Progressive Taxation as an Automatic Destabilizer under Endogenous Growth*

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April 29, 2015

Abstract

It has been shown that in an otherwise standard one-sector real business cycle model with an indeterminate steady state under laissez faire, sufficiently progressive income taxation can stabilize the economy against aggregate fluctuations caused by agents' animal spirits. In this paper, we show that this earlier finding can be overturned within an identical model which allows for sustained endogenous growth. Specifically, progressive taxation may operate like an automatic destabilizer that leads to equilibrium indeterminacy and sunspot-driven cyclical fluctuations in an endogenously growing macroeconomy. This instability result is obtained under two tractable progressive tax policy formulations that have been considered in the literature.

Keywords: Progressive Income Taxation, Automatic Stabilizer, Equilibrium Indeterminacy, Endogenous Growth.

JEL Classification: E62, O41.

* *Preliminary and Incomplete.* Part of this research was conducted while Guo was a visiting research fellow at the Institute of Economics, Academia Sinica, whose hospitality is greatly appreciated. Of course, all remaining errors are our own.

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1 Introduction

[To be Completed]

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes its equilibrium conditions under a fiscal policy rule that exhibits continuously increasing average and marginal tax rates. Section 3 investigates the local stability properties associated with the economy's balanced growth path(s). Section 4 analytically examines the interrelations between equilibrium (in)determinacy and linearly progressive taxation within our endogenously growing macroeconomy. Section 5 concludes.

2 The Economy

Our analysis begins with incorporating a progressive fiscal policy rule *a la* Guo and Lansing (1998), which exhibits continuously increasing average and marginal tax rates, into the endogenous-growth version of Benhabib and Farmer's (1994, section 5) one-sector representative agent macroeconomy in continuous time. Households live forever, and derive utility from consumption and leisure. The production side consists of a social technology that displays increasing returns-to-scale due to positive productive externalities from aggregate capital and labor inputs. The government balances the budget each period by spending its tax revenue on goods and services that do not contribute to the households' utility or the firms' production. We assume that there are no fundamental uncertainties present in the economy.

2.1 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm i produces output y_{it} according to a Cobb-Douglas production function

$$y_{it} = x_t k_{it}^{\alpha} h_{it}^{1-\alpha}, \;\; 0 < \alpha < 1, \tag{1}$$

where k_{it} and h_{it} are capital and labor inputs, respectively, and x_t represents positive productive externalities that are taken as given by each individual firm. As in Benhabib and Farmer (1994), we postulate that externalities take the form

$$x_t = k_t^{1-\alpha} h_t^{(1-\alpha)\chi}, \;\; \chi \geq 0, \tag{2}$$

where k_t and h_t denote the economy-wide levels of capital and labor services. In a symmetric equilibrium, all firms make the same decisions such that $k_{it} = k_t$ and $h_{it} = h_t$, for all i and t. As a result, (2) can be substituted into (1) to obtain the following aggregate increasing returns-to-scale production function for total output y_t:

$$y_t = k_t \, h_t^{(1-\alpha)(1+\chi)}. \tag{3}$$

Notice that the economy exhibits sustained economic growth because the social technology (3) displays linearity in physical capital. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the representative firm's profit maximization problem are given by

$$r_t = \alpha \frac{y_t}{k_t}, \tag{4}$$

$$w_t = (1-\alpha)\, \frac{y_t}{h_t}, \tag{5}$$

where r_t is the capital rental rate and w_t is the real wage rate.

2.2 Households

The economy is also populated by a unit measure of identical infinitely-lived households, each of which maximizes a discounted stream of utilities over its lifetime

$$\int_0^\infty \left\{ \log c_t - A \frac{h_t^{1+\gamma}}{1+\gamma} \right\} e^{-\rho t} dt, \quad A > 0, \tag{6}$$

where c_t is consumption, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho > 0$ is the subjective rate of time preference. The budget constraint faced by the representative household is

$$c_t + i_t = (1-\tau_t)\,(r_t k_t + w_t h_t)\,, \tag{7}$$

where i_t is gross investment, and τ_t represents a proportional income tax rate. Investment adds to the stock of physical capital according to the following law of motion:

$$\dot{k}_t = i_t - \delta k_t, \quad k_0 > 0 \;\; \text{given}, \tag{8}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

In terms of the income tax rate, we adopt the sustained-growth version of Guo and Lansing's (1998, p.485, footnote 4) nonlinear tax formulation and postulate τ_t as

$$\tau_t = 1 - \eta \left(\frac{y_t^*}{y_t} \right)^{\phi}, \ \eta \in (0,1), \ \phi \in [0,1), \tag{9}$$

where y_t $(= r_t k_t + w_t h_t)$ is the household's taxable income, and y_t^* denotes a benchmark level of income that is taken as given by the representative agent. In our model with ongoing growth, y_t^* is set equal to the level of per capita output on the economy's balanced growth path (BGP) whereby $\frac{\dot{y}_t^*}{y_t^*} = \theta$ for all t.[1] The parameters η and ϕ govern the level and slope of the tax schedule, respectively. When $\phi > (<) 0$, the tax rate τ_t is monotonically increasing (decreasing) with the household's income y_t, *i.e.* agents with income above y_t^* face a higher (lower) tax rate than those with income below y_t^*. When $\phi = 0$, all households face the constant tax rate $1 - \eta$ regardless of the level of their taxable income.

With regard to the progressivity features of the above tax structure, we note that the marginal tax rate τ_{mt}, defined as the change in taxes paid by the household divided by the change in its taxable income, is given by

$$\tau_{mt} = \frac{\partial(\tau_t y_t)}{\partial y_t} = \tau_t + \eta\phi \left(\frac{y_t^*}{y_t} \right)^{\phi}. \tag{10}$$

In this paper, our analyses are restricted to the environment in which the government does not have access to lump-sum taxes or transfers, hence $\tau_t > 0$ and $\tau_{mt} > 0$ are imposed. We also require $\tau_t < 1$ to ensure that the government can not confiscate all productive resources, and $\tau_{mt} < 1$ so that households have an incentive to provide labor and capital services to firms. Along the economy's balanced-growth equilibrium path with $y_t = y_t^*$, these considerations imply that $\eta \in (0,1)$ and $\frac{\eta - 1}{\eta} < \phi < 1$, where $\frac{\eta-1}{\eta} < 0$. Next, in order to satisfy the second-order conditions for the representative household's dynamic optimization problem, its budget constraint (7) needs to be jointly concave in the state and control variables, *i.e.* k_t, c_t and h_t. It turns out that this requirement, together with $0 < \eta < 1$ and $\phi < 1$, yields a more restrictive lower bound on the tax-slope parameter $\phi \geq 0$. Given these restrictions on η and ϕ, it is straightforward to show that when $\phi > 0$, the marginal tax rate (10) is higher than the average tax rate given by (9). In this case, the tax schedule is said to be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the level of $1 - \eta$, thus the tax

[1] In order for a balanced-growth equilibrium to exist in our model economy, the household's taxable income y_t in equilibrium needs to grow at the same rate as the baseline level of income y_t^*. The constant growth rate θ for y_t^* will be endogenously determined through the model's equilibrium conditions (see equation 21).

schedule is "flat". Notice that the the original Benhabib-Farmer economy without income taxation corresponds to our model under $\eta = 1$ and $\phi = 0$.

