Dynamics of FDI and Welfare with Product Cycle

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Abstract

This paper extends the product cycle model of Grossman and Helpman(1991a) to explore the effect of FDI on innovation and imitation through product cycle mechanism. We explores the effect of two types of FDI, one-way FDI that there is only investment from developed countries to developing countries and two-way FDI that includes investment from developing countries to establish R&D centers in developed countries. Our model yields very rich results concerning the effect of changing the relative size of north to south on innovation and imitation, and the policy effect of R&D subsidy. These results are very different from the case of no FDI, because the link between two countries' labor market is more complex when there is FDI that creates a co-movement of labor demand in these two types of countries if R&D activity or manufacture activity experience a shock in a single country. Finally we find that the welfare level in case of one-way FDI is bigger than that in case of one-way FDI.

Keywords: Dynamic welfare; FDI; Product cycle; Quality ladder; Industrial Policy

JEL classification: F14; F21; L62; P63; P33

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1.Introduction

Trade and income inequality has become a very hot topic recently, with ever rising income inequality both between north and south and among individual countries, the effect of globalization, most notably trade and FDI, is under careful review by policy makers all over the world. The race of Innovation and imitation is the major force driving the evolving pattern of north-south trade, especially for countries like China, whose export products to developed countries has been undergoing fast quality upgrading. So a product cycle model with quality upgrading as the outcome of R&D activity is the best choice to study the impact of FDI on innovation, imitation and the dynamics of north-south wage gap.

Product life cycle hypothesis is firstly formalized by Krugman in a framework of trade model in his 1979 paper. And he proposed that product cycle hypothesis is a very good approach to introduce technology innovation into trade theory. Krugman(1979)views product cycle hypothesis as a good approach to incorporate technology innovation into trade theory. With the multiple framework developed in later literatures, this approach can provide valuable insight into the question of how would innovation and technology transfer impact trade patterns and income gap, and also provides a useful framework to study the regime effect.

FDI can affect the regime effect and through the linkage between the labor market in north and south. Traditional research on the effect of FDI focuses on the FDI flows from developed countries to developing countries, a main feature of FDI flows since 1980s. but recent years witness the reverse trend of FDI from developing countries to developed countries, which mainly focuses on R&D sector to utilize the human capital and innovation network in developed countries. This new trend may have different impact on innovation and income gap through the product cycle dynamics. Because in one-way FDI, the expansion of production of northern products would raise the labor demand in south and push up the wage and squeeze out labor in R&D sector. But R&D activity in south do not have impact on Northern labor market. But in case of two-way FDI, R&D activity by southern firms would raise both the labor demand in north and south due to more southern products invented.

Furthermore, The impact of FDI on product cycle and innovation, both one-way and two-way, is critical to know thoroughly about the effect of FDI on trade gains, economic growth and welfare. How would FDI affect income inequality through product cycle? the answer to this question could provide new insight to policy concerns about globalization.

So in our paper we first extend the quality ladder model of Grossman and Helpman(1991b) to study the effect of FDI on innovation and imitation, and on regime effect through the product cycle dynamics, then we explore how would two-way FDI alter these results. On top of that we study the policy impact of subsidy on innovation and imitation.

Our study is most closely related to the strand of literature that formalizes the product cycle hypothesis to study north-south trade issues. Krugman(1979)'s work establishes the fundamental questions of the product cycle model, how is north-south trade pattern determined, the effect of innovation and imitation on income gap between north and south. So innovation rate and imitation rate is assumed exogenous in their work. This approach is extended by Dollar(1986) and explains factor price equalization hypothesis. The regime effect is opposite to Krugman(1979)'s work, increasing labor supply in north would raise northern relative wage in the short run and reduce northern wage in the long run. Jensen&Thursby(1986) find that in steady state enlarging southern labor supply would reduce the ratio of innovation to technology transfer.

