and Berger, 2004\)](#page-36-4) is used. That is, we estimate the posterior inclusion probability $P(r_{mj} = 1|y)$ for each covariate j within cluster m from the Monte Carlo sample shown in equation (2) . We claim covariate j should be included into the model for cluster m once the posterior probability $P(r_{mj} = 1|y)$ is greater than or equal to 0.5.

To evaluate the performance of our proposed method, we consider the following measures: the accurate rate of grouping observations (ARG), the accuracy of classification of variables (ACC), the true positive rate (TPR), and the false positive rate (FPR). ARG is the rate of grouping the observations in the correct subpopulation. ACC is the ratio of variable truly classified to the number of variables. TPR is defined as the ratio of the number of true variables identified to the true number of active variables, and FPR the number of inactive variables to the number of inactive variables. These measures provide the statistical

$corr(x_i, x_j)$			0.5		0.75		0.95	
	t -Error	Normal t -Error		Normal t -Error		Normal t -Error		Normal
\rm{ARG}	0.95	0.93	0.93	0.90	0.88	0.83	0.80	0.75
ACC	0.99	0.96	0.99	0.94	0.97	0.93	0.88	0.80
TPR	1.00	0.98	0.98	0.93	0.95	0.92	0.90	0.81
FPR.	0.01	0.01	$0.02\,$	0.08	0.01	0.01	0.11	0.20

Table 1: Simulation Study

The performance of our proposed variable selection approach under different cross correlations between covariates. t-Error indicates the regression errors follow a Student-t distribution with the degree of freedom of $v = 5$, whereas Normal indicates the regression errors follow a normal distribution.

assessment of the proposed approach for variable selection. Additionally, we investigate the effect of outliers on the variable selection problem. To this end, we compare all the above measures with our proposed variable selection approach while ω is assumed to be fixed and equal to 1 in equation [\(1\)](#page-9-0).

The ARG, ACC, TPR, and FPR over 100 simulation runs for four different pair correlations between covariates are summarized in Table [1.](#page-16-0) The performance of classification by applying our proposed approach produces comparable results. Even there exists high collinearity in the design matrix, our proposed model is still able to identify important variables and well classify the observations. Additionally, when the simulated data contains observations with longer than normal tails or atypical observations, the use of mixture model with *t*-distribution for error terms leads to fewer misallocations.

5 Empirical Results

In order to make a comparable study, we apply our proposed Bayesian framework to the measures of exchange market pressures (EMP) analyzed in [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4). [Feld](#page-37-4)[kircher et al.](#page-37-4) [\(2014\)](#page-37-4) consider a large number of leading indicators that have been discussed in the early warning literature, covering a wide range of different factors, including data on financial conditions, foreign reserve adequacy, macroeconomic policies, institutional features, monetary policy regimes and more. The dataset is balanced and the candidate covariates are measured annually as of end-2007.^{[4](#page-17-1)} In total, there are 58 potential leading indicators of EMP for a broad global sample of 149 countries. We use the same variable names as in [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4), and the full name of each variable can be found in the Appendix (Table [A1\)](#page-16-0).^{[5](#page-17-2)} The dependent variable of interest is the EMP index on a quarterly basis.^{[6](#page-17-3)} The EMP measure consists of the percentage change in the exchange rate (positive values denote percentage depreciation) and percentage loss of reserves.[7](#page-17-4) Higher values of EMP indicate greater pressure of exchange market. Figure [2](#page-18-0) presents the distribution of EMP during the recent crisis across regions of countries. It is remarkable how extreme and widespread across regions of countries were external pressures, ranging from highs experienced in Slovak Republic (101%), Venezuela (92%) and Estonia (88%) to low values for countries such as China, Bolivia and Hong Kong. This observation is consistent with [Aizenman and Hutchison](#page-36-5) [\(2012\)](#page-36-5), who find that there is considerable heterogeneity in their response. Emerging markets differ most from other country groups in the adjustment mechanism. With the "fear of reserve

⁴The data are available from http://feldkircher.gzpace.net/pages/replication_JIMF.RData.