As in Guo and Lansing (1998), we postulate that agents take into account the way in which the tax schedule affects their net earnings when they decide how much to work, consume and invest over their lifetimes. Consequently, it is the marginal tax rate of income τ_{mt} that governs the household's economic decisions. The first-order conditions for the representative household with respect to the indicated variables and the associated transversality condition (TVC) are

$$c_t \quad : \quad \frac{1}{c_t} = \lambda_t, \tag{11}$$

$$h_t \quad : \quad \frac{A h_t^{\gamma}}{\lambda_t} = \underbrace{\eta(1-\phi)\left(\frac{y_t^*}{y_t}\right)^{\phi}}_{(1-\tau_{mt})} \underbrace{(1-\alpha)\frac{y_t}{h_t}}_{w_t}, \tag{12}$$

$$k_t \quad : \quad \lambda_t \left[\underbrace{\eta(1-\phi)\left(\frac{y_t^*}{y_t}\right)^{\phi}}_{(1-\tau_{mt})} \underbrace{\alpha\frac{y_t}{k_t}}_{r_t} - \delta \right] = \rho\lambda_t - \dot{\lambda}_t, \tag{13}$$

$$\text{TVC} \quad : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0, \tag{14}$$

where $\lambda_t > 0$ is the the Lagrange multiplier on the budget constraint (7), (12) equates the slope of the household's indifference curve to the after-tax real wage, (13) is the consumption Euler equation, and (14) is the transversality condition. Notice that under the restrictions on η and ϕ specified above, equations (11)-(13) are not only necessary, but also sufficient conditions for the unique global maximum of the household's optimization problem.

2.3 Government

The government sets the tax rate τ_t according to (9), and balances its budget each period. Hence, its instantaneous budget constraint is given by

$$g_t = \tau_t y_t, \tag{15}$$

where g_t is public spending on goods and services. With the government, the aggregate resource constraint for the economy is

$$c_t + \dot{k}_t + \delta k_t + g_t = y_t. \tag{16}$$

2.4 Balanced Growth Path

We focus on the economy's balanced growth path(s) along which labor hours are stationary; whereas output, consumption, and physical capital all grow at a common constant rate θ. To facilitate the subsequent dynamic analyses, we adopt the variable transformation $z_t \equiv \frac{c_t}{k_t}$. Our model's equilibrium conditions (with $\frac{\dot{y}_t^*}{y_t^*} = \theta$ imposed) can then be collapsed into the following autonomous dynamical system:

$$\frac{\dot{h}_t}{h_t} = \frac{\left(1-\phi-Ah_t^{1+\gamma}\right)z_t-\phi\left(\theta+\delta\right)-\rho}{\left(1-\phi\right)\left(1-\alpha\right)\left(1+\chi\right)-1-\gamma}, \tag{17}$$

$$\frac{\dot{z}_t}{z_t} = \left\{1+\frac{A[\left(1-\phi\right)\alpha-1]h_t^{1+\gamma}}{\left(1-\phi\right)\left(1-\alpha\right)}\right\}z_t-\rho. \tag{18}$$

An interior balanced-growth equilibrium is characterized by a pair of positive real numbers (h^*, z^*) which satisfy $\dot{h}_t = \dot{z}_t = 0$. It is straightforward to derive from (17) and (18) that z^* is the solution(s) to the following nonlinear equation:

$$z^* = \rho+\eta\left[1-\alpha\left(1-\phi\right)\right]\left[\frac{Az^*}{\eta\left(1-\phi\right)\left(1-\alpha\right)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \equiv f\left(z^*\right), \tag{19}$$

and that the corresponding expressions for employment h^* together with the common rate of economic growth θ are

$$h^* = \left[\frac{Az^*}{\eta\left(1-\phi\right)\left(1-\alpha\right)}\right]^{\frac{1}{(1-\alpha)(1+\chi)-1-\gamma}}, \tag{20}$$

and

$$\theta = \frac{\alpha\left(1-\phi\right)z^*-\rho}{1-\alpha\left(1-\phi\right)}-\delta. \tag{21}$$

For the existence and number of the economy's interior balanced growth path(s), we use the right-hand side of (19) to obtain

$$f'\left(z^*\right) = \frac{\eta\left(1-\alpha\right)\left(1+\chi\right)\left[1-\alpha\left(1-\phi\right)\right]}{\left[\left(1-\alpha\right)\left(1+\chi\right)-1-\gamma\right]z^*}\left[\frac{Az^*}{\eta\left(1-\phi\right)\left(1-\alpha\right)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \lesseqgtr 0 \text{ when } \left(1-\alpha\right)\left(1+\chi\right)-1-\gamma \lesseqgtr 0, \tag{22}$$

and

$$f''(z^*) = \frac{(1+\gamma) f'(z^*)}{[(1-\alpha)(1+\chi) - 1 - \gamma] z^*} > 0, \tag{23}$$

regardless of whether $f'(z^*)$ is positive or negative. Therefore, the equilibrium z^* can be located from the (possibly more than one) intersection(s) of $f(z^*)$ and the 45-degree line in the positive quadrant. Section 3 below will show that the number of BGP's in our model is governed by the sign of $f'(z^*)$ or $(1-\alpha)(1+\chi) - 1 - \gamma$, which turns out to be identical to that in Benhabib and Farmer's (1994) economy under laissez faire.

3 Macroeconomic (In)stability

In terms of the local stability properties of a balanced-growth equilibrium path, we analytically compute the Jacobian matrix J of the dynamical system, defined by (17) and (18), evaluated at (h^*, z^*). The determinant and trace of the Jacobian are

$$Det = \frac{\alpha\phi(1-\phi)(1+\gamma)(z^*-\rho) z^*}{[1-\alpha(1-\phi)][(1-\phi)(1-\alpha)(1+\chi) - 1 - \gamma]}, \tag{24}$$

$$Tr = \rho - \frac{(1-\phi)(1-\alpha)(1+\gamma)(z^*-\rho)}{[1-\alpha(1-\phi)][(1-\phi)(1-\alpha)(1+\chi) - 1 - \gamma]}. \tag{25}$$

The equilibrium dynamics of our model's balanced growth path(s) are determined by comparing the eigenvalues of J that have negative real parts to the number of initial conditions in the dynamical system (17)-(18), which is zero because h_t and z_t both are non-predeterminded jump variables.[2] As a result, the BGP displays saddle-path stability and equilibrium uniqueness when both eigenvalues have positive real parts. If one or two eigenvalues have negative real parts, then the balanced-growth equilibrium path is locally indeterminate (*i.e.* a sink) and can be exploited to generate endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

In the reminder of this section, we examine the existence and number of the economy's interior balanced-growth equilibrium path(s), as well as their associated local dynamics, in three parametric configurations.