Segerstrom&Dinopoulos(1990) try to endognenize innovation rate by introducing Schumpeter(1942)'s description of product innovation and find that relative wage rises with the size of southern labor force in case of incomplete specialization but falls in case of incomplete specialization. Grossman and Helpman introduce the concept of R&D race(Lee and Wilde(1980)) to endogenize innovation and imitation and build a new framework of product cycle hypothesis (Grossman& Helpman(1989)). They introduce the quality ladder concept in define product innovation in this framework, instead of the expansion of horizontal differentiated product in Krugman's approach (Grossman&Helpman(1989)), and revisit these basic questions in product cycle literature (Grossman &Helpman (1991b)), this framework is widely used to study issues like economic growth(Grossman &Helpman(1991a), Jones(1995b), Segerstrom(1998), Gustafsson &Segerstrom(2007)), intellectural property right protection and technology transfer(Helpman(1993), Glass&Saggi (1996,2002), Sener (2006)), Borota(2012)), Glass&Saggi(2002)), outsourcing(Lai et al.(2009)) or the effect of trade liberalization or IPR on income inequality (Gustafsson&Segerstrom(2008,2011)) and on technology catchup (Borota(2012)).

Our work is also related to the strand of literature that studies the impact of IPR protection and FDI using the product cycle framework. Product cycles hypothesis is always adopted to study the impact of IPR protection on growth and welfare, trade and FDI are always viewed as channels of technology transfer. Two central research question of this strand of literature is the impact of IPR protection on innovation and imitation and how would strengthening IPR protection affect the wage gap between north and south. Glass and Saggi(1996) distinguish two types of consumers and FDI in terms of quality level and conclude that find that larger size of north relative to south would increase the rate of innovation and decrease the extent of high-quality FDI and bigger subsidy to imitation would increase the extent of high-quality FDI and increase the rate of innovation. Some works find that stronger IPR protection hurts the south and innovation, Helpman(1993) finds that stronger IPR protection would hurt the south and would hurt the north if imitation rate is low. Lai(1998) finds that stronger IPR protection limits innovation, imitation and lower southern wage if imitation is the channel of production transfer, but has contrary effect if both imitation and FDI are channels of production transfer. Glass&Saggi(2002b) endogenize FDI decision and find that stronger IPR protection cause intensities of imitation to fall and crowds out FDI and reduce innovation. On the contrary, Dinopolous & Segerstrom(2010) find that stronger IPR protection would increase innovation rate and shrink north-south wage gap, which is in accordance with empirical studies of Branstetter(2006) and Sala-i-Martin(2006). Our research do not focus on the effect of IPR protection, but reveals how one-way FDI and two-way FDI would affect the innovation rate, imitation rate and north-south income gap.

This paper revisits the basic questions of a product cycle model and is comparable to existing literatures, like the effect of relative size of North to South on innovation, imitation and wage gap between North and South. And we find that in case of one-way FDI, increasing the size of South would accelerate both innovation rate and imitation rate, but increasing the size of north would only accelerate innovation rate. But in case of two-way FDI, expanding the size of South would slow down innovation by northern firms and accelerate innovation by southern firms, while expanding the size of North would accelerate northern innovation but has ambiguous effect on southern innovation. Relative wage is immune from the change in relative size of North to South. Then we further explore the policy effect of R&D subsidy and find that subsidy to innovation of northern firms do not necessarily promote innovation rate, and subsidy to R&D activity generates different impact on innovation and imitation in case of one-way FDI and two-way FDI. In one-way FDI subsidy to southern firms promote innovation by northern firms but in the presence of two-way FDI this promotion effect is uncertain. R&D subsidy for southern firms works in its targeted direction only when there is two-way FDI. Finally, we compared the welfare level in both cases and find that The welfare of a representative consumer in case of one-way FDI is bigger than that in case of one-way FDI. That is because the weighted average quality adjusted price level is higher in case of two-way FDI than in case of one-way FDI while the expenditure in both cases are the same.

