Economic Growth, Patent Race, and the Distribution of R&D Firms

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Abstract

The paper constructs a general equilibrium model model where the rate of technical progress and the distribution of R&D expenditure by heterogeneous research firms are simultaneously determined. Using the model, we explore the effects of the following policy measures on those two endogenous variables: (i) subsidies to flow variable R&D costs, (ii) subsidies to flow fixed R&D costs, (iii) an increase in entrant firms into a series of patent races, and (iv) an increase in the supply of human capital as inputs to R&D. Contrasting results are demonstrated. For example, subsidies to flow variable R&D costs promotes technical progress and induces the exit of R&D firms with low R&D productivity. That is, the policy leads to the situation where technological progress accelerates through R&D by "elite" firms. On the other hand, the opposite result holds if subsidies are applied to flow fixed R&D costs.

Keyword: R&D, Patent race, technical progress, heterogeneous firms, firm distribution

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1 Introduction

It is widely recognized that the primary engine of long-run growth in modern economies is technological progress mainly driven by research and development (R&D). Roughly a half of GDP per capita growth is attributed to technological progress, and this fact is felt throughout our daily life (e.g. smartphones and medical instruments). In this regard, the role of governments in promoting innovation is of primary importance by affecting the incentives of firms to conduct R&D.

The purpose of this paper is to re-examine the impact of government policy on R&D in the private sector, using a model of endogenous technological progress. In particular, we emphasize the explicit role of heterogeneity of R&D firms, which is neglected in the literature on dynamic general equilibrium models. There are many precursory studies on policy effects.[1] A departure of the present paper from them is to examine the determination of the distribution of R&D firms as well as R&D expenditure in the analysis of policy effects. That is, we explore how policy affects the firm and R&D expenditure distribution in addition to the issue of whether or not government policy promotes innovation. In doing so, we identify types of firms that conduct R&D (and types of firms that do not), the amount of R&D investment each of them does, the number of such firms.

Policy measures considered in the present paper are (i) subsidies to flow variable expenditure on R&D, (ii) subsidies to flow fixed expenditure on R&D, (iii) an increase in the number of entrant firms into R&D (increasing competition), and (iv) an increase in the supply of human capital used in R&D.

The importance of explicitly considering the distribution of R&D firms and their expenditure is three-fold. First, the pace of technical progress in an economy is determined by R&D activities of *all* firms rather than a small number of "big" firms. Naturally, the analysis of the R&D firm distribution is imperative to understand the economy-wide effects of public policy. Second, industrial policy like R&D tax/subsidy is very much likely to affect incentives of all firms in *different* ways. Such policy effects may be realized directly for some and indirectly for others, e.g. in response to changes in R&D expenditure of the directly affected rival firms. Capturing those differences is important to understand the aggregate effects of public policy. Third, industrial policy is also likely to cause exit/entry of R&D firms. Though analysis based on homogeneous firms can deal with the issue, it cannot allow us to identify types of firms which start or stop investing in R&D when policy changes. Such information is certainly important for policy makers.

In the model, patent races continuously take place across industries, driving growth in the long run.[2] This allows us to examine the afore-mentioned issues in a dynamic general

[1] For example, see Hall and Rosenberg (2010) and Reinganum (1989).

[2] Pioneering studies on a patent race include Lee and Wilde (1980), Loury (1979), Dasgupta and Stiglitz (1980) and Reinganum (1982). Partial equilibrium models in those studies contributed to the development of general equilibrium models of endogenous technical progress. In particular, Lee and Wilde (1980) laid the foundation of what is now called the quality-ladder models of Aghion and Howitt

equilibrium model. One of our key results shows that subsidies to flow variable R&D costs increase the rate of technical progress. This is due to the *cleansing effect*, which results from two factors; firms with relatively low R&D productivity exit patent races and the remaining firms expand R&D investment. That is, innovation accelerates due to increased R&D investment by "elite" firms. On the other hand, the reverse of the cleansing effect arises in the case of subsidies to flow fixed R&D costs. Innovation decelerates with the entry of low productivity firms. Quantitatively those two policy measures are diametric in the sense that nothing changes if subsidies to flow variable *and* fixed R&D costs are raised simultaneously.[3] This result indicates that R&D subsidies, which are often taken as a useful tool to correct market failures in R&D, have a pitfall in assessing their impact on innovation.

Given the above results, we consider the following questions. Should different R&D firms be subsidized (or taxed) at the same rate? If not, how differently? One approach to tackle the questions is to consider them in terms of achieving social optimum. Instead, we assess the issue, taking a balanced government budget as a constraint. More specifically, we explore the case where subsidies to flow variable R&D costs are financed out of taxes on flow fixed R&D costs. Note that the both policy are pro-growth. We argue that the approach considered here is important because the source of financing subsidies is often neglected in the literature for the reason of simplification. Indeed, our model offers a suitable framework to explore the approach. Our analysis demonstrates that firms with high productivity should be subsidized and low productivity firms be taxed in *net*. This implies that resources are required to shift from low to high productivity firms. In this sense, our result is in line with minimization of overall R&D costs in an economy as a whole.

We also analyze the effect of a measure to increase the number of firms entering a patent race. An important message of the result is that it generates a cleansing effect like subsidies to flow variable R&D costs: low productivity firms are forced to exit and innovation accelerates. In contrast, boosting human capital used as inputs for R&D accelerates innovation by inducing low productivity firms to enter a patent race.

The structure of the paper is as follows. Section 2 develops the model. Equilibrium conditions are derived in Section 3, which also explores the effects of industrial policy. Section 4 concludes.

(1992) and Grossman and Helpman (1991a) where technical progress is endogenized by introducing profit-seeking monopoly firms. Our model belongs to this literature. See Aghion, Akcigit, and Howitt (2014) for a survey.

[3]It assumes that the initial rates of subsidies to flow variable and fixed R&D costs are the same.

2 The Model

2.1 Consumers, Final and Intermediate Goods

Our model is built on variety expansion models of endogenous technical progress developed by Romer (1990) and Grossman and Helpman (1991b, Ch 3). However, ours departs from theirs in R&D technology in order to introduce a series of patent races.

For simplicity, we assume that there are two types of workers, skilled and unskilled. Although they are different in skill levels, they share the same preferences. The utility function is

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt \tag{1}$$

where ρ the subjective rate of time preference and C_t is final consumption. Utility maximization requires the Euler condition $\dot{E}_t/E_t = r_t - \rho$ where r_t is the interest rate and E_t is consumption expenditure. Following Grossman and Helpman (1991b, Ch 3), consumption expenditure is taken to be the numeraire ($E_t = 1$), so that $r_t = \rho$.