 5 For details of definitions and sources for these variables, see [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4).

⁶In this paper, we focus on two versions of EMP to capture the different aspects of external pressures facing each market during the crisis period of 2007Q3–2010Q2. The first measure is the maximum EMP during the crisis $(EMPu_{max})$, the second is the maximum EMP normalized to the average pre-crisis EMP $(EMPu_{max.0006})$. While our main discussion is based on $EMPu_{max}$, we use $EMPu_{max.0006}$ to check the consistency and robustness of our variable selection results.

⁷See, e.g., [Aizenman and Hutchison](#page-36-5) [\(2012\)](#page-36-5); [Aizenman et al.](#page-36-6) [\(2010\)](#page-36-6) for the detail on EMP.

loss", the absorption of the shock facing emerging markets was mainly through exchange rate depreciation rather than international reserves depletion.

Figure 2: The distribution of EMP during the global financial crisis across regions of countries.

As the first step, we use information criteria based on the model's log likelihood to determine the number of mixture clusters m . As such, we estimate the finite mixture model for several clusters.^{[8](#page-18-1)} In addition to the information criteria such as the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and the integrated completed likelihood information criterion (ICL; [Biernacki et al.](#page-36-7) [\(2000\)](#page-36-7)), we also consider the Corrected Akaike Information Criterion (CAIC) and the Akaike Information Criterion 3 (AIC3) for robustness as in [Owen et al.](#page-39-9) [\(2009\)](#page-39-9). The results in Table [2](#page-20-0) show that all the information criteria are

 8 To avoid the local maximum in the EM algorithm, we try 10,000 starting values and report the estimation results with the highest log-likelihood value. We also standardize all but dummy variables prior to analysis to facilitate convergence when the number of the considered variables is large.

consistently in favor of the model with two clusters $(m = 2)$ over the linear model $(m = 1)$. This result suggests that the assumption of parameter homogeneity for cross-country EMP may be unrealistic. In particular, there is strong evidence against the homogeneous parameter model $(m = 1)$. This result is in line with other studies on heterogeneity of exchange rate movements, such as [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4) and [Fratzscher](#page-38-9) [\(2009\)](#page-38-9). Figure [3](#page-20-1) show the cluster memberships for the countries included in our data set based on the posterior cluster membership probabilities. A country is assigned to a particular cluster if its estimated posterior probability of being in this cluster is greater than that of being in others.

The figure shows that the majority of countries belongs to cluster 1, while nearly 10% of countries are in another cluster.^{[10](#page-19-1)} It is worth noting the cluster memberships do not match with pre-defined regional segmentation or country-specific characteristics. Of those in cluster 2, many countries are severely affected by sharp drops in primary commodity exports due to falling prices and demand for their commodities (e.g., Pakistan, Venezuela, Malawi, and Belarus). The decline in export earnings along with withdrawal of short-term foreign capital are always accompanied by serious balance of payments problems. In addition, cluster 2 includes four countries adopting the euro during the crisis period (Malta, Cyprus, Estonia and Slovak Republic) which are entered as the dummy variable of euroAdopt in [Feldkircher](#page-37-4) [et al.](#page-37-4) $(2014).$ $(2014).$ ^{[11](#page-19-2)}

A fixed-width approach was taken in which the MCMC scheme ran for 1 million iterations

⁹ It should be noted that the models with more than two clusters fail to converge.

¹⁰There are 14 countries in cluster 2 spread across a number of regions, including Europe (Malta and Cyprus), Asia (Australia, Pakistan and Sri Lanka), Latin America (Ecuador and Venezuela), Africa (Malawi, Seychelles and Zambia), CIS (Belarus and Ukraine) and CEEC (Estonia and Slovak Republic).

¹¹Interestingly, while this dummy is identified as a robustly important indicator with substantial posterior probability in [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4), it only receives marginal support from our analysis. That is, the results from our proposed variable selection indicates this country-specific dummy variable does not have a significant effect on EMP. Therefore, the finite mixture model with 2 clusters can uncover the heterogeneity in EMP.