The paper is organized as follows, the next session introduces the model. We first explore the equilibrium of one-way FDI and conduct comparative static analysis about regime effect and policy implication. Then we explore the same issues in the equilibrium of two-way FDI.

2. The model

2.1 Consumer behavior

Consumers worldwide share identical preferences. They seek to maximize an additively separable intertemporal utility function of the form

(1) $$\mathrm{U} = \int_0^{\infty} e^{-\rho t} logU(t)dt$$

where ρ represents the discount factor. instantaneous utility is given by

(2) $$\log\mathrm{U(t)} = \int_0^{\infty} \log[\Sigma_m q_m(\beta) x_{mt}(\beta)]\, d\beta$$

$\beta \in [0,1]$ is continuum of products and $x_{mt}(\beta)$ represents consumption of good β of quality m at time t.

Quality level m of product β provides quality $q_m(\beta) = \lambda^m$ and λ >1 , the increment of quality are common to all products and are exogenously given. $q_{m-1}(\beta) < q_m(\beta)$. All products start at time t=0 and quality level m=0, so the initial quality is assumed to be

$$q_0(m) = \lambda^0 = 1$$

Every consumer maximizes utility subject to an inter-temporal budget constraint

(3) $$\int_0^{\infty} e^{-R(t)} E(t)dt \le \mathrm{A}(0) + \int_0^{\infty} e^{-R(t)} Y(t)dt$$

where $\mathrm{R(t)} = \int_0^t r(s)ds$ is the cumulative interest up to time t, and E(t) is the aggregate expenditure of all consumers in a country. A(0) is the aggregate value of initial asset holdings by consumers.

Y(t) is aggregate income of consumers in a country at time t.solution follows(G&H(1989b)):

(4) $$\frac{\dot{E}}{E} = r - \rho$$

A consumer's maximization problem can be broken into three stages: the allocation of lifetime wealth across time, the allocation of expenditure at each instant across products, and the allocation of expenditure at each instant for each product across available quality levels. Consumers are indifferent between quality level m and m-1 if the relative price equals quality difference.

Our preference setup means different products are completely substitutable, and from this property we can derive that only the product with the best quality will be consumed. It means at any time, only the product with the best quality can survive on market and it grabs the whole market. This setup makes way for the R&D race on supply side and Bertrand competition as market structure.

2.2 Firm behavior

we follow the basic setup of Grossman& Helpman(1991b), that labor is the only factor of production, firm can produce one unit of output using one unit of labor. And each firm produces one type of product.

Northern firms can develop new products(quality improvement) through innovation, southern firms develop new products only through imitation due to technology constraint or human capital constraint, they cannot do innovation. Successful Innovation means the invention of new product that is at least one step higher than the current best quality. While successful imitation means inventing a product that catches up with the current best quality on market.

Product of different qualities on the quality ladder can be seen as different varieties of the same product.

There are in total three types of firms: northern firms that have exclusive abilities to invent the best quality variety of a product and compete with another northern firm, northern firms that have exclusive ability to invent the best quality variety of a product and compete with a southern firm that can only imitate the current best quality variety; southern firms that are able to produce the state-of-the-art product only through imitation. Their numbers are respectively n_{NS}, n_{NN} and n_S, $n_{NN} + n_{NS} + n_S = 1$.

Consumers only buy the top quality for any product, so only firms that are able to produce the top quality of a product β can grab the market. All firms compete as price-setting Bertrand oligopolists, a firm would be driven out of market if only any of its competitor can product the same product and charge a price equal or lower than its marginal cost. Let's define the firm that can produce the top quality variety of any product as the leader, then all other firms can take the market from industry leader by either imitating and producing the current best quality variety of this product and setting the price at the marginal cost of the current leader, or upgrade the quality level of current product and set the quality-adjusted price at the marginal cost of the current leader.