Final output is produced under perfect competition, using intermediate products. To be more specific, we assume a continuum of intermediate goods industries $j \in [0, 1]$, in each of which k_{tj} varieties are produced at time t. y_{tji} denotes the amount of variety product i in industry j. Given these assumptions, the production function of final output Y_t is assumed to take the following form:

$$Y_t = \left(\int_0^1 \sum_{i=1}^{k_{tj}} y_{tji}^{\alpha} dj \right)^{1/\alpha}, \qquad 1 > \alpha > 0. \tag{2}$$

Technical progress is captured by an increase in k_{tji}, which occurs whenever innovation occurs in a series of patent races. Given that the production function (2) is CES in nature, derived demand for intermediate goods can be shown to take the form of

$$y_{tji} = \frac{p_{tji}^{-\frac{1}{1-\alpha}}}{\int_0^1 \sum_{i'=1}^{k_{tj}} p_{tji'}^{-\frac{\alpha}{1-\alpha}} dj}. \tag{3}$$

where p_{tji} is the price of intermediate goods y_{tji}.

Turning to the intermediate goods industry, products can be produced only after their blueprints are successfully created through R&D. Since those new ideas are protected by patents, monopoly firms produce the goods, facing the demand (3) with the price elasticity being $-1/(1-\alpha)$. Assume that one unit of the intermediate goods is produced with one worker. Then, firms set the following monopoly price

$$p_{tj} = \frac{\omega_t}{\alpha} \equiv p_t \tag{4}$$

where ω_t is unskilled wages. Using (4), demand for a variety good (3) is reduced to

$$y_{tji} = \frac{1}{p_t K_t} \equiv y_t, \qquad \text{where } K_t = \int_0^1 k_{tj} dj. \tag{5}$$

K_t is the total number of variety intermediate goods. The results (4) and (5) allow us to derive monopoly profits per variety:

$$\pi_{tj} = \frac{1-\alpha}{K_t} \equiv \pi_t. \tag{6}$$

Note that profits in (6) are the same for all products across industries. Let V_t denote the sum of discounted future profits arising from selling a variety intermediate good. It is determined by

$$\rho V_t = \pi_t + \dot{V}_t. \tag{7}$$

V_t is interpreted as the value of a single patent. Since V_t is independent of the industry index j, R&D is conducted in all intermediate goods industries $j \in [0,1]$ in equilibrium if it is active in any one of them.

2.2 Patent Races of Heterogeneous Firms

Firms have to invest and succeed in R&D in order to create a new intermediate product and earn monopoly profits protected by patents. Firms compete in a race to obtain a patent for new variety products in a given intermediate goods industry. The first firm which generates innovation in a patent race becomes monopoly in the industry, and others which fail cannot produce goods.[4]

2.2.1 Assumptions

Heterogeneity of R&D firms is due to differences in research productivity, denoted by a. In particular, we assume that such heterogeneity arises probabilistically. Firms do not know their own true productivity before entry into a patent race, and it is revealed only after entry. R&D productivity levels are distributed according to the following distribution function:

$$Z(a), \qquad a \in [0, a_H], \qquad 0 < a_H < \infty, \tag{8}$$

which is assumed to be known. On the other hand, we assume that rival firms' R&D productivity levels are not observable even during a patent race. We assume that N firms enter a patent race whenever it starts afresh. For simplicity, N is taken as exogenously given.[5]

[4] The basic structure of a patent race is based on Lee and Wilde (1980).

[5] In principle, N can be endogenized by introducing fixed entry costs. But, we focus on the case of a fixed N because it is sufficient to establish our key results.

After entry, firms incur flow R&D costs which consist of variable and fixed components. Consider a firm with productivity a. Let $R_{tj}(a)$ be the number of R&D workers the firm choose to employ in industry j. Use f to denote a fixed number of R&D workers required as long as the firm stays in a patent race irrespective of whether or not the variable number of R&D workers $R_{tj}(a)$ is positive. More specifically, if the firm employs $R_{tj}(a)+f$ workers, then innovation occurs at a Poisson arrival rate of

$$ah(R_{tj}(a)), \qquad h(R_{tj}(a)) \equiv R_{tj}(a)^{\mu}, \quad 0 < \mu < 1. \tag{9}$$

In the R&D technology (9), there is no term representing externalities which are an important characteristic. Instead, we make the following assumption. When a firm succeeds in R&D, it is able to creates blueprints for λK_t number of variety products. A parameter $\lambda > 0$ captures positive externalities in R&D, and it plays an important role in sustaining the rate of technical progress in the long run. Therefore, a winner in a patent race achieves the firm value $\lambda K_t V_t$.

Note that because of the presence of flow fixed costs f, relatively low productivity firms exit the race after their productivity levels are observed, and relatively high productivity firms stay in the race. In other words, there exists the threshold level of productivity, denoted by A_{tj} ($0 < A_{tj} \leq a_H$), at which firms are indifferent between continuing and exiting the race. This threshold productivity level is an important factor in determining the distribution of R&D firms in equilibrium.

A patent race ends once innovation occurs, and another race immediately starts with the entry of another set of N firms.[6] Note that although the intermediate goods industries are symmetric in structure, the number of varieties is different across those industries since innovation occurs stochastically.

2.2.2 Optimal Decisions

R&D firms with productivity a are called firms a. The value of R&D firm a, denoted by $v_{tj}(a)$, is determined by the following recursive equation:

$$\rho v_{tj}(a) = [\lambda K_t V_t - v_{tj}(a)]ah(R_{tj}(a)) - w_t[(1-s_R)R_{tj}(a) + (1-s_f)f] - v_{tj}(a)I_{tj}^{-1}. \tag{10}$$

The first term on the right-hand side is the expected flow benefit of R&D (an expected increment of the firm's value), and the second term gives flow R&D costs with w_t as skilled wages. There are two policy variables in the flow costs. s_R is the rate of subsidies to variable costs, and s_f is the rate of subsidies to fixed costs. I_{tj}^{-1} in the third term is the expected Poisson rate of rival firms succeeding in R&D. That is, $-v_{tj}(a)I_{tj}^{-1}$ captures the risk of the firm a losing in the patent race with its value being reduced to nil. Note that the Poisson rate I_{tj}^{-1} is the same for all firms competing in the race in industry j because

[6] Following the literature, it is assumed that incumbent firms which earn profits in the intermediate goods industries do not engage in R&D.

of the assumption that rival firms' R&D productivity levels are unobservable (incomplete information).