Number of Clusters log-likelihood AIC CAIC AIC3			-BIC -	-ICL
	-595.82	1311.64 1551.88 1371.64 1491.88 1491.88		
	-149.83	541.67 1026.15 662.67 905.14 905.35		

Table 2: Determination of the Number of Mixture Clusters

This table present values of the information criteria for the finite mixture model with different number of clusters. The number in bold style indicates the preferred model with smallest information criteria.

Figure 3: The posterior cluster memberships in the finite mixture model with two clusters.

to ensure MCSEs were 0.001 or smaller [\(Flegal et al., 2008\)](#page-37-8).^{[12](#page-20-2)} The posterior samples are then used to estimate the posterior quantity of interest. For the sake of illustration, we only present the highest ranked leading indicators for the cross-country EMP, and the full results can be found in Appendix (Table [A2\)](#page-20-0).^{[13](#page-20-3)} For each variable we report its associated posterior inclusion

 12 In our case, the largest MCSE of the posterior probability of the indicator variable equal to 1 was less than 0.001, indicating that a sufficient number of samples were drawn.

¹³Although [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4) consider interaction terms between the pre-crisis inflation and several variables, allowing for such interactions would lead to an impractically large model/variable space for our current application.

probability and the posterior mean of regression coefficients in Table [3.](#page-24-0) The variables are ordered based on the posterior inclusion probabilities. We consider $v = 5, 25, 100$ to see whether the results are robust to deviations from a normal distribution. If outliers are more likely to exist, a smaller value of v should be considered. While the results of variable selection are generally similar between $v = 25$ and $v = 100$, the rankings and the posterior estimates are somewhat different when $v = 5$, particularly, the sign of the regression coefficient can change with the degrees of freedom. For example, for cluster 2, $dom.credit_06$ is negatively correlated with EMP for $v = 5$, whereas the opposite is produced for $v = 25$ and $v =$ 100. Examples of this type include $ext{.debt.exp_06}, ext{.debt.getp_06}, kof_cultProx_06,$ dGap 0006, and so on. These observations suggest the outliers should be accommodated and so we restrict our main discussion to the case of $v = 5^{14}$ $v = 5^{14}$ $v = 5^{14}$.

In Table [3,](#page-24-0) if we use the threshold value of 0.5 for posterior probability, there are 5 and 15 leading indicators across two clusters of countries, respectively, that are signifi-cantly correlated with external market pressures.^{[15](#page-21-1)} Although two clusters have a different set of important risk factors for the early warning models, variables shared in common include the growth rate in GDP per capita (chg_rgdpcap0006), the share of money supply in GDP (money.gdp 06), and the globalization indicators ($kof_poltGlob_06$ and kof infFlows 06). Of these variables, similar marginal effects are observed with different magnitudes across two groups of countries. Money supply, however, presents an opposite effect on the EMP between two clusters. The posterior estimates in Table [3](#page-24-0) show that an increase in money supply reduces pressure on the exchange market in cluster 1 countries. By contrast, for countries in cluster 2, money supply constitutes a waste of resources for the economy, subsequently amplifying the pressure on the exchange market. Finally, other variables

¹⁴Full results can be found in Appendix [A.1](#page-32-0)

¹⁵For cluster 2, tradeExposureEU15.gdp -0006 and imp -0206 are marginally significant with the posterior probability close to 0.5.

that have been previously flagged as important determinants of EMP, such as imbalances in the current account, international reserves or real exchange rate misalignment—although having their expected signs—do not appear robust in our data.