The marginal cost of a norther firm is w_N, the marginal cost of a southern firm is w_S, $w_N > w_S$, so southern firms can profit by simply imitating the current best quality for a product β and set the price at w_N. But the only way for northern firms to take the market is upgrading the current best quality. Once a northern firm successfully improves the quality of existing product, it can grab the whole market from its competitor by setting the quality adjusted price at the marginal cost of the current producer of this product.

2.2.1 Technology for innovation and imitation:

According to Lee&Wilde(1980),the probability of individual research success subjects to a Poisson process and depends solely on current level of R&D activity. τ is the intensity of R&D or imitation, as well as the probability of taking a step up the quality ladder. So τ_N is the R&D intensity taken by a northern firm targeting at a northern product. τ_S is the R&D intensity taken by a northern firm targeting at a southern product. μ is the imitation intensity taken by a southern firm targeting at a northern product.

We distinguish two categories of northern firms, leaders and followers. Leaders are those who have most recently upgraded the quality of a state-of-the-art product through R&D activities, $q_t(\beta)$. They enjoy a cost advantage in developing the next generation of product with the substantial product specific knowledge they accumulated in recent success in R&D. Followers are those that may have developed previous generations of product β , but are trying to upgrade the best quality of product β , $q_t(\beta)$. So we set two technologies for innovation in north, one for quality improvement by leaders, one for quality improvement by followers. The labor requirement for R&D activities at intensity τ is $a_{NL}\tau$ for a leader, and $a_{NF}\tau$ for a follower, $a_{NF} > a_{NL}$. While the labor requirement for imitation at intensity μ is $a_M\mu$.

The incentive for firms to undertake research activity is the future profit stream produced by devoting labor resources into R&D activity. So the maximization of future values requires the expected gains not exceed the R&D cost. For northern firms that target at southern products, it is

(5) $$v_{NS}\tau_S \le a_{NL}w_N\tau_S \quad \text{with equality for } \tau_S > 0.$$

For northern firms that target at northern products, it is

(6) $$v_{NN}\tau_N \le a_{NF}w_N\tau_N \quad \text{with equality for } \tau_N > 0$$

For southern firms, it is

(7) $$v_S\mu \le a_S w_S\mu \quad \text{with equality for } \mu > 0$$

2.3 Equilibrium of one-way FDI

To simplify the analysis, we can set $w_S = 1$, so northern wage becomes w , the relative wage between north and south. Northern firms can invest in South by moving their manufacture to South to take advantage of cheap labor there. Here in our model the FDI decision is exogenous because, unlike the setup in Glass and Saggi(2002) where multinational firms enjoy the cost advantage of South but are also easier to be imitated, we do not assume any tradeoff concern for FDI decision , So in our model there is no reason for a northern firm not to move its manufacture activity to south. We make the FDI exogenous so that we can focus on the pure effect brought by FDI and do not need to distract the forces that determine the equilibrium proportion of northern firms and multinational firms.

When northern firms invest in South, they need to pay some additional operation cost. According to Hymer(1976) and Markusen(1995), multinational firms have additional cost when they move their production to another country, this cost disadvantage may arise from the difficulty to organize production in host country with technology developed in parent company, and the induced communication cost, language barrier, unfamiliarity with local business practice and being out of established business and government networks. To outweigh this inherent cost disadvantage, multinational companies must have some ownership advantage in the form of exclusive intangible assets to confer some market power. So the marginal cost of multinational firms would be lower than production in north, but still a little bit higher than southern firms. Let's set this marginal manufacture cost of multinational firms to be θw, with the condition $0 < \theta < 1$. The marginal cost by northern firms is w, while marginal cost of southern firms is 1 .