Taking I_{tj}^{-1} as given, firm a chooses $R_{tj}(a)$ to maximize the right-hand side of (10). The first-order condition is

$$[\lambda K_t V_t - v_{tj}(a)]\, ah'(R_{tj}(a)) = (1 - s_R)w_t. \tag{11}$$

Its left-hand side gives the expected marginal benefit of employing an additional R&D worker, and the associated marginal cost is on the right-hand side.

To explore some implications of this result, we use (10) and (11) to derive

$$\left(\lambda K_t V_t - \frac{\lambda K_t V_t ah(R_{tj}(a)) - w_t[(1 - s_R)R_{tj}(a) + (1 - s_f)f]}{\rho + ah(R_{tj}(a)) + I_{tj}^{-1}}\right) ah'(R_{tj}(a)) = (1 - s_R)w_t. \tag{12}$$

Let $X_{tj}(a)$ denote the value of $R_{tj}(a)$ which satisfies (12) (see (28)). $X_{tj}(a)$ reflects the firm's optimal decision on R&D employment, taking rival firms' decisions. It is easy to show that a firm with a higher a employs more workers.[7]

Next, substitute $X_{tj}(a)$ into (10) to obtain

$$v_{tj}(a) = \frac{\lambda K_t V_t ah(X_{tj}(a)) - w_t[(1 - s_R)X_{tj}(a) + (1 - s_f)f]}{\rho + ah(X_{tj}(a)) + I_{tj}^{-1}}. \tag{13}$$

This equation allows us to examine how the R&D firm value $v_{tj}(a)$ changes as productivity a changes. An increase in a impacts on the firm value directly via a on the right-hand side of (13) and indirectly through $X_{tj}(a)$. Due to the envelope theorem, however, the indirect effect via the $X_{tj}(a)$ can be ignored. Therefore, taking I_{tj}^{-1} as given, the following result holds:

$$\left.\frac{\partial v_{tj}(a)}{\partial a}\right|_{\text{constant } I_{tj}^{-1}} > 0. \tag{14}$$

This demonstrates that firms with higher R&D productivity have a greater value of competing in a patent race.

Based on the above results, we next characterize the types of firms which continue investing in flow R&D expenditure after entry. A patent race starts with N firms. At the beginning of the race, those firms observe their productivity a and form the expectation of their firm value $v_{tj}(a)$. It is obvious that firms stay in the race as long as its value $v_{tj}(a)$ is positive. On the other hand, firms exit the race for $v_{tj}(a) < 0$. The threshold productivity A_{tj} at which firms are indifferent between staying and exiting a patent race is defined by

$$v_{tj}(A_{tj}) = 0, \tag{15}$$

[7] This result can be established by totally differentiating (12) and using the envelope theorem.

which means

$$\lambda K_t V_t A_{tj} h(X_{tj}(A_{tj})) = w_t[(1-s_R)X(A_{tj}) + (1-s_f)f]. \quad (16)$$

This condition determines the threshold productivity A_{tj}.[8]

In addition, making use of (11) and (16), we can show the following result:

$$X_{tj}(A_{tj}) = \frac{\mu}{1-\mu}\frac{1-s_f}{1-s_R}f \equiv \bar{X}_A \quad (17)$$

$$\frac{\lambda K_t V_t}{w_t} = \frac{(1-s_R)^{\mu}(1-s_f)^{1-\mu}F}{A_{tj}} \equiv \tilde{\Lambda}(A_{tj}) \quad (18)$$

where $F \equiv \frac{f^{1-\mu}}{\mu^{\mu}(1-\mu)^{1-\mu}}$. There are two implications regarding (18). First, the threshold productivity is the same for all industries, i.e. $A_{tj} \equiv A_t$. Second, the threshold A_t depends on the value of a patent V_t. Intuitively, an increase in the value of a patent boosts the value $v_{tj}(a)$ of all firms competing in the patent race, so that even relatively low productivity firms can afford to stay in the race, incurring flow R&D costs. Because of this property, the effective value of a "prize" $\tilde{\Lambda}$ in the patent race is negatively related to the threshold productivity A_t. That is, a greater prize of winning in a patent race induces even lower productivity firms to invest in flow R&D. This mechanism accords with our intuition. But, we will show that there is a counter-acting effect in the following analysis.

2.3 Labor Market

There are two types of workers, skilled and unskilled. The latter type of workers are employed for the production of intermediate goods only. Using (4), (5) and L to denote the number of unskilled workers, the following full-employment condition holds:

$$L = \int_0^1 \sum_{i=1}^{k_{tj}} y_{tji} dj = K_t y_t = \frac{\alpha}{\omega_t}. \quad (19)$$

Skilled workers, on the other hand, are used for R&D only. Using H to denote their total number, the following condition holds for their full employment.

$$H = N \int_0^1 \int_{A_{tj}}^{a_H} [R_{tj}(a) + f] dZ(a) dj. \quad (20)$$

On the right-hand side is the sum of variable and fixed R&D workers employed for flow R&D activities in the continuum of the intermediate goods industries $j \in [0,1]$

[8]The "interior" value of A_{tj} exists if a_H is sufficiently large.

2.4 Technical Progress

The number of firms making flow R&D investment is $N\left[1-Z\left(A_{tj}\right)\right]$, because firms with productivity greater than the threshold A_{tj} stay in a patent race and others exit. Therefore, the average Poisson rate of innovation in industry j, denoted by ι_{tj}, is given by

$$\iota_{tj}=\int_{A_{tj}}^{a_H} ah(R_{tj}(a))\frac{dZ(a)}{1-Z(A_{tj})}. \tag{21}$$

Because the industry-wide Poisson rate I_{tj} is equivalent to the product of the number of active firms and the average Poisson rate, we have

$$I_{tj}=N\left[1-Z(A_{tj})\right]\iota_{tj}. \tag{22}$$

This equation is equivalent to the sum of the Poisson rates of all firms conducting flow R&D, given the assumption that R&D productivity a is randomly distributed.

Similarly, the average Poisson rate of rival firms is

$$I_{tj}^{-1}=\left[N(1-Z(A_{tj}))-1\right]\iota_{tj}. \tag{23}$$

Note that this rate is the same for all firms, since R&D productivity levels of rival firms are unobservable. Making use of the above equations, the following relations can be derived

$$I_{tj}^{-1}=m(A_{tj};N)I_{tj} \tag{24a}$$

$$m(A_{tj};N)=1-\frac{1}{N\left[1-Z(A_{tj})\right]},\qquad \frac{\partial m}{\partial A_{tj}}<0,\quad \frac{\partial m}{\partial N}>0. \tag{24b}$$

This equation relates the Poisson rate of rival firms to the industry-wide Poisson rate.