To check the consistency and robustness for the results in Table [3,](#page-24-0) we consider an alternative EMP measure, $EMPu_{max,0006}$, as the response variable. This variable is the maximum EMP normalized to the pre-crisis EMP average. As shown in Table [4,](#page-25-0) two striking observations can be made. First, while the rankings are generally similar, there are more significant covariates with the PIP above 0.5, particularly in cluster 1^{16} 1^{16} 1^{16} For example, the average pre-crisis inflation rate, **infl_0006**, is ranked 10 with the PIP of 0.611, but it only receives a moderate support when the maximum EMP is considered. This variable is also one of a few robust leading indicators found in [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4). Our result also support the positive role of the price stability in containing the external market pressures as in [Feld](#page-37-4)[kircher et al.](#page-37-4) [\(2014\)](#page-37-4). Second, the average pre-crisis EMP, **EMP 0006**, is robustly important and negatively correlated with the EMP during the crisis. This result is also consistent with [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4). Overall, the results from Table [4](#page-25-0) corroborate the findings of Table [3](#page-24-0) even when a different measure of EMP is considered.

The robust FMR model allows us to identify outliers based on the posterior estimate of $E(\omega|y)$ shown in Figure [4.](#page-23-0) An observation is considered as a potential outlier when its corresponding estimate $E(\omega|y)$ is greater than 2.5, which implies its variation is 2.5 times larger than the average level across all the observations in the data set. The countries recognized as outliers include China, Mauritania, Seychelles, Venezuela, and the U. S. It is interesting to note that four out of five outlying countries are emerging/developing markets which include the severely affected countries suffered from the abrupt declines in commodity

¹⁶The list includes **EMP**_0006, infl_0006, openness_0206, kof_persCont_06, merchTrade.gdp_0006, imp 0206, dGap 0006Exo, exp 0206, tradeExp.US.gdp 0006, and dom.credit 06.

Figure 4: Countries that have high estimates of ω 's.

exports and the least affected country of China with the buffer of international reserves. Finally, the U. S. was the epicenter of the recent global financial crisis.

		Cluster 1			Cluster 2	
Rank	Variable	PM	PIP	Variable	PM	PIP
1	chg_r gdp $cap0006$	0.066	0.881	chg_r gdp $cap0006$	0.131	0.727
$\overline{2}$	kof_poltGlob_06	0.083	0.812	openness_0206	0.174	0.641
3	$dGap_0006$	0.069	0.545	merchTrade.gdp_0006	0.115	0.598
4	money.gdp_06	-0.033	0.517	kof_infFlows_06	0.094	0.570
5	kof_infFlows_06	0.044	0.504	$tradeExposureEU15_0006$	0.096	0.561
6	$dGap_0006Exo$	0.055	0.486	kof_poltGlob_06	0.042	0.529
7	kof_overallGlob_06	0.034	0.477	kof_overallGlob_06	0.055	0.527
8	infl_0006	0.139	0.412	dom.credit_06	-0.011	0.523
9	kof_persCont_06	0.037	0.405	money.gdp_06	0.004	0.522
10	dom.credit_06	-0.022	0.399	kof_persCont_06	0.079	0.512
11	outputGap_06Exo	0.111	0.389	adv.claims.gdp_06	0.068	0.507
12	openness_0206	0.008	0.380	petrol.to.Exp_0006	0.203	0.507
13	invRate.gdp_0006	0.084	0.376	dGap_0006Exo	0.108	0.502
14	merchTrade.gdp_0006	-0.009	0.374	ext.debt.exp_06	0.002	0.496
15	imp_0206	0.028	0.358	$tradeExposureEU15.gdp_0006$	0.139	0.495
16	$exp_0.0206$	-0.043	0.348	imp_0206	0.086	0.494
17	ext.debt.exp_06	0.001	0.325	$ext.debt.gdp_06$	0.004	0.492
18	trade.balance_0206	-0.053	0.316	int.res.ext.debt_06	0.031	0.470
19	$intres.gdp_0$	-0.051	0.310	genGovDebt.gdp_06	0.067	0.465
20	kof_cultProx_06	-0.023	0.307	$\exp_0.0206$	0.091	0.461
				.		