2.3.1 Profit of three types of firms

Northern firms that compete with a southern firm upgrade the quality of current southern product and for one step ahead, and set the quality-adjusted price of upgraded product at the marginal cost of its southern competitor. So the price charged by this type of northern firm is λ .So the profit of northern firms that innovate southern products is

(8) $$\pi_{NS} = \frac{(\lambda-\theta w)E}{\lambda} \ .$$

Northern firms that compete with another northern firm produce the product whose quality is one step higher than his competitor, so the quality adjusted price he charges would be the same with his competitor's marginal cost. They set the price at $\lambda\theta w$, their marginal cost is θw. The profit of northern firms that innovate northern products is

(9) $$\pi_{NN} = \frac{(\lambda-1)E}{\lambda}$$

Southern firms charge a quality-adjusted price equal to the marginal cost of its northern competitor, the price they charge is θw . The profit of southern firms is

(10) $$\pi_S = (\theta w - 1)E/\theta w.$$

In steady state, expected return to equity must equal the risk of total capital loss. So $\frac{\pi}{v} + \frac{\dot{v}}{v} = f + r$, where f denotes the risk of losing market. This is called no arbitrage condition. For the three types of firms, the no arbitrage conditions are respectively :

(11) $$\frac{\pi_{NS}}{v_{NS}} + \frac{\dot{v_{NS}}}{v_{NS}} = r + \mu + \tau_N$$

(12) $$\frac{\pi_{NN}}{v_{NN}} + \frac{\dot{v_{NN}}}{v_{NN}} = r + \mu + \tau_N$$

(13) $$\frac{\pi_S}{v_S} + \frac{\dot{v_S}}{v_S} = r + \tau_S$$

2.3.2 Steady state equilibrium:

In steady state, $\frac{\dot{v_j}}{v_j} = \frac{\dot{E}}{E} = 0$. So $r = \rho$ in steady state, because from $E = E(0)e^{-(r-\rho)t} we\ derive$ $\frac{\dot{E}}{E} = r - \rho$.

Applying the non-arbitrage condition(11)(12) and (13) with profit function, we get

(14) $$\frac{\pi_{NS}}{a_{NL}w} = \rho + \mu + \tau_N$$

(15) $$\frac{\pi_{NN}}{a_{NF}w} = \rho + \mu + \tau_N$$

(16) $$\frac{\pi_S}{a_S} = \rho + \tau_S$$

Let's define η as the extent of imitation, $\eta = \mu n_N$. And further define the aggregate innovation rate $\tau = \tau_N n_N + \tau_S n_S$.

In steady state, the number of products "flowing" out of this country must equal to the number of products "flowing" into the country. So firstly, the number of northern products copied by south must equal to the number of southern products innovated by north, that is $\mu(n_{NN} + n_{NS}) = \tau_S n_S$. n_{NN} is the number of northern firms that innovate northern products, n_{NS} is the number of northern firms that innovate southern products. As each firm produces one product, n_{NN} and n_{NS} can be seen as the number of products that upgraded from northern products and southern products. Secondly, within northern firms, the measure of products produced by northern firms that compete with another northern firm would face the risk of being copied by southern firms, the measure of these "outflow" of products must match the measure of northern products successfully innovated by these northern firms. That is $\mu n_{NN} = \tau_N n_{NS}$, where $n_{NN} + n_{NS} = n_N$.

Taking advantage of the two steady state condition we can derive that $n_{NN} = \frac{n_N(\tau-\eta)}{\tau}$, $n_{NS} = \frac{n_N\eta}{\tau}$ and $\tau_S = \frac{\eta}{1-n_N}$, $\tau_N = \frac{\tau-\eta}{n_N}$.(see appendix A for details)

If we put subsidy into this analysis, government of each country subsidizes the innovation activity and imitation activity by burdening part of research cost (a fraction of $1 - s_D$ for northern firms and $1 - s_M$ for south firms).