[2] Notice that k_0 does not introduce an initial condition to the dynamical system (17)-(18) because the period-0 values of h_0 and c_0 are both endogenously determined.

3.1 When $0 < \phi < 1$ and $(1-\alpha)(1+\chi) - 1 - \gamma < 0$

In this case, the fiscal policy rule (9) is progressive with $\phi \in (0,1)$, and the degree of productive externalities from labor hours in firms' production can be zero ($\chi = 0$). Based on (22) and (23), Figure 1 depicts that $f(z^*)$ is a downward-sloping and convex curve which intersects the 45-degree line once in the positive quadrant; hence there exists a unique balanced-growth equilibrium characterized by z^*. Regarding local dynamics, it is straightforward to show that the determinant (24) of the model's Jacobian matrix J is negative, indicating that the BGP exhibits equilibrium indeterminacy and belief-driven growth fluctuations.[3] On the contrary, Benhabib and Farmer (1994) find that the same parameterization yields local determinacy and saddle path stability without income taxation ($\eta = 1$ and $\phi = 0$). These results together imply that in sharp contrast to a conventional automatic stabilizer, progressive taxation may destabilize an endogenously growing macroeconomy by generating cyclical fluctuations driven by agents' animal spirits.

The intuition for the above indeterminacy result can be understood through the model's phase diagram illustrated in Figure 2. Using (17) and (18), we find that the equilibrium loci $\dot{h}_t = 0$ and $\dot{z}_t = 0$ are upward sloping, and that the $\dot{z}_t = 0$ locus is steeper than the positively-sloped stable arm (denoted as SS), followed by $\dot{h}_t = 0$. Next, start from a particular balanced growth path (h^*, z^*), and suppose that agents become optimistic about the economy's future. Acting upon this anticipation, households will invest more and consume less today, which in turn lead to another dynamic trajectory $\left\{h_t', z_t'\right\}$ that begins at (h_0', z_0') with $h_0' < h^*$ and $z_0' < z^*$. Figure 2 shows that for this alternative path to become a self-fulfilling equilibrium, the after-tax return on investment $(1-\tau_{mt})MPK_t$ must be monotonically increasing along the transitional path SS as the consumption-to-capital ratio $z_t \equiv \frac{c_t}{k_t}$ rises. From (3)-(5) and (9)-(12), it can be shown that

$$sign\left\{ \left. \frac{d\left[(1-\tau_{mt})MPK_t\right]}{dz_t} \right|_{SS} \right\} = sign\left\{ \frac{1}{z_t} + \frac{1+\gamma}{h_t} \underbrace{\left. \frac{dh_t}{dz_t} \right|_{SS}}_{\text{positive}} \right\} > 0. \tag{26}$$

As a consequence, agents' initial rosy expectation is validated under progressive income taxation.

[3] It can be shown that anlong any balanced-growth equilibrium path, $z^* > \rho$ because every term on the right-hand-side of equation (19) is positive. Moreover, since $0 < \phi < 1$ and $(1-\alpha)(1+\chi) - 1 - \gamma < 0$, the second bracket term in the denominator of (24), *i.e.* $[(1-\phi)(1-\alpha)(1+\chi) - 1 - \gamma]$, is negative.

3.2 When $0 < \phi < 1$ and $(1-\alpha)(1+\chi) - 1 - \gamma > 0$

Figure 3 shows that under progressive income taxation, $f(z^*)$ in this formulation is an upward-sloping convex curve with a positive vertical intercept ($= \rho$). Hence, the number of intersections between $f(z^*)$ and the 45-degree line in the positive quadrant can be zero, one or two. We proceed with first deriving the critical level of tax progressivity, denoted as $\hat{\phi}$, at which $f(z^*)$ is tangent to the 45-degree line such that there exists a unique BGP characterized by $\hat{z}$ and thus the equilibrium growth rate $\theta(\hat{z})$. Using (22) with $f'(\hat{z}) = 1$ and (19) evaluated at $\hat{z}$, it is straightforward to show that

$$\hat{z} = \frac{\rho(1-\alpha)(1+\chi)}{1+\gamma}, \tag{27}$$

and that $\hat{\phi} \in (0,1)$ is the unique solution to the following equation:[4]

$$\frac{\rho\left[(1-\alpha)(1+\chi) - 1 - \gamma\right]}{\eta(1+\gamma)\left[1 - \alpha\left(1 - \hat{\phi}\right)\right]} = \left[\frac{A\rho(1+\chi)}{\eta\left(1-\hat{\phi}\right)(1+\gamma)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}}. \tag{28}$$

Next, we find that an increase in the tax progressivity ϕ shifts the locus of $f(z^*)$ upwards because

$$\frac{\partial f(z^*)}{\partial \phi} = \frac{\eta\left[(1-\alpha)(1+\chi) - \alpha(1-\phi)(1+\gamma)\right]}{(1-\phi)\left[(1-\alpha)(1+\chi) - 1 - \gamma\right]}\left[\frac{Az^*}{\eta(1-\phi)(1-\alpha)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} > 0, \tag{29}$$

which in turn implies that our model possesses no (two) balanced growth path(s) provided $\phi > (<)\ \hat{\phi}$.[5] Hence, any small deviation from the balanced growth path with $\hat{z}$ and $\theta(\hat{z})$ will lead to the BGP's disappearance, or the emergence of dual BGP equilibria. This result indicates that the economy undergoes a saddle-node bifurcation which may cause the hard loss of equilibrium stability, *i.e.* a radical qualitative change in the behavior of the dynamical system (17)-(18) takes place, as the tax-slope parameter passes through the threshold value $\hat{\phi}$.

Figure 3 also shows that when the tax progressivity $\phi < \hat{\phi}$, there exist two interior balanced-growth equilibrium paths in our model characterized by z_1^* and z_2^*, where $z_1^* < \hat{z} < z_2^*$. Given $(1-\alpha)(1+\chi) - 1 - \gamma > 0$ within this specification, an increase in the consumption-to-capital

[4] Notice that the left-hand-side of (28) is increasing with respect to $\hat{\phi}$, whereas the right-hand-side is monotonically decreasing. It follows that there will be a unique intersection that determines $\hat{\phi}$.

[5] Since $0 < \alpha < 1$, the bracket term in the numerator of (29) is greater than $(1-\alpha)(1+\chi) - (1-\phi)(1+\gamma)$, which can be rewritten as $(1-\alpha)(1+\chi) - 1 - \gamma + \phi(1+\gamma) > 0$.

ratio leads to a higher level of hours worked (see equation 20). This in turn raise the marginal product of capital and the equilibrium growth rate (see equation 21), thereby $\theta(z_1^*) < \theta(\hat{z}) < \theta(z_2^*)$. To help understand the resulting local stability properties, we substitute (3) into the logarithmic version of the labor-market equilibrium condition (12), and find that the slope of the after-tax equilibrium wage-hours locus is given by $(1-\phi)(1-\alpha)(1+\chi)-1$, while the slope of the household's labor supply curve is γ (≥ 0). It turns out that the relative steepness of these two curves in the labor market plays an important role in affecting the local dynamics around both balanced-growth equilibria.