Table 3: Results of Robust Variable Selection for $EMPu_{max}$

Note. The table represents a snapshot of the full results and presents the posterior inclusion probability (PIP) of the 20 highest ranked variables across two clusters. PM stands for the posterior mean of the regression coefficient. The estimation of the regression coefficient is based on the Rao-Blackwellized estimators. The degrees of freedom for t-errors assumed to be $v = 5$ in our robust variable selection approach. The variables are ordered by their posterior probabilities. The full name of each variable refers to the Appendix (Table [A1\)](#page-16-0).

		Cluster 1				Cluster 2	
Rank	Variable	PM	PIP	Variable	PM	PIP	
1	kof_poltGlob_06	0.104	0.850	$chg_r gdpcap0006$	0.187	0.805	
$\overline{2}$	chg_r gdp $cap0006$	0.044	0.828	openness_0206	0.206	0.700	
3	money.gdp_06	-0.046	0.772	$tradeExposureEU15_0006$	0.140	0.682	
4	$dGap_0006$	0.092	0.753	merchTrade.gdp_0006	0.139	0.650	
5	EMP ₋₀₀₀₆	-0.558	0.740	kof_infFlows_06	0.100	0.605	
6	kof_infFlows_06	0.057	0.708	money.gdp_06	0.061	0.570	
7	kof_overallGlob_06	-0.020	0.656	dom.credit_06	-0.012	0.569	
8	invRate.gdp_0006	0.196	0.656	kof_poltGlob_06	0.022	0.567	
9	outputGap_06Exo	0.204	0.640	kof_overallGlob_06	0.040	0.560	
10	infl_0006	0.249	0.611	kof_persCont_06	0.074	0.546	
11	openness_0206	0.032	0.611	imp_0206	0.112	0.541	
12	kof_persCont_06	0.057	0.597	ext.debt.exp_06	-0.003	0.539	
13	merchTrade.gdp_0006	-0.024	0.588	genGovDebt.gdp_06	0.094	0.537	
14	imp_0206	0.043	0.583	petrol.to.Exp_0006	0.195	0.528	
15	$dGap_0006Exo$	0.032	0.562	$ext.debt.gdp_06$	-0.023	0.521	
16	$exp_0.0206$	-0.017	0.540	tradeExposureEU15.gdp_0006	0.127	0.520	
17	tradeExp. US. gdp.0006	-0.147	0.502	$dGap_0006Exo$	0.064	0.520	
18	dom.credit_06	-0.020	0.501	adv.claims.gdp_06	0.011	0.513	
19	kof_cultProx_06	-0.036	0.490	int.res.ext.debt_06	0.058	0.501	
20	$chg.money. gdp_0006$	0.026	0.465	\exp_0206	0.097	0.501	
\cdots				\cdots			

Table 4: Results of Robust Variable Selection for $EMPu_{max.0006}$

Note. The table represents a snapshot of the full results and presents the posterior inclusion probability (PIP) of the 20 highest ranked variables across two clusters. PM stands for the posterior mean of the regression coefficient. The estimation of the regression coefficient is based on the Rao-Blackwellized estimators. The degrees of freedom for t-errors assumed to be $v = 5$ in our robust variable selection approach. The variables are ordered by their posterior probabilities. The full name of each variable refers to the Appendix (Table [A1\)](#page-16-0).

6 Conclusion

In this paper, we consider a robust Bayesian variable selection to the FMR models in the context of the recent global financial crisis. In the process we have demonstrated when the design matrix is not of full rank, an alternative to the g-prior should be used to circumvent the collinearity problem. This work has the potential to have a broad and immediate impact on the variable selection problem when the data is heterogeneously distributed across different subgroups of interest.

Our results are more optimistic than those of [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4) and [Rose and](#page-39-0) [Spiegel](#page-39-0) [\(2010,](#page-39-0) [2011,](#page-39-1) [2012\)](#page-39-2), who investigate which of the previously suggested early warning indicators are effective in explaining the cross-country incidence of the late-2000's crisis. Rose and Spiegel find that equity prices are relatively useful in explaining crisis incidence, but in general their message is skeptical. In comparison to [Frankel and Saravelos](#page-37-0) [\(2012\)](#page-37-0), who present more optimistic findings concerning the usefulness of early warning indicators (specifically they report that the level of reserves and real appreciation are effective leading indicators), we find different indicators more useful for different types of countries.