The three equations can be further converted into:

(17) $$\frac{(\lambda - \theta w)en_N}{\lambda a_{NL}} = (\rho n_N + \tau)(1 - s_D)$$

(18) $$\frac{(\lambda - 1)en_N}{\lambda a_{NF}} = (\rho n_N + \tau)(1 - s_D)$$

(19) $$\frac{(\theta w - 1)e(1 - n_N)}{\theta a_S} = (\rho(1 - n_N) + \eta)(1 - s_M)$$

To close the model, labor market needs to be cleared. In case of one way FDI, Norther labor force is devoted to R&D sector only, including R&D carried out by northern leaders and northern followers. The labor demand of R&D by northern firms that innovate southern products is $a_{NL}\eta$ (because the intensity of northern R&D targeting at southern products is $\tau_S n_S$, which equals η, and the labor requirement per unit of R&D by such firms is a_{NL}). The labor demand of R&D by northern firms that innovate northern products is $a_{NF}(\tau-\eta)$, because the intensity of northern R&D targeting at northern products is $\tau_N n_N$, which equals $\tau-\eta$, and the labor requirement per unit of R&D by such firms is a_{NF}.

So the labor market clear equation in north is

$$L_N = a_{NL}\eta + a_{NF}(\tau-\eta) \tag{20}$$

Southern labor force is devoted to three sectors, southern imitation activity, manufacture of southern products and manufacture of northern products. The labor demand of southern imitation is $a_S\eta$, because the aggregate extent of imitation is μn_N, which equals η, and the labor requirement per unit of imitation is a_S. the labor demand of manufacture by southern firms is $\frac{(1-n_N)e}{\theta}$ .The labor demand of manufacture by northern followers is $\frac{n_N(\tau-\eta)e}{\lambda\tau\theta}$. The labor demand of manufacture by northern leaders is $\frac{n_N\eta ew}{\lambda\tau}$.

So the labor market clear equation of south is

$$L_S = a_S\eta + \frac{(1-n_N)e}{\theta} + \frac{n_N(\tau-\eta)e}{\lambda\tau\theta} + \frac{\eta n_N ew}{\lambda\tau} \tag{21}$$

Compared to the case where there is no FDI, the introduction of FDI increases the labor demand in south, shrinks the labor demand in north.

Then the non-arbitrage condition for the three types of firms and the labor market equilibrium equations (17)-(21)can be converted into the a system of five equations with five endogenous variables, η, τ, n_N, e *and* w:

$$L_S = a_S\eta + \frac{(1-n_N)e}{\theta} + \frac{n_N(\tau-\eta)e}{\lambda\tau\theta} + \frac{\eta n_N ew}{\lambda\tau}$$

$$L_N = a_{NL}\eta + a_{NF}(\tau-\eta)$$

$$\frac{(\lambda-\theta w)en_N}{\lambda a_{NL}} = (\rho n_N + \tau)(1-s_D)$$

$$\frac{(\lambda-1)en_N}{\lambda a_{NF}} = (\rho n_N + \tau)(1-s_D)$$

$$\frac{(\theta w-1)e(1-n_N)}{\theta a_S} = (\rho(1-n_N)+\eta)(1-s_M)$$

Using equation (17) and (18) we can derive the equilibrium relative wage w, then the remaining endogenous variables are η, τ, n_N, e .

Now total differentiation of the above equations with respect to η, n_N, τ, e, using L_S, L_N, s_M, s_D as shift parameters, let $\beta = E/\lambda\tau$, yields the following system:

$$dL_S = \left[a_S + \beta n_N\frac{\theta w-1}{\theta}\right]d\eta - \frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta]dn_N + \frac{\beta n_N\eta(1-\theta w)}{\theta\tau}d\tau + [\frac{1-n_N}{\theta} + \frac{n_N(\tau-\eta)}{\lambda\tau\theta} + \frac{n_N\eta}{\lambda\tau}]de \tag{23}$$

$$dL_N = (a_{NL}-a_{NF})d\eta - a_{NF}d\tau \tag{24}$$

(25) $$\frac{\tau}{n_N}(1-s_D)dn_N-(1-s_D)d\tau+\frac{\lambda-1}{\lambda a_{NF}}n_N de=-b_1 ds_D$$