3.2.1 When $0 < \phi < \hat{\phi} < 1$ and $(1-\phi)(1-\alpha)(1+\chi)-1 < \gamma$

In this case, the tax progressivity is lower than $\hat{\phi}$ such that the model economy exhibits two interior balanced-growth equilibrium paths; and higher than the critical level $\phi^c \equiv 1 - \frac{1+\gamma}{(1-\alpha)(1+\chi)}$ such that the after-tax equilibrium wage-hours locus is flatter than the labor supply curve. As a result, $0 < \phi^c < \phi < \hat{\phi} < 1$ within this specification. Using (24), it is immediately clear that the two eigenvalues of the model's Jacobian matrix J are of opposite signs ($Det < 0$). Therefore, both BGP's are locally indeterminate that may lead to macroeconomic instability, which in turn implies that progressive income taxation operates like an automatic destabilizer raising the magnitude of business cycle fluctuations. We also find that the intuition for this indeterminacy result is identical to that in section 3.1, demonstrated by the phase diagram in Figure 2, when our model economy possesses a unique balanced-growth equilibrium. Moreover, this finding turns out to be exactly opposite to that obtained in Guo and Lansing (1998, p. 488) – a progressive tax policy (9) which satisfies the condition $(1-\phi)(1-\alpha)(1+\chi)-1 < \gamma$ will eliminate sunspot-driven fluctuations in the no-sustained-growth version of Benhabib and Farmer's (1994) economy with an indeterminate steady state under sufficiently strong increasing returns-to-scale in aggregate production.

3.2.2 When $0 < \phi < \hat{\phi} < 1$ and $(1-\phi)(1-\alpha)(1+\chi)-1 > \gamma$

In this case, $0 < \phi < \phi^c < \hat{\phi} < 1$ thus (i) there exists two interior BGP equilibria in the economy, and (ii) the after-tax equilibrium wage-hours locus is positively-sloped and steeper than the labor supply curve. Since $0 < \alpha, \phi < 1$ and $\gamma \geq 0$, together with $z_2^* > z_1^* > \rho > 0$ (see footnote ??) and $(1-\phi)(1-\alpha)(1+\chi)-1 > \gamma$, the Jacobian matrix J for this configuration possesses a positive determinant ($Det > 0$). Using (25), (27) and $z_1^* < \hat{z} < z_2^*$ as seen in Figure 3, it is straightforward to show that $Tr(z_2^*) < Tr(\hat{z}) < Tr(z_1^*)$, where $Tr(\hat{z})$ denotes the Jacobian's trace evaluated at $z^* = \hat{z}$ given by

$$Tr(\hat{z}) = \frac{\rho\phi\left[\alpha(1-\phi)(1-\alpha)(1+\chi)-1-\gamma\right]}{\left[1-\alpha(1-\phi)\right]\left[(1-\phi)(1-\alpha)(1+\chi)-1-\gamma\right]} \gtrless 0. \tag{30}$$

Without being able to obtain the analytical expressions of z_1^* and z_2^* from solving equation (19), we can not derive the exact condition that governs the local stability properties for this version of our model. As a result, numerical experiments are conducted to quantitatively explore the economy's equilibrium dynamics. Per the parameterization that is commonly adopted in the RBC-based indeterminacy literature, the capital share of national income, α, is chosen to be $\frac{1}{3}$; the time discount rate, ρ, is set equal to 0.01; the capital depreciation rate, δ, is fixed at 0.025; the household's labor supply elasticity, γ, is calibrated to be 0 (*i.e.* indivisible labor); and the preference parameter, A, is normalized to 1. In addition, we set the degree of productive externalities from hours worked $\chi = 0.6$,[6] and the tax-level parameter $\eta = 0.8$ based on the average value of Chen and Guo's (2013) year-by-year point estimates from the 1966-2005 U.S. federal individual income tax schedule.

Given these baseline parameter values, we find that $\phi^c = 0.0625$ and $\hat{\phi} = 0.9689$; and that the requisite condition $(1-\phi)(1-\alpha)(1+\chi)-1 > \gamma$ is satisfied for all positive values of the tax progressivity $\phi < \phi^c$. Next, since the bracket term in the numerator of (30), $\alpha(1-\phi)(1-\alpha)(1+\chi)-1-\gamma$, is now smaller than zero, $Tr(\hat{z})$ and thus $Tr(z_2^*)$ both will be negative. This implies that in the neighborhood of the BGP associated with z_2^* and $\theta(z_2^*)$, the model's Jacobian matrix J possesses a negative trace and a positive determinant. Therefore, the high-growth equilibrium path is a sink that exhibits indeterminacy and sunspots. On the other hand, we numerically verify that $Tr(z_1^*) > 0$ under the benchmark parameterization, hence the low-growth BGP associated with z_1^* and $\theta(z_1^*)$ displays saddle-path stability and equilibrium uniqueness in that both eigenvalues have positive real parts. As it turns out, the two interior balanced-growth equilibria in Benhabib and Farmer's (1994) laissez-faire economy also exhibit exactly the same local stability properties.

Figure 4 presents the phase diagram for the indeterminate high-growth BGP characterized by z_2^* and $\theta(z_2^*)$. As in Figure 2, the positively-sloped $\dot{h}_t = 0$ locus is flatter than $\dot{z}_t = 0$; however, the associated upward-sloping stable arms, denoted as SS_1 and SS_2 with

[6] Given $\alpha = \frac{1}{3}$ and $\gamma = 0$, the minimum level of labor externalities in firms' production that satisfies the condition needed for the possibility of multiple interior BGP's, $(1-\alpha)(1+\chi)-1-\gamma > 0$, is $\chi_{\min} = 0.51$. The (in)stability results reported in this subsection remain qualitatively unchanged over the range of $\chi \in [0.51, 0.8]$, where $\chi = 0.8$ leading to $(1-\alpha)(1+\chi) = 1.2$ is considered by Benhabib and Farmer (1994, p. 38) in their quantitative analysis.

each corresponding to a negative real eigenvalue,[7] are the flattest. When the representative household deviates from the original balanced-growth equilibrium (h^*, z^*) and lowers today's consumption because of its optimism about the economy's future, the resulting dynamic trajectory $\left\{h_t', z_t'\right\}$ will begin at (h_0', z_0') with $h_0' < h^*$ and $z_0' < z^*$. Figure 4 shows that when $z_t \equiv \frac{c_t}{k_t}$ increases monotonically along a convergent transitional path, the equilibrium after-tax marginal product of capital $(1-\tau_{mt})MPK_t$ must be rising in order to justify $\left\{z_t', x_t'\right\}$ as a self-fulfilling equilibrium path. Using (26), we find that this requisite condition is satisfied, *i.e.* $\frac{d[(1-\tau_{mt})MPK_t]}{dz_t} > 0$, along either SS_1 or SS_2, hence agents' initial optimistic expectations are validated.