In summary, we are confident that Bayesian variable selection approach to a mixture of regression models provides an important application to uncovering underlying structure in covariates, and identify the determinants of external market pressures. We demonstrate the practicality, efficacy and feasibility of a general Bayesian solution to the variable selection problem in mixture regression models. We also expect that the Bayesian framework can complement the growing empirical literature of early warning systems of crisis events. A

possible extension of this work would be to relax the assumption of the fixed number of clusters, that is, assuming M is unknown. A starting point is to adopt a hierarchical Bayesian nonparametric mixture model to estimate M based on posterior probability [\(Yau and Holmes,](#page-40-2) [2011\)](#page-40-2). Extensions to the subpopulation distribution, other than the normal, for inference involving mixed data types would be a promising direction for future research.

Appendix

A Posterior distribution and full conditionals

In this section, we provide the details of derivation of full conditional distribution of each parameter with the sampling and prior distributions specified in Section [3.](#page-8-0) The joint posterior distribution is derived as follows. For notation convenience, we set $\theta = (\theta_1, \dots, \theta_M)$, $\theta_m =$ $(\beta_m, \sigma_m^2, \rho_m, \omega_m, r_m)$ with $\beta_m = (\beta_{m1}, \ldots, \beta_{mp}), \omega_m = (\omega_{m1}, \ldots, \omega_{mn_i}), r = (r_{m1}, \ldots, r_{mp}),$ G_m contains the members in component m, and y_m is a vector consisting of observations in component m , and x_m is the corresponding design matrix. The complete likelihood of the mixture regression model is given by

$$
\ell(y, z | \theta) = \prod_{i=1}^{n} \rho_{z_i} f(y_i | \theta_{z_i}) = \prod_{m=1}^{M} \rho_m^{n_m} \left[\prod_{i \in G_m} f(y_i | \theta_{z_i}) \right].
$$

Combining the likelihood and the priors $\pi(\theta)$, we have the posterior distribution as follows

$$
p(\theta, z|y) \propto \prod_{m=1}^{M} \rho_m^{n_m} \left[\prod_{i \in G_m} f(y_i|\theta_{z_i}) \right] \pi(\theta).
$$

The explicit representation of posterior distribution is

$$
p(\theta, z|y) \propto p(y, z|\beta, \sigma^2, r, \omega, \rho)p(\beta|r, \tau^2)p(\sigma^2)p(\nu)p(\rho)
$$

\n
$$
\propto \prod_{m=1}^{M} \rho_m^{n_m} \left(\prod_{i \in G_m} \left(\frac{1}{\sigma_m^2 \omega_i} \right)^{1/2} \right) \exp \left\{ - \frac{(y_m - X_m(r_m)\beta_m(r_m))' \Omega_m^{-1} (y_m - X_m(r_m)\beta_m(r_m))}{2\sigma_m^2} \right\}
$$

\n
$$
\times \prod_{m=1}^{M} \exp \left\{ - \frac{\beta'_m(r_m)\Lambda_m^{-1}\beta_m(r_m)}{2} \right\}
$$

\n
$$
\times \prod_{m=1}^{M} \left(\frac{1}{\sigma_m^2} \right)^{a_{m_0}/2+1} \exp \left\{ - \frac{b_{m_0}}{2\sigma_m^2} \right\}
$$

\n
$$
\times \prod_{m=1}^{M} \prod_{\{j: r_m \neq 0\}} \left(\frac{1}{\tau_{mj}^2} \right)^{a_{r_m j_0}/2+1} \exp \left\{ - \frac{b_{r_m j_0}}{2\tau_{m j}^2} \right\}
$$