Next, let us consider the growth of final output. Making use of (19), the production function (2) can be reduced to $Y_t=K_t^{\frac{1-\alpha}{\alpha}}L$. Hence, its growth rate is $\frac{1-\alpha}{\alpha}\frac{\dot{K}_t}{K_t}$. To calculate it, note that the number of variety goods increases by λK_t in each industry whenever innovation occurs at a rate of I_{tj}. Given a continuum of intermediate goods industries, we can write $\dot{K}_t=\int_0^1\lambda K_t I_{tj}dj$ or

$$\frac{\dot{K}_t}{K_t}=\lambda\int_0^1 I_{tj}dj. \tag{25}$$

3 Steady State Analysis

3.1 Equilibrium Conditions

This section focuses upon steady state, dropping the subscripts t from variables. Innovation follows a stochastic process. Because of this assumption, the timing and the number of variety products differ across the intermediate goods industries. Other endogenous variables, on the other hand, are the same for all industries due to the symmetric structure of the model. This feature allows us to drop the industry subscript j from all variables other than the number of varieties goods k_{tj}.

We derive equilibrium conditions which determine two variables (I, A). For this, substitute (18) and (24a) into (12) and rewrite it as

$$\left(\Lambda(A; s_R, s_f) - \frac{\Lambda(A; s_R, s_f) a h(R(a)) - R(a) - \frac{1-s_f}{1-s_R} f}{\rho + a h(R(a)) + m(A; N) I} \right) a h'(R(a)) = 1 \tag{26}$$

where

$$\Lambda(A; s_R, s_f) \equiv \frac{\tilde{\Lambda}(A)}{1-s_R}, \qquad \frac{\partial \Lambda}{\partial A} < 0, \quad \frac{\partial \Lambda}{\partial \left(\frac{1-s_f}{1-s_R}\right)} > 0 \tag{27}$$

and $\tilde{\Lambda}(A)$ is defined in (18). Hence, due to the envelope theorem and (26), the following result can be verified:

$$\begin{aligned} & R(a) = X\left(a, I, A; s_R, s_f, N\right), \\ & \frac{\partial X(a)}{\partial k_1} > 0, \quad k_1 = a, I, N, s_R, \qquad \frac{\partial X(a)}{\partial k_2} < 0, \quad k_2 = A, s_2. \end{aligned} \tag{28}$$

This equation shows the employment of R&D workers optimally chosen by firms a. Note that the signs of the derivatives with respect s_R and s_f are different, meaning that those two policy variables have opposite effects on $R(a)$. The implications of this result will be explored later.

Next, substitute (28) into (21) and rewrite the resulting equation, using (22) and (24a) to obtain

$$I = N \int_A^{a_H} a h(X(a, I, A; s_R, s_f)) dZ(a) \tag{29}$$

This equation determines the industry-wide Poisson rate I, taking the threshold productivity A as given. It is called the *R&D equilibrium* condition because it is interpreted to summarize optimal behaviors of all firms conducting flow R&D in all intermediate goods industries.

To succinctly express the relation between the two variables (I, A) in (29), rewrite it as

$$I = \Phi(A; N, s_R, s_f), \quad \frac{\partial \Phi(A)}{\partial A} < 0, \quad \frac{\partial \Phi(A)}{\partial N} > 0, \quad \frac{\partial \Phi(A)}{\partial \left(\frac{1-s_f}{1-s_R}\right)} > 0. \tag{30}$$

The signs of the derivatives are due to the assumption regarding "stability" of the model:[9]

$$1 - N\int_{A}^{a_H} ah'(X(a))\frac{\partial X(a)}{\partial I}dZ(a) > 0. \tag{31}$$

The R&D equilibrium condition (30) is downward-sloping in Figure 1. Recalling that it determines I for a given A, suppose that the threshold productivity A falls. Then, relatively lower productivity firms stay in a patent race and conduct flow R&D, boosting the industry-wide Poisson rate of innovation. This gives rise to a negative relationship between A and I.[10]

To derive another equilibrium condition, substitute (28) into (20) to derive the skilled labor market condition in terms of two variables (I, A).

$$H = N\int_{A}^{a_H} [X(a, I, A; s_R, s_f) + f]dZ(a). \tag{32}$$

It is depicted as an upward-sloping curve in Figure 1. To understand intuition behind the slope, suppose that the Poisson rate I rises. It requires a greater number of skilled workers in flow R&D (see (28)), which results in an increase in skilled wages, i.e. higher flow R&D costs. Consequently, relatively lower productivity firms can no longer afford flow R&D and exit a patent race, raising the threshold A. Such adjustment instantaneously occurs until the labor market condition holds.

The two endogenous variables (I, A) are determined in equilibrium in the system of two equations (30) and (32). In Figure 1, the intersection E_0 point between the two curves define steady state equilibrium.[11]

3.2 Policy Effects

3.2.1 R&D Subsidies

This section considers the effects of two different types of R&D subsidies. s_R is the rate of subsidies to variable flow R&D expenditure, and s_f is the equivalent to fixed flow R&D expenditure. The following proposition summarizes the results concerning the former industrial policy.

Proposition 1. *Following an increase in* s_R*,*

[9] Although there may be cases where (31) does not hold, we focus upon a "normal" in which (31) is met. (31) is essentially equivalent to the assumption that Lee and Wilde (1980) uses to prove Theorem 1 of their paper.

[10] Furthermore, there is an additional channel where the following chained effects work: $I \uparrow \Rightarrow v(a) \downarrow \Rightarrow [V - v(a)] \uparrow \Rightarrow R(a) \uparrow$ for $a \in [A, a_H]$ (the third "$\Rightarrow$" is due to (11)). This again leads to a higher I.

[11] To derive a condition which determines skilled wages, note that w_t, $R_t(a)$, A_t and $v(a)$ are constant in steady state. Therefore, inspecting (11) shows that $\dot{V}_t/V_t = -\dot{K}_t/K_t$. Using this, (7) is rewritten as $V_t = \pi_t/(\rho + \dot{K}_t/K_t)$. This equation, (6) and (25) allows us to reexpress (18) as $\frac{\lambda(1-\alpha)}{(\rho+\lambda I)w} = \frac{(1-s_R)^{\mu}(1-s_f)^{1-\mu}F}{A}$ Thus, equilibrium skilled wage w is defined in this equation once (I, A) are determined.