3.3 When $\phi = 0$

In this case, the tax schedule (9) becomes flat with $\tau_t = \tau_{mt} = 1 - \eta$ for all t. Resolving our model with $\phi = 0$ yields the following single differential equation in $z_t \equiv \frac{c_t}{k_t}$ that describes its equilibrium dynamics:

$$\frac{\dot{z}_t}{z_t} = \eta\left(\alpha - 1\right)\left[\frac{Az_t}{\eta\left(1-\alpha\right)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} + z_t - \rho. \tag{31}$$

Following the same procedure as in section 2.4, an interior balanced-growth equilibrium is characterized by a positive real number z^* that satisfies $\dot{z}_t = 0$, which leads to

$$z^* = \rho + \eta\left(1-\alpha\right)\left[\frac{Az^*}{\eta\left(1-\alpha\right)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \equiv g\left(z^*\right), \tag{32}$$

where

$$g'\left(z^*\right) = \frac{\eta\left(1-\alpha\right)^2\left(1+\chi\right)}{\left[\left(1-\alpha\right)\left(1+\chi\right)-1-\gamma\right]z^*}\left[\frac{Az^*}{\eta\left(1-\alpha\right)}\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \lesseqgtr 0 \text{ when } \left(1-\alpha\right)\left(1+\chi\right)-1-\gamma \lesseqgtr 0, \tag{33}$$

and

$$g''(z^*) = \frac{\left(1+\gamma\right)g'\left(z^*\right)}{\left[\left(1-\alpha\right)\left(1+\chi\right)-1-\gamma\right]z^*} > 0. \tag{34}$$

We then linearize (31) around z^* and find that the model's equilibrium dynamics are determined by the eigenvalue $\left[1 - g'\left(z^*\right)\right]z^*$. Similar to Figure 1, $g(z^*)$ is a downward-sloping and

[7] Given the baseline parameterization mentioned above, the two eigenvalues associated with the high-growth BGP are found to be real and negative. This result continues to hold when χ takes alternative values from the interval [0.51, 0.8].

convex curve when $(1-\alpha)(1+\chi)-1-\gamma<0$, hence the economy possesses a unique balanced-growth equilibrium that turns out to a saddle path because the associated eigenvalue is positive and there is no given initial condition in equation (31). When $(1-\alpha)(1+\chi)-1-\gamma>0$, it is straightforward to show the existence of two interior BGP equilibria (similar to Figure 3) with $z_1^*<z_2^*$, $0<g^{'}(z_1^*)<1$ and $g^{'}(z_2^*)>1$. As a result, the high-growth equilibrium is a sink in that $\left[1-g^{'}(z_2^*)\right]z_2^*<0$, whereas the low-growth equilibrium is a saddle due to $\left[1-g^{'}(z_1^*)\right]z_1^*>0$. These findings illustrate that our endogenously growing macroeconomy under flat income taxation exhibits the same local stability properties as those in Benhabib and Farmer's (1994) otherwise identical model under laissez faire.

4 Linearly Progressive Taxation

Dromel and Pintus (2007) point out that the feature of continuously increasing average and marginal tax rates *a la* equation (9) is not consistent with the progressive tax policies observed in many developed countries, hence they incorporate an alternative fiscal formulation into Benhabib and Farmer's (1994) indeterminate one-sector real business cycle model under laissez faire and no endogenous growth. Specifically, a constant marginal tax rate is imposed on the household's taxable income when it exceeds a fixed exemption threshold, namely linearly progressive taxation is levied. As in Guo and Lansing (1998), these authors find that the economy will be immune to sunspot-driven cyclical fluctuations when the exemption threshold is larger than a critical level, or when the associated tax progressivity is sufficiently high.

In this section, we adopt the time-varying version of Dromel and Pintus' (2007) linearly progressive tax formulation and then examine its (de)stabilization effects within the endogenously growing macroeconomy described in section 2. The budget constraint faced by the representative household is now changed to

$$c_t+\dot{k}_t+\delta k_t=y_t-\underbrace{\tau\left(y_t-E_t\right)}_{Tax\ Paid},\quad E_0>0\ \text{ given,} \tag{35}$$

where y_t $(=r_tk_t+w_th_t)$ is the household's taxable income, and E_t represents the exemption threshold that is postulated to grow continuously at the same rate as per-capita output on the economy's balanced growth path, *i.e.* $\frac{\dot{E}_t}{E_t}=\frac{\dot{y}_t^*}{y_t^*}=\theta$ for all t. As in Dromel and Pintus (2007), our analyses below are restricted to the environment with $y_t>E_t$ and a constant marginal tax rate $\tau\in(0,1)$ that is higher than the corresponding average tax rate given by $\tau\left(1-\frac{E_t}{y_t}\right)$. It follows that the tax schedule under consideration here is progressive.

Next, it is straightforward to show that (i) the equilibrium conditions for this specification can be represented by the following autonomous dynamical system in terms of $x_t \equiv \frac{E_t}{y_t}$ and $z_t \equiv \frac{c_t}{k_t}$ with no given initial condition:

$$\frac{\dot{x}_t}{x_t} = \theta + \rho + \delta - \alpha (1-\tau) \left[\frac{A z_t}{(1-\tau)(1-\alpha)} \right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} - \frac{1+\gamma}{(1-\alpha)(1+\chi)-1-\gamma} \frac{\dot{z}_t}{z_t}, \tag{36}$$

$$\frac{\dot{z}_t}{z_t} = z_t - \rho - [(1-\tau)(1-\alpha) - \tau x_t] \left[\frac{A z_t}{(1-\tau)(1-\alpha)} \right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}}; \tag{37}$$

(ii) the existence and number of the economy's interior balanced growth path(s) are governed by

$$z^* = \rho + \frac{\tau E_0}{k_0} + (1-\tau)(1-\alpha) \left[\frac{A z^*}{(1-\tau)(1-\alpha)} \right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \equiv m(z^*), \tag{38}$$

where

$$m'(z^*) = \frac{(1-\tau)(1-\alpha)^2 (1+\chi)}{[(1-\alpha)(1+\chi) - 1 - \gamma] z^*} \left[\frac{A z^*}{(1-\tau)(1-\alpha)} \right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \lesseqgtr 0 \text{ when } (1-\alpha)(1+\chi) - 1 - \gamma \lesseqgtr 0, \tag{39}$$

and

$$m''(z^*) = \frac{(1+\gamma) m'(z^*)}{[(1-\alpha)(1+\chi) - 1 - \gamma] z^*} > 0; \tag{40}$$

and (iii) the determinant and trace of the resulting Jacobian matrix are

$$Det = -\frac{\tau (1-\alpha)(1+\chi) x^*}{\alpha (1-\tau) [(1-\alpha)(1+\chi) - 1 - \gamma]} \left[\frac{\alpha (1-\tau)(z^* - \rho)}{(1-\tau)(1-\alpha) + \tau x^*} \right]^2, \tag{41}$$

$$Tr = z^* - \frac{\left\{ (1-\tau)(1-\alpha)^2 (1+\chi) + \tau [(1-\alpha)(1+\chi) - 1 - \gamma] x^* \right\}}{\alpha (1-\tau) [(1-\alpha)(1+\chi) - 1 - \gamma]} \left[\frac{\alpha (1-\tau)(z^* - \rho)}{(1-\tau)(1-\alpha) + \tau x^*} \right]. \tag{42}$$