\n
$$
\times \prod_{i=1}^{M} \rho_m^{\alpha_m - 1}
$$

\n
$$
\times \prod_{i=1}^{n} \left(\frac{1}{\omega_i} \right)^{a_{\omega_{m} j_0}/2+1} \exp \left\{ - \frac{b_{\omega_{m} j_0}}{2\omega_i} \right\}
$$

\n
$$
\times \prod_{m=1}^{M} \prod_{j=1}^{p} (d_{m,j})^{r_{m,j}} (1 - d_{m,j})^{1 - r_{m,j}},
$$

where $n_m = #\{i \in G_m\}$, the number of members in component m , Ω_m is a diagonal matrix of components, ω_i corresponding to i in component m, and Λ is also a diagonal matrix of components τ_j^2 when $r_{mj} \neq 0$ in component m.

The posterior quantities of interest are the probability of a variable included in the model and its expected estimate of regression coefficient, $p(r_{mj} = 1|y)$ and $E(\beta_{mj}|y)$, respectively. These quantities are analytically intractable and must be approximated with Monte Carlo methods. We will describe a particular MCMC method in the subsequent section. A naïve approach would be to construct an MCMC sampler having the full posterior $p(\theta, z|y)$ as the invariant density. Next, we give the conditional probability of each parameter. The

parameters in each component are then updated individually using a Gibbs sampler (where available), or a Metropolis-Hastings sampling algorithm. For ease of notation, we drop all required parameters in each conditional distribution.

1. The conditional probability of latent variable z_i is

$$
p(z_i = m|y) \propto \rho_m \phi(X_i(r_m)\beta_m(r_m), \omega_i \sigma_m^2),
$$

where $\phi(\mu_{z_m}, \sigma_{z_m}^2)$ stands for the normal density function with mean μ_{z_m} and variance $\sigma_{z_m}^2$.

2. The conditional distribution of ρ follows a Dirichlet distribution given by

$$
\rho \sim \text{Dirichlet}(n_1 + \alpha_1, \ldots, n_m + \alpha_m).
$$

3. The conditional distribution of σ_m^2 is

$$
p(\sigma_m^2|y) \propto \left(\frac{1}{\sigma_m^2}\right)^{n_m/2} \exp\left\{-\frac{\left(y_m - X_m(r_m)\beta_m(r_m)\right)'\Omega_m^{-1}\left(y_m - X_m(r_m)\beta_m(r_m)\right)}{2\sigma_m^2}\right\}
$$

$$
\times \left(\frac{1}{\sigma_m^2}\right)^{a_{m_0}/2 + 1} \exp\left\{-\frac{b_{m_0}}{2\sigma_m^2}\right\}
$$

that is, σ_m^2 has an inverse Gamma distribution given by

$$
\sigma_m^2 \sim IG\left(\frac{a_m}{2}, \frac{b_m}{2}\right),\,
$$

where

$$
a_m = n_m + a_{m_0}
$$

$$
b_m = (y_m - X_m(r_m)\beta_m(r_m))' \Omega_m^{-1} (y_m - X_m(r_m)\beta_m(r_m)) + b_{m_0}
$$

4. The conditional distribution of $\beta_{mj}(r_{mj})$ when $r_{mj} \neq 0$ is

$$
p(\beta_m(r_m)|y) \propto \exp\left\{-\frac{(y_m - X_m(r_m)\beta_m'(r_m))\,\Omega_m^{-1}\,(y_m - X_m(r_m)\beta_m(r_m))}{2\sigma_m^2} - \frac{\beta_m'(r_m)\Lambda_m^{-1}\beta_m(r_m)}{2}\right\}
$$

$$
\propto \exp\left\{-\frac{\beta_m'(r_m)\,[X_m'(r_m)\Omega^{-1}X_m(r_m) + \sigma_m^2\Lambda_m^{-1}]\,\beta_m(r_m) - 2y_m'\Omega^{-1}X_m(r_m)\beta_m(r_m)}{2\sigma_m^2}\right\}
$$

$$
\propto \exp\left\{-\frac{1}{2}(\beta_m(r_m) - \mu_m)'\Sigma_m^{-1}(\beta_m(r_m) - \mu_m)\right\},
$$

.

where $\mu_m = \sum_m X'_m(r_m) \Omega_m^{-1} y_m / \sigma_m^2$ and $\Sigma_m^{-1} = (X'_m(r_m) \Omega_m^{-1} X_m(r_m) + \sigma_m^2 \Lambda_m^{-1}) / \sigma_m^2$.