(i) the threshold R&D productivity A rises (the number of active firms $N[1 - Z(A)]$ decreases),

(ii) the industry-wide Poisson rate I increases, and

(iii) the total and average number of variable R&D workers $R(a)$ employed by active firms for $a \in [A, a_H]$ increases.

Proof. See Appendix A. □

In Figure 1, the initial equilibrium is given at a point E_0, which moves to a point E_1 following a rise in the rate of subsidy s_R. To understand this result, consider (12) which determines the optimal number of variable flow R&D workers for firms. A higher subsidy rate s_R reduces the marginal cost of R&D directly (see (12)). Indirectly, it also decreases the expected marginal benefit of R&D via the effective "prize" of a patent race $\tilde{\Lambda}$ which is realized through the product market (see (26) and (18)). Because the former effect dominates the latter effect, firms increase variable R&D workers. But, this fact generates two opposite effects. First, a greater employment of variable R&D workers raise the industry-wide Poisson rate I, causing a rightward shift in the R&D equilibrium condition in Figure 1. This is the partial equilibrium effect. On the other hand, a greater demand for variable R&D workers creates an upward pressure for skilled wages, given a fixed supply of skilled workers. This induces relatively low productivity firms to exit a patent race, leading to a higher threshold productivity A. This second effect is reflected in an upward shift of the (32) curve. It is the general equilibrium effect of the subsidy policy which works via the labor market.

The subsidy policy unambiguously raises the threshold productivity A because both of the partial and general equilibrium effects of the subsidy policy reinforce each other (Proposition 1-(i)). On the other hand, those two effects work on the industry-wide Poisson rate I in opposite directions. The partial equilibrium effect of the policy tends to boost the Poisson rate, whereas it tends to fall due to the general equilibrium effect. Since the former effect outweighs the latter effect, the industry-wide Poisson rate I unambiguously rises (Proposition 1-(ii)).

Regarding changes in the employment of variable R&D workers $R(a)$, the result is ambiguous in general. A rise in s_R and a resulting higher I both encourages firms to employ more skilled workers (see (28)). On the other hand, a higher threshold A which results from the policy change reduces to employ skilled workers, counteracting the aforementioned effect. However, it can be easily established that the total number of flow variable workers and its average are both unambiguously increase (Proposition 1-(iii)).

Next, let us consider a higher rate of subsidy to flow fixed costs. The results are summarized in the next proposition.

Proposition 2. *Following a rise in s_f,* (*i*) *the threshold productivity A falls (the number of active firms $N[1-Z(A)]$ increases),*

(*ii*) *the industry-wide Poisson rate I drops, and*

(*iii*) *the total and average numbers of variable R&D workers $R(a)$ employed by active firms for $a \in [A, a_H]$ decrease.*

Proof. See Appendix B. □

This is diametric to Proposition 1. The marginal cost of R&D is independent of s_f. On the other hand, as in the case of s_R, the expected marginal benefit of R&D is a decreasing function of s_f via the product market. Thus, firms tend to reduce R&D employment with a drop in the Poisson rate I for a given threshold A. This is reflected in a leftward shift of the (30) curve (the R&D equilibrium condition) in Figure 1. On the other hand, the (32) curve (the labor market condition) shifts downward. Intuitively, the reduction of demand for flow variable R&D workers tends to reduce skilled wages. This makes it possible for relatively lower productivity firms, which did not invest in flow R&D before, start doing so. As a result, the threshold productivity A falls.

It is clear from Figure 1 that the threshold A unambiguously drops, given the directions of shifts of the equilibrium curves, as explained above (Proposition 2-(i)). The figure also shows that the industry-wide Poisson rate falls with the entry of less productive firms (Proposition 2-(ii)).

Having established the above two propositions, we next explore the case where subsidies are applied to both of flow variable and fixed costs simultaneously without distinguishing them. The result is summarized below:

Proposition 3. *Suppose $s \equiv s_R = s_f$. A change in s affects none of the threshold A, the Poisson rate I and variable R&D employment $R(a)$.*

This proposition indicates that the effects of two types of subsidies s_R and s_f exactly cancel each other because they are qualitatively and quantitatively opposite in their effects. This intuition can also be confirmed in the fact that (26) is independent of those policy variables. Proposition 3 contrasts with results reported in the precursory studies including Aghion and Howitt (1992) and Grossman and Helpman (1991b). In the literature on endogenous technical progress, R&D subsidies are shown to promote innovation and instrumental in restoring the socially optimal level of R&D investment. The difference in policy implications is due to the assumption of heterogeneity in R&D firms.

3.2.2 Entry of Firms

Due to the assumption of the Poisson process of innovation, the average length of time for each patent race is $1/I$. Once a race ends in a given industry, a new patent race starts afresh with an entry of N firms. This section considers how the degree of competition measured by N affects innovation and the firm distribution.

Proposition 4. *As the number of entrant firms N in each patent race increases,*

(i) *the threshold productivity A increases (and the number of active firms $N[1 - Z(A)]$ increases),*

(ii) *the industry-wide Poisson rate I increases, and*

(iii) *the total and average numbers of variable R&D workers $R(a)$ employed by active firms for $a \in [A, a_H]$ increase.*

Proof. See Appendix C. □

The results are explained using Figure 2. The two curves shift in the same directions as in the case of a higher subsidy rate to flow variable costs s_R, but for different reasons. Consider the R&D equilibrium condition (30). As entrant firms rise in number, there are more firms which stay in the race and invest in flow R&D, intensifying competition. This means a higher risk of losing a race, thereby reducing the value $v(a)$ of each and every active firm. As a result, the expected marginal benefit of R&D increases (see (11)), causing a rightward shift of the (30) curve. With other things equal, both of the threshold A and the Poisson rate I increase. This is the partial equilibrium effect of the policy change. The general equilibrium effect is realized via the skilled labor market condition (32). More entrant firms boost demand for skilled workers, which is amplified because of intensified R&D competition, mentioned above. This raises skilled wages, causing relatively low productivity firms to exit. This causes the threshold productivity A to rise with an upward shift of the the (32) curve.

It is not clear whether the direct or indirect effects dominates. If the former is dominant, equilibrium is established at E_3. In the opposite case, equilibrium settles at E_3'. In other words, a higher A due to the partial equilibrium effect is reinforced by the general equilibrium effect. On the other hand, those two effects work in opposite directions regarding their impact on the Poisson rate I, hence its overall change is ambiguous.