When $(1-\alpha)(1+\chi) - 1 - \gamma < 0$, these exists a unique balanced-growth equilibrium path in that $m(z^*)$ is a negatively-sloped and convex curve (similar to Figure 1). Given $z^* > \rho > 0$ (see equation 38), the BGP expressions of all other endogenous variables can be easily derived.[8] In

[8] It can be shown that along the economy's balanced growth path, $h^* = \left[\frac{A z^*}{(1-\tau)(1-\alpha)} \right]^{\frac{1}{(1-\alpha)(1+\chi)-1-\gamma}}$, $x^* = \frac{E_0}{k_0 (h^*)^{(1-\alpha)(1+\chi)}}$ and $\theta = \alpha (1-\tau) (h^*)^{(1-\alpha)(1+\chi)} - \delta - \rho$.

addition, we find that the Jacobian's determinant for this configuration is positive ($Det > 0$), and that the sign of the corresponding trace (42) is determined by

$$Tr\left(z^*\right) \gtrless 0 \text{ when } E_0 \gtrless E^c \equiv \frac{k_0 \left\{ \underbrace{\frac{\rho\left[(1-\alpha)(1+\chi)-1-\gamma\right]}{1+\gamma}}_{\text{negative}} - (1-\alpha)(1+\gamma)\left(h^*\right)^{(1-\alpha)(1+\chi)} \right\}}{\tau \left\{ 1 - \underbrace{\frac{\rho\left[(1-\alpha)(1+\chi)-1-\gamma\right]}{(1-\tau)(1-\alpha)(1+\gamma)\left(h^*\right)^{(1-\alpha)(1+\chi)}}}_{\text{negative}} \right\}}, \tag{43}$$

where $E^c < 0$. Since the exogenously-given initial level of exemption threshold E_0 is strictly higher than zero, the only feasible case will be $Tr\left(z^*\right) > 0$, which implies that both eigenvalues have positive real parts. It follows that as in Benhabib and Farmer's (1994) laissez-faire counterpart, the economy's unique balanced-growth equilibrium under linearly progressive taxation continues to display saddle path stability without the possibility of endogenous cyclical fluctuations.

On the other hand, Figure 5 depicts that the number of balanced growth paths can be zero, one, or two when $(1-\alpha)(1+\chi)-1-\gamma > 0$. As in section 3.2, we use (39) with $m'(\hat{z}) = 1$ and (38) evaluated at $\hat{z}$ to obtain

$$\hat{z} = \frac{(1-\alpha)(1+\chi)}{1+\gamma}\left(\rho + \frac{\tau \hat{E}}{k_0}\right), \tag{44}$$

where $\hat{E}$ is the unique solution to the following equation:[9]

$$\rho + \frac{\tau \hat{E}}{k_0} = \frac{(1-\tau)(1-\alpha)(1+\gamma)}{(1-\alpha)(1+\chi)-1-\gamma}\left[\frac{A(1+\chi)}{(1-\tau)(1+\gamma)}\left(\rho + \frac{\tau \hat{E}}{k_0}\right)\right]^{\frac{(1-\alpha)(1+\chi)}{(1-\alpha)(1+\chi)-1-\gamma}} \equiv \Psi\left(\rho + \frac{\tau \hat{E}}{k_0}\right). \tag{45}$$

Figure 5 also shows that an increase in E_0 shifts the locus of $m(z^*)$ upwards because of a higher vertical intercept, thus two balanced-growth equilibria characterized by $z_1^* < \hat{z} < z_2^*$ will emerge when $E_0 < \hat{E}$. In this case, the model's Jacobian matrix possesses a negative determinant *a la* (41), indicating that the two eigenvalues are of opposite signs. Therefore, both

[9] It is straightforward to show that the plot of $\Psi(\cdot)$ on the right-hand-side of (45) is an upward-sloping and convex curve that begins at the origin. As a result, this locus will intersect the 45-degree line once in the positive quadrant, which in turn determines the unique $\hat{E}$.

BGP's exhibit equilibrium indeterminacy and belief-driven growth fluctuations, which in turn implies that linearly progressive taxation may also operate like an automatic destabilizer in our endogenously growing macroeconomy. Intuitively, when households become optimistic and decide to raise their investment expenditures today, it can be shown that the aforementioned mechanism that makes for multiple equilibria, *i.e.* an increase in the equilibrium after-tax marginal product of capital, will generate convergent trajectories toward the original balanced growth path. As a result, agents' initial rosy anticipation about the economy's future is validated.

5 Conclusion

[To be Completed]

References

[1] Baier, S.L. and G. Glomm (2001), "Long-Run Growth and Welfare Effects of Public Policies with Distortionary Taxation," *Journal of Economic Dynamics and Control* 25, 2007-2042.

[2] Barro, R.J. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98, S103-S125.

[3] Barro, R.J. and X. Sala-i-Martin (1992), "Public Finances in Models of Economic Growth," *Review of Economic Studies* 59, 645-661.

[4] Basu, Susanto and J.G. Fernald (1997), "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy* 105, 249-283.

[5] Benhabib, J. and R.E.A. Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory* 63, 19-41.

[6] Benhabib, J. and R.E.A. Farmer (1996), "Indeterminacy and Sector-Specific Externalities," *Journal of Monetary Economics* 37, 421-444.

[7] Burnside, C. (1996), "Production Function Regression, Returns to Scale, and Externalities," *Journal of Monetary Economics* 37, 177-201.

[8] Cazzavillan, G. (1996), "Public Spending, Endogenous Growth, and Endogenous Fluctuations," *Journal of Economic Theory* 71, 394-415.

[9] Chen, B.-L. (2006), "Public Capital, Endogenous Growth, and Endogenous Fluctuations," *Journal of Macroeconomics* 28, 768-774.

[10] Chen, S.-H. and J.-T. Guo (2013), "Progressive Taxation and Macroeconomic (In)stability with Productive Government Spending," *Journal of Economic Dynamics and Control* 37, 951-963.

[11] Dromel, N.L. and P.A. Pintus (2007), "Linearly Progressive Income Taxes and Stabilization," *Reserach in Economics* 61, 25-29.

[12] Economides, G., H. Park and A. Philippopoulos (2011), "How Should the Government Allocate its Tax Revenuers between Productivity-Enhancing and Utility-Enhancing Public Goods?" *Macroeconomic Dynamics* 15, 336-364.

[13] Futagami, K., Y. Morita and A. Shibata (1993), "Dynamic Analysis of an Endogenous Growth Model with Public Capital," *Scandinavian Journal of Economics* 95, 607-625.

[14] Greiner, A. (2006), "Progressive Taxation, Public Capital, and Endogenous Growth," *FinazArchiv* 62, 353-366.

[15] Greiner, A. (2007), "An Endogenous Growth Model with Public Capital and Sustainable Government Debt," *Japanese Economic Review* 58, 345-361.