That is,

$$
\beta_m(r_m)|y \sim N(\mu_m, \Sigma_m).
$$

5. The conditional distribution of τ_{mj}^2 is

$$
p(\tau_{mj}^2|y) \propto \exp\left\{-\frac{\beta_{mj}^2(r_{mj})}{2\tau_{mj}^2}\right\} \left(\frac{1}{\tau_{mj}^2}\right)^{a_{\tau_{mj_0}}/2+1} \exp\left\{-\frac{b_{\tau_{mj_0}}}{2\tau_{mj}^2}\right\}.
$$

That is,

$$
\tau_{mj}^2 \sim IG\left(\frac{a_{\tau_{mj_0}} + 1}{2}, \frac{\beta_{mj}^2(r_{mj}) + b_{\tau_{mj_0}}}{2}\right).
$$

6. The conditional distribution of ω_i is, given $i \in G_m$,

$$
p(\omega_i|y) \propto \left(\frac{1}{\omega_i}\right)^{1/2} \exp\left(-\frac{\left[y_i - x_{im}(r_{mi})\beta_{mj}(r_{mj})\right]^2}{2\omega_i \sigma_m^2}\right) \left(\frac{1}{\omega_i}\right)^{a_{\omega_{mj_0}}/2+1} \exp\left\{-\frac{b_{\omega_{mj_0}}}{2\omega_i}\right\}
$$

$$
\omega_i \sim IG\left(\frac{a_{\omega_{m j_0}}}{2}, \frac{\left[y_i - x_{im}(r_{mi})\beta_{mj}(r_{mj})\right]^2/\sigma_m^2 + b_{\omega_{mj_0}}}{2}\right).
$$

7. The conditional probability of r_{mj} is

$$
p(r_{mj}=1|r_{m,(-j)},y) = \frac{p(r_{m,j}=1|r_{m,(-j)},y)}{p(r_{mj}=1|r_{m,(-j)},y) + p(r_{mj}=0|r_{m,(-j)},y)}.
$$

where

$$
p(r_{m,j} = 1 | r_{m,(-j)}, y) \propto \exp \left\{ -\frac{(y_m - X_m(r_m)\beta_m(r_m))'\,\Omega_m^{-1}\,(y_m - X_m(r_m)\beta_m(r_m))}{2\sigma_m^2} \right\}
$$

$$
\times \exp \left\{ -\frac{\beta_m'(r_m)\Lambda_m^{-1}\beta_m(r_m)}{2} \right\} \times \left(\frac{1}{\tau_{mj}^2}\right)^{a_{\tau_{mj_0}}/2} \exp \left\{ -\frac{b_{\tau_{mj_0}}}{2\tau_{mj}^2} \right\}
$$

$$
\times \prod_{j=1}^p (d_{m,j})^{r_{m,j}} (1 - d_{m,j})^{1 - r_{m,j}},
$$

and $r_{m,(-j)}$ denotes a vector of r_{m} excluding $r_{mj}.$

A.1 Tables

Table A1: Variable Description for Cross-Country Exchange Market Pressures (Variables are measured the average over 2000–2006 unless stated otherwise)

Table $A1$ – *Continued from previous page*

Note. The original source of this table is from Table A2 in [Feldkircher et al.](#page-37-4) [\(2014\)](#page-37-4).

approach. The variables are ordered by the posterior probabilities. The full name of each variable refers to Table A1. approach. The variables are ordered by the posterior probabilities. The full name of each variable refers to Table [A1.](#page-16-0)

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