3.2.3 Skilled Workers

Skilled workers only can serve as inputs for R&D. In this sense, they may be interpreted as human capital in a narrow sense. Then, how does its increase, e.g. through education, affect equilibrium?

Proposition 5. *Following an increase in H,*

(1) *the threshold productivity A falls (the number of active firms $N[1 - Z(A)]$ decreases),*

(2) *the industry-wide productivity I increases, and*

(3) *all of active firms for $a \in [A, a_H]$ expands flow variable R&D workers $R(a)$ (the total and average numbers of variable R&D workers $R(a)$ increase).*

	s_R	s_f	$s_R = s_f$	N	H
I	$+$	$-$	0	$\pm$	$+$
A	$+$	$-$	0	$+$	$-$
$N\int_0^1 R(a)dZ(a)$ $\int_0^1 R(a)\frac{dZ(a)}{1-Z(A)}$	$+$	$-$	0	$+$	$+$

Table 1: Summary of comparative statics results. $N\int_0^1 R(a)dZ(a)$ is the total flow variable R&D employment, and $\int_0^1 R(a)\frac{dZ(a)}{1-Z(A)}$ is the average employment.

Proof. See Appendix D. □

To understand this proposition, note that the supply of skilled workers H affects the labor market condition (32) only. In Figure 2, the (32) curve only shifts downward, giving rise to the results Proposition 5-(1) and (2). It is also easy to confirm an unambiguous increase in $R(a)$ for all active firms, using (28).

Intuitively, a greater supply of skilled workers cause skilled wages to fall, reducing the marginal cost of R&D. This induces relatively low productivity firms to start investing in flow R&D, lowering the threshold productivity A. In addition, this leads to a greater effective "prize" for a winner of a patent race through the product market, as (18) shows. This fact incentivizes active firms to boost R&D employment, accelerating technical progress.

The growth literature shows that an increase in the supply of skilled workers is conducive to innovation, and it is called the scale effect. Though the prediction is not supported (e.g. Jones (1995)), education for that purpose draws much attentions in most of economies. In addition to the scale effect prediction, our result demonstrates that boosting human capital generates incentives for low productivity firms to start investing in R&D. This is because skilled wages become lower, reducing flow fixed R&D costs.

3.3 Self-financed R&D Policy

Table 1 summarizes the effects of changes in policy variables. It shows that the qualitative effects (i.e. signs) of subsidies to flow variable and fixed costs, s_R and s_f, are opposite. Their quantitative effects are also opposite in the sense that the net effects are nil if the both policy variables marginally change by the same amount for $s = s_R = s_f$ initially. A higher s_R is pro-innovation, whereas a higher s_f is anti-innovation. Figure 3 illustrates such differences in terms of the firm distribution. The vertical axis measures the number of firms, and R&D productivity is on the horizontal axis. The area under the distribution is equivalent to the number of firms which stay in a patent race and invests in flow R&D. An increase in s_R induces lower productivity firms to exit a patent race and makes it possible for "elite" firms only to contribute to accelerated innovation. In contrast, a greater s_f retards innovation, giving a greater incentives for lower productivity firms to join a patent race.

The above results have two important implications for policy makers. First, In the literature, theoretical analysis is often conducted to demonstrate that R&D subsidies are best used to correct market failures which are prevalent in R&D. Our result shows that this prediction fails if the policy is applied to flow fixed costs only, and it even worsens the situation. Second, the literature usually ignores the *source* of financing R&D subsidies in developing normative implications for the reason of simplification. Our paper gives a possible justification for such omission. To understand this claim, note that in our model, the result of subsidizing flow variable R&D costs can be replicated by *taxing* flow fixed R&D expenditure. That is, the combined subsidy-tax policy measures, which promote innovation in net, can be self-financed with neutral fiscal stance. In this sense, policy analysis in the literature may be justified if flow variable and fixed R&D costs are clearly distinguished.

To explore this second implication further, note that the following budget constraint should hold for self-financed R&D subsidy/tax policy:

$$w(-s_f)N[1 - Z(A)]f = ws_R N \int_A^{a_H} R(a)dZ(a) \tag{33}$$

where $-s_f > 0$ is now interpreted as a tax rate on flow fixed R&D costs. The left-hand side of (33) is the tax revenue, and the right-hand side gives government spending on subsidies to flow variable R&D costs. Suppose that the tax rate $-s_f$ is endogenously adjusted for the balanced budget constraint (33) to hold in equilibrium.[12] Rearranging (33) gives

$$w(-s_f)f = ws_R\tilde{R}(A) \tag{34}$$

where $\tilde{R}(A) \equiv \int_A^{a_H} R(a)\frac{dZ(a)}{1-Z(A)}$ is the average flow variable R&D workers. This equation shows that gross tax payments that each R&D firm makes $w(-s_f)f$ are equal to subsidies to a firm with the average flow variable R&D employment, $ws_R\tilde{R}(A)$. Therefore, net tax payments are given by $w(-s_f)f - ws_R R(a) = ws_R(\tilde{R}(A) - R(a))$.[13]

This demonstrates that firms with flow variable employment lower than the average are taxed in net, whereas those with the above average employment are subsidized in net.[14] Empirical studies show that the social rate of return is much higher than the

[12] In this extended case, three endogenous variables are (I, A, s_f). To solve it, substitute (32) into (33) to derive

$$-sf = s_R\left(\frac{H}{N[1 - Z(A)]f} - 1\right).$$

(I, A, s_f) are determined by the system of three equations (30), (32) and the above equation.

[13] Using the equation in footnote 12, one can verify the following:

$$-\frac{ds_f}{s_f} = \frac{ds_R}{s_R} + \left(\frac{s_R}{s_f} + 1\right)\frac{z(A)}{1 - Z(A)}dA$$

for $s_f \neq 0$ and $s_R \neq 0$. This means that the proportionate rate of changes in s_f must be greater than that of s_R when the rate of subsidy s_R is changed. It means that a change in the tax rate must be greater than that of the subsidy rate in order for the budget constraint (33) to hold.

[14] Note that firms with $\tilde{R}(A)$ are not necessarily those with the average R&D productivity for $a \in$

private counterpart, implying underinvestment in R&D. To correct the market failure, as explained above, the literature argues that R&D subsidies should be used. Following this widely recognized argument, suppose that self-financed subsidies are used to address the issue of R&D underinvestment. According to our analysis, it would be necessary to tax relatively lower productivity firms and subsidize subsidize relatively higher productivity firms to improve welfare.