[16] Glomm G. and B. Ravikumar (1994), "Public Investment in Infrastructure in a Simple Growth Model," *Journal of Economic Dynamics and Control* 18, 1173-1187.

[17] Glomm G. and B. Ravikumar (1997), "Productive Government Expenditures and Long-Run Growth," *Journal of Economic Dynamics and Control* 21, 183-204.

[18] Guo, J.-T. and K.J. Lansing (1998), "Indeterminacy and Stabilization Policy," *Journal of Economic Theory* 82, 481-490.

[19] Harrison, S.G. (2001), "Indeterminacy in a Model with Sector-Specific Externalities," *Journal of Economic Dynamics and Control* 25, 747-764.

[20] Hu, Y., R. Ohdoi and K. Shimomura (2008), "Indeterminacy in a Two-Sector Endogenous Growth Model with Productive Government Spending," *Journal of Macroeconomics* 30, 1104-1123.

[21] Hendricks, L. (2001), "Growth, Death, and Taxes," *Review of Economic Dynamics* 4, 26-57.

[22] Jones, L.E. and R. Manuelli (1990), "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy* 98, 1008-1038.

[23] Jonsson, M. (2007), "The Welfare Cost of Imperfect Competition and Distortionary Taxation," *Review of Economic Dynamics* 10, 576-594.

[24] King, R.G. and S. Rebelo (1990), "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy* 98, S126-S150.

[25] Li, W. and P-D. Sarte (2004), "Progressive Taxation and Long-Run Growth," *American Economic Review* 94, 1705-1716.

[26] Palivos, T., C.Y. Yip and J. Zhang (2003), "Transitional Dynamics and Indeterminacy of Equilibria in an Endogenous Growth Model with a Public Input," *Review of Development Economics* 7, 86-98.

[27] Park, H. and A. Philippopoulos (2002), "Dynamics of Taxes, Public Services, and Endogenous Growth," *Macroeconomic Dynamics* 6, 187-201.

[28] Pecorino, P. (1993), "Tax Structure and Growth in a Model with Human Capital," *Journal of Public Economics* 52, 251-271.

[29] Perli, R. (1998), "Indeterminacy, Home Production, and the Business Cycle: A Calibrated Analysis," *Journal of Monetary Economics* 41, 105-125.

[30] Rebelo, S. (1991), "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy* 99, 500-521.

[31] Schmitt-Grohé, S. and M. Uribe (1997), "Balanced-Budget Rules, Distortionary Taxes and Aggregate Instability," *Journal of Political Economy* 105, 976-1000.

[32] Slobodyan, S. (2006), "One Sector Models, Indeterminacy and Productive Public Spending," CERGE-EI Working Paper Series No. 293.

[33] Song, E.Y. (2002), "Taxation, Human Capital and Growth," *Journal of Economic Dynamics and Control* 26, 205-216.

[34] Turnovsky, S.J. (1996), "Fiscal Policy, Adjustment Costs, and Endogenous Growth," *Oxford Economic Papers* 48, 361-381.

[35] Turnovsky, S.J. (1997), "Fiscal Policy in a Growing Economy with Public Capital," *Macroeconomic Dynamics* 1, 615-639.

[36] Turnovsky, S.J. (1999), "Productive Government Expenditures in a Stochastically Growing Economy," *Macroeconomic Dynamics* 3, 544-570.

[37] Weder, Mark (2000), "Animal Spirits, Technology Shocks and the Business Cycle," *Journal of Economic Dynamics and Control* 24, 273-295.

[38] Wen, Yi (1998), "Capacity Utilization Under Increasing Returns to Scale," *Journal of Economic Theory* 81, 7-36.

[39] Yamarik, S. (2001), "Nonlinear Tax Structures and Endogenous Growth," *Manchester School* 69, 16-30.

[40] Zhang, J. (2000), "Public Services, Increasing Returns, and Equilibrium Dynamics," *Journal of Economic Dynamics and Control* 24, 227-246.

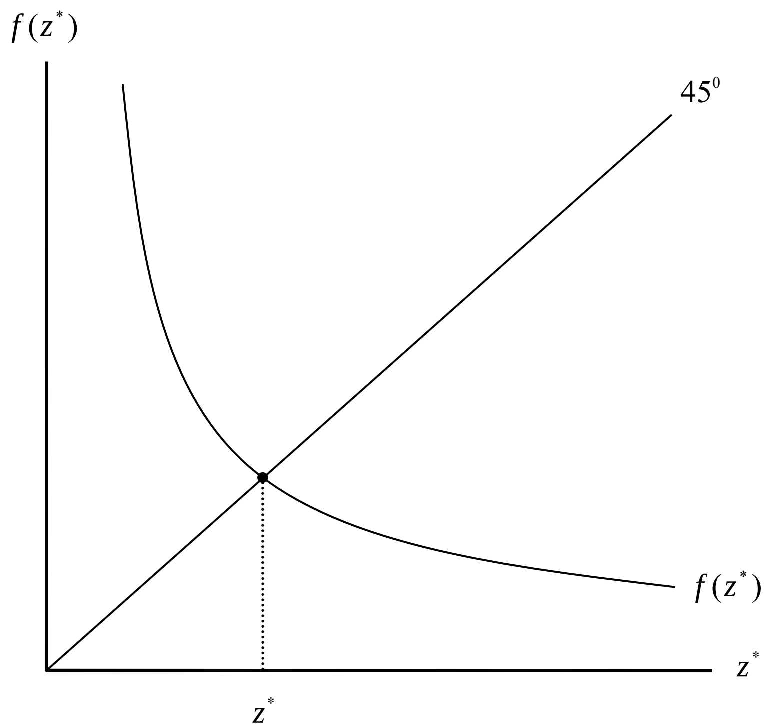

Figure 1. When 0 < ϕ < 1 and (1-α)(1+χ) - 1 - γ < 0: Unique BGP

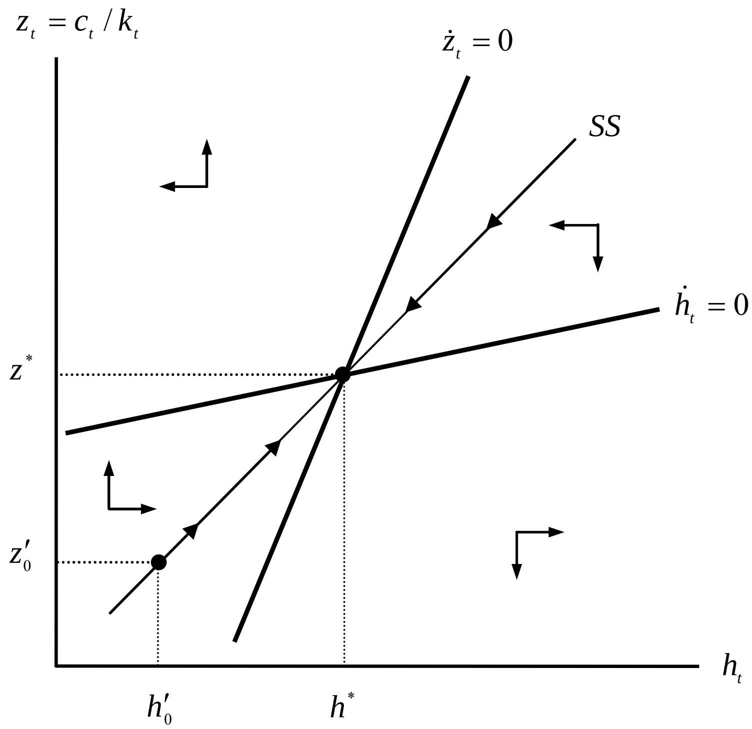

Figure 2. When 0 < φ < 1 and (1-α)(1+χ) - 1 - γ < 0: Indeterminacy

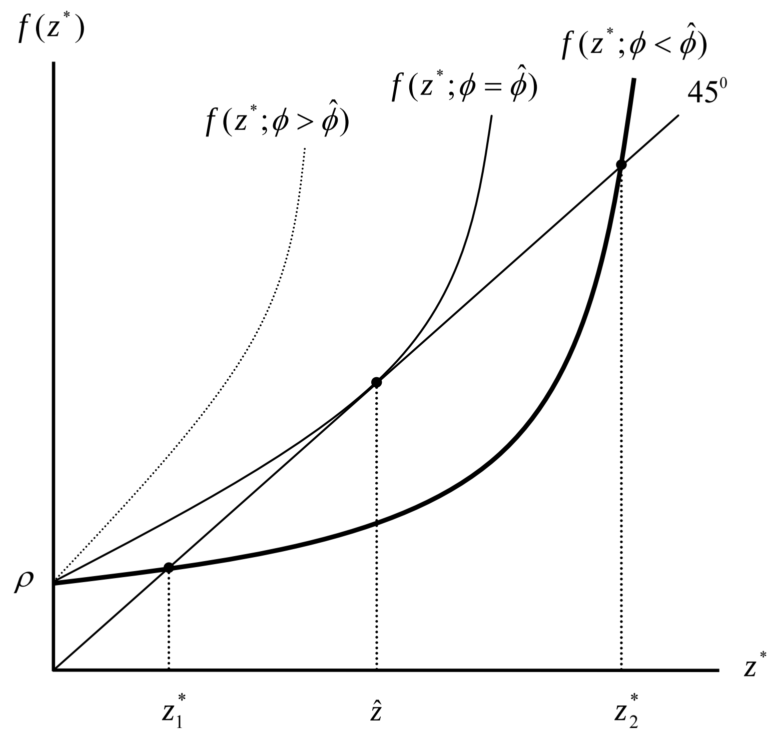

Figure 3. When 0 < ϕ < 1 and (1-α)(1+χ) - 1 - γ > 0: Possible Multiple BGP's

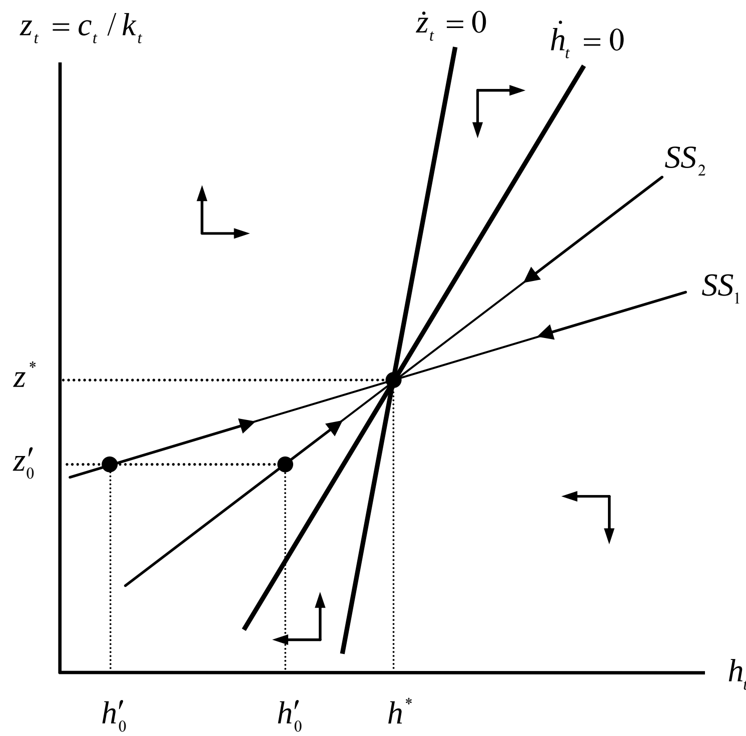

Figure 4. When 0 < $\hat{\phi}$ < φ < 1 and (1- φ)(1-α)(1+χ) - 1 - γ > 0: Indeterminacy of the High-Growth BGP

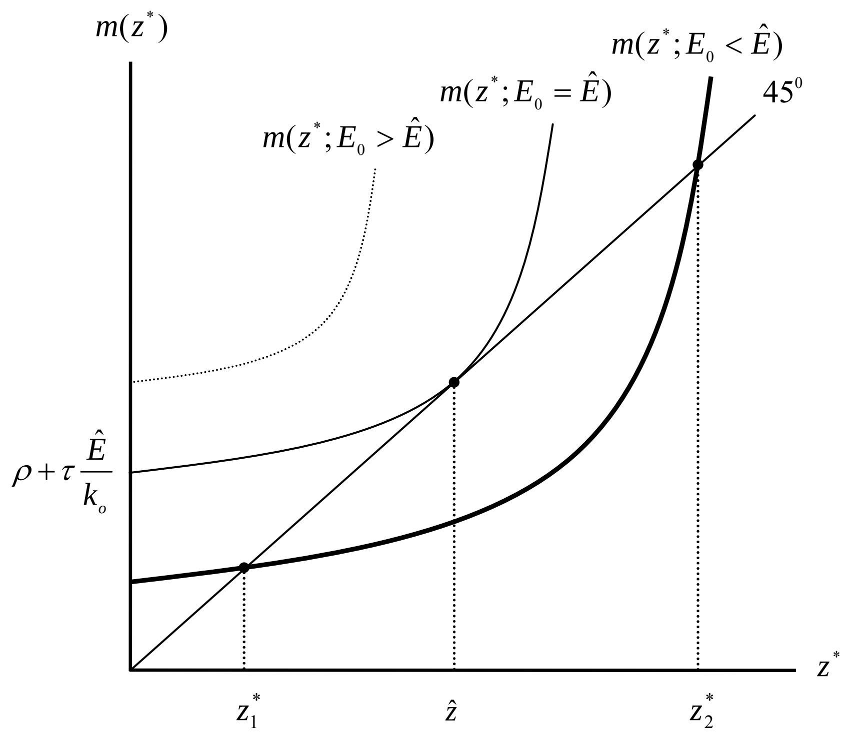

Figure 5. Linearly Progressive Tax and (1-α)(1+χ) - 1 - γ > 0: Possible Multiple BGP's