4 Conclusion

The paper developed a model of endogenous technical progress where heterogeneous firms compete in patent races. In equilibrium, the rate of technical change and the distribution of R&D firms and their research expenditure are determined. Using the model, we identified several interesting results concerning the effects of government policy. In particular, subsidies to flow variable expenditure and policy to intensify competition with more firms entry generate the cleansing effect in the sense that relatively low productivity firms exit patent races and innovation accelerates. On the other hand, subsidies to flow fixed R&D expenditure gives rise to the opposite result. We also explored the issue of whether firms with high (or low) productivity should be subsidized (or taxed) in net when the industrial policy is self-financed. Our analysis demonstrated that relatively high productivity firms should be more favorably treated than lower productivity firms.

Finally, we comment on possible extensions of the model. First, the distribution of R&D firms and their expenditure can be further endogenized. The firm distribution is characterized by the distribution function $Z(a)$, the threshold productivity A and the number of entrant firms N. The second variable A only is enogenized in the paper, but in principle N can also be endgenized. One possible way is to assume a free entry process at the beginning of each patent race with fixed sunk costs, say f_N. Free entry continues until the expected benefit of entry into a patent race is equalized to the fixed cost. By so doing, the "hight" of the firm distribution in Figure 3 would respond to policy changes, and new insight could be gained. Second, the current paper focussed upon steady state. But, off steady state analysis is required to examine the stability properties of steady state equilibrium. In addition, dynamic changes in the distribution in response to policy changes may generate interesting insights.

$[A, a_H]$, given a general form of the distribution function $Z(a)$.

Appendix

A. Proof of Proposition 1

Totally differentiating (32) and (29) gives

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} dI \\ \\ dA \end{bmatrix} = \begin{bmatrix} M_1^S \\ M_2^S \end{bmatrix} dS + \begin{bmatrix} M_1^N \\ M_2^N \end{bmatrix} dN + \begin{bmatrix} M_1^H \\ M_2^H \end{bmatrix} dH \tag{A1}$$

$$S = \frac{1-s_f}{1-s_R}, \qquad P_{11} = N \int_A^{a_H} \frac{\partial X(a)}{\partial I} dZ(a) > 0, \tag{A2}$$

$$P_{12} = -NX(A)z(A) + N \int_A^{a_H} \frac{\partial X(a)}{\partial A} dZ(a) - Nz(A)f < 0, \tag{A3}$$

$$P_{21} = 1 - N \int_A^{a_H} ah'(X(a)) \frac{\partial X(a)}{\partial I} dZ(a) > 0, \tag{A4}$$

$$P_{22} = NAh(R(A))z(A) - N \int_A^{a_H} ah'(X(a)) \frac{\partial X(a)}{\partial A} dZ(a) > 0, \tag{A5}$$

$$M_1^S = -N \int_A^{a_H} \frac{\partial X(a)}{\partial S} dZ(a) < 0, \qquad M_2^S = N \int_A^{a_H} ah'(X(a)) \frac{\partial X(a)}{\partial S} dZ(a) > 0, \tag{A6}$$

$$M_1^N = -\int_A^{a_H} X(a) dZ(a) - \int_A^{a_H} \frac{\partial X(a)}{\partial N} - [1 - Z(A)]f < 0, \tag{A7}$$

$$M_2^N = \int_A^{a_H} ah(X(a)) dZ(a) + N \int_A^{a_H} ah'(X(a)) \frac{\partial X(a)}{\partial N} dZ(a) > 0, \tag{A8}$$

$$M_1^H = 1, \qquad M_2^H = 0, \tag{A9}$$

where $z(a)$ is the density function of productivity a. The following can be confirmed:

$$D = P_{11}P_{22} - P_{21}P_{12} > 0. \tag{A10}$$

Resutl (i): Using (A1), it is easy to show $D_A^S = P_{11}M_2^S - P_{21}M_1^S > 0$. By Cramer's rule,

$$\frac{dA}{dS} = \frac{D_A^S}{D} > 0 \qquad \Rightarrow \qquad \frac{dA}{ds_R} = \frac{dA}{dS}\frac{dS}{ds_R} > 0. \tag{A11}$$

Result (ii): Using (A1),

$$D_I^S = M_1^S P_{22} - M_2^S P_{12} = N^2 \left\{ z(A) \int_A^{a_H} \frac{\partial X(a)}{\partial S} \Theta(a, A) dZ(a) + \Delta(A) \right\} \tag{A12}$$

where

$$\Theta(a, A) = ah'(X(a))[\bar{X}_A + f] - Ah(\bar{X}_A), \tag{A13}$$

$$\Theta(A, A) = 0, \tag{A14}$$

$$\frac{\partial \Theta(a, A)}{\partial a} = \frac{\partial \Theta(a, A)}{\partial ah'(X(a))} \frac{\partial ah'(X(a))}{\partial a} = [\bar{X}_A + f] \frac{\partial ah'(X(a))}{\partial a} > 0. \tag{A15}$$

The sign of (A15) can be confirmed, using (11). Hence, $\Theta(a, A) > 0$. In addition,

$$\begin{aligned}\Delta(A) &= \int_A^{a_H} \frac{\partial X(a)}{\partial S} dZ(a) \int_A^{a_H} a h'(X(a)) \frac{\partial X(a)}{\partial A} dZ(a) \\ &\quad - \int_A^{a_H} a h'(X(a)) \frac{\partial X(a)}{\partial S} dZ(a) \int_A^{a_H} \frac{\partial X(a)}{\partial A} dZ(a) \end{aligned} \tag{A16}$$

$$\begin{aligned} &= \int_A^{a_H} \frac{\partial X(a)}{\partial S} dZ(a) \int_A^{a_H} a h'(X(a)) \frac{\partial X(a)}{\partial A} dZ(a) \\ &\quad - \int_A^{a_H} a h'(X(a)) \frac{\partial X(a)}{\partial A} \frac{\partial A}{\partial S} dZ(a) \int_A^{a_H} \frac{\partial X(a)}{\partial S} \frac{\partial S}{\partial A} dZ(a) \end{aligned} \tag{A17}$$

$$= 0 \tag{A18}$$

where $\frac{\partial A}{\partial S}$ is given in (A11). Using this result, we have $D_I^S > 0$, and

$$\frac{dI}{dS} = \frac{D_I^S}{D} > 0 \qquad \Rightarrow \qquad \frac{dA}{ds_R} = \frac{dA}{dS} \frac{dS}{ds_R} > 0. \tag{A19}$$

Result (iii): Rearranging (32) gives

$$\int_A^{a_H} R(a) \frac{dZ(a)}{1 - Z(A)} = \frac{H}{N[1 - Z(A)]} - f. \tag{A20}$$

This shows that as the average flow variable employment on the left-hand side increases as A increases. Hence, (A20) along with Result (i) proves the result.

B. Proof of Proposition 2

Result (i): From (A11), $\dfrac{dA}{ds_2} = \dfrac{dA}{dS} \dfrac{dS}{ds_2} < 0.$

Result (ii): From (A19), $\dfrac{dA}{ds_f} = \dfrac{dA}{dS} \dfrac{dS}{ds_f} < 0.$

Result (iii): It is obvious from Result (i) and (A20).

C. Proof of Proposition 3

Result (i): From the second equation of (A2), (A4), (A7) and (A8),

$$\frac{dA}{dN} = \frac{P_{11} M_2^N - P_{21} M_1^N}{D} > 0.$$

Result (ii): From (A3), (A5), (A7) and (A8),

$$\frac{dI}{dN} = \frac{M_1^N P_{22} - M_2^N P_{12}}{D} \gtreqless 0.$$

Result (iii): It is obvious from Result (i) and (A20).

D. Proof of Proposition 4

Result (i): From the second equation of (A2), (A4) and (A9),

$$\frac{dA}{dH} = \frac{P_{11}M_2^H - P_{21}M_1^H}{D} = -\frac{P_{21}}{D} < 0.$$

Result (ii): From (A3), (A5) and (A9),

$$\frac{dI}{dH} = \frac{M_1^H P_{22} - M_2^H P_{12}}{D} = \frac{P_{22}}{D} > 0.$$

Result (iii): It is obvious from Result (i) and (A20).

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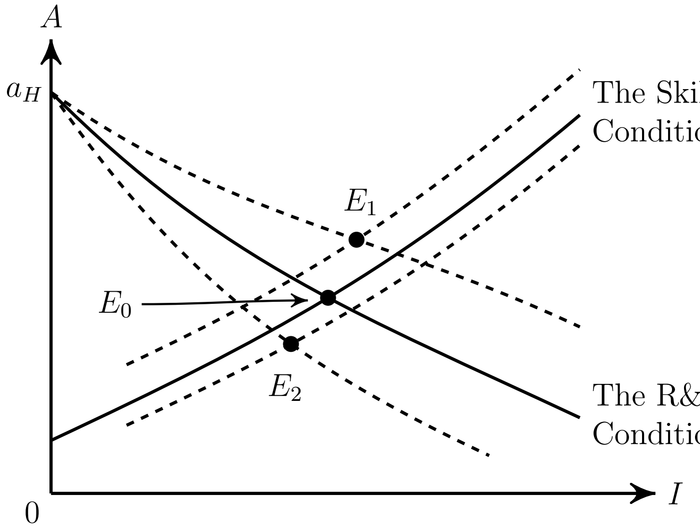

Figure 1: Following an increase in s_R, the rate of subsidies to flow variable R&D costs, equilibrium moves from E_0 to E_1. On the other hand, it moves to E_2 after an increase in s_f, the rate of subsidies to flow fixed R&D costs.

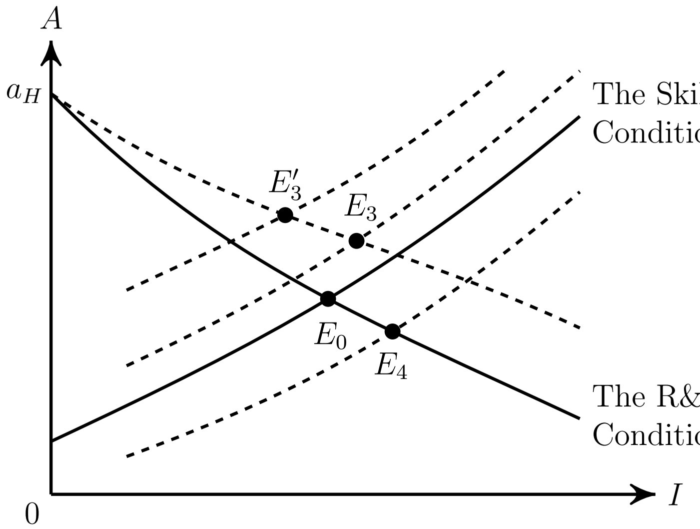

Figure 2: An increase in the number of entrant firms N moves equilibrium to E_3 or E_3'. Equilibrium moves to E_4 after the number of skilled workers H increases.

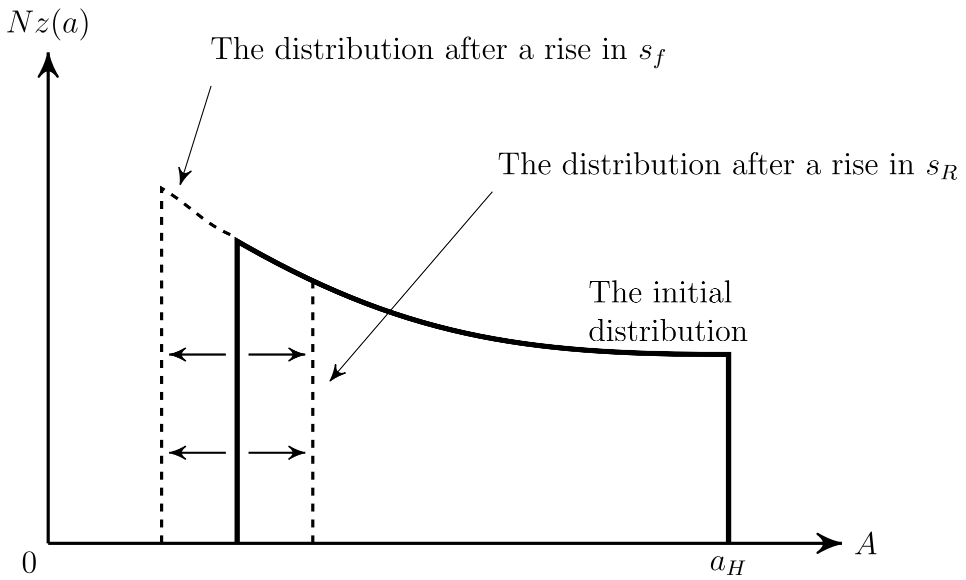

Figure 3: The initial distribution of R&D firms is given by thick lines, and the area inside the lines is equivalent to the number of firms which invest in flow R&D. As the rate of subsidies to flow variable R&D costs s_R increases, the distribution shrinks. On the other hand, a higher rate of subsidies to flow fixed costs s_f expands the distribution.