(26) $$-(1-s_M)d\eta-\frac{\eta}{1-n_N}(1-s_M)dn_N+\frac{\theta w-1}{\theta a_S}(1-n_N)de=-b_2 ds_M$$

So we have $A\begin{pmatrix} d\eta \\ dn_N \\ d\tau \\ de \end{pmatrix}=\begin{pmatrix} dL_S \\ dL_N \\ -b_1 ds_D \\ -b_2 ds_M \end{pmatrix}$

Where $A=\begin{pmatrix} a_S+\beta n_N\frac{\theta w-1}{\theta} & -\frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta] & \frac{\beta n_N\eta(1-\theta w)}{\theta\tau} & \frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau} \\ a_{NL}-a_{NF} & 0 & a_{NF} & 0 \\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}}n_N \\ -(1-s_M) & -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{pmatrix}$

And $b_1=\eta+(1-n_N)\rho$, $b_2=\eta+n_N\rho$

We calculate that $|A|<0$ (See appendix B and C)

2.3.3 Comparative Statics

First, we derive the relative wage of north to south:

(27) $$w=\frac{1}{\theta}[\lambda-\frac{a_{NL}}{a_{NF}}(\lambda-1)]$$

Relative wage between north and south is irrelevant with labor supply in the two countries, this is in accordance with the result in Grossman&Helpman(1991b) and Glass&Saggi(2002),but in contrast with literatures adopting Krugman's framework, like Grossman&Helpman (1991a), Dollar(1986) etc.

Lemma1: *relative wage of north to south is irrelevant with relative size of north to south. The bigger the intertemporal knowledge spillover(Jones,1995b), or the smaller additional cost induced by moving manufacture to South, the larger the wage gap between North and South.*

By solving the above system, we have the following comparative statics:

$$\frac{d\eta}{dL_S}>0 \text{ and } \frac{d\tau}{dL_S}>0$$

$$\frac{d\tau}{dL_N}>0$$

Proposition 1: *In case of one-way FDI, in the long run, an expansion of labor supply in South would accelerate the innovation rate ($d\tau/dL_S>0$) and increase the aggregate extent of imitation ($d\eta/dL_S>0$). An expansion of labor supply in North would raise innovation rate, but has ambiguous effect on* *imitation.*

Proof. see appendix D

Remarks:

An expansion of labor supply in south means more resources can be devoted into imitation by southern firms, so the aggregate extent of imitation, η, rises. As a result, the number of products produced in south, n_S, increases and northern firms produce less kinds of products than before. This would release some labor resources that would be absorbed by R&D sector, so innovation intensity τ rises. Then the number of northern products would expand as a result. So in new steady state, the number of northern products could be more or less than before the expansion of south. The rise of the number of northern products provides more targets for southern firms to imitate, so the level of aggregate imitation will not fall.

An expansion of labor supply in north means more resources could be devoted to manufacture and R&D, so the R&D intensity τ increases, as a result there will be more northern products and less southern products, so the aggregate extent of imitation falls if imitation intensity stays unchanged. But the overall effect of the proportion of north to south products is a reduction of labor hired by manufacture sector in south. Because producing one more kind of southern product requires more labor than producing one more kind of northern product. So the excess labor in south would be absorbed in R&D sector and lead to a bigger scale of imitation, resulting in an rise of n_S and η. So in new steady state, the effect of northern expansion on n_N and η cannot be predetermined by intuition only. And the result shows they are actually uncertain.

Effect of subsidy:

$$\frac{\mathrm{d}n_N}{ds_D} < 0$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}s_D} > or < 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}s_D} > or < 0$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}s_D} > or < 0$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}s_M} > 0$$

$$\frac{\mathrm{d}n_N}{ds_M} > 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}s_M} > 0$$

$$\frac{d\mu}{ds_D} > or < 0$$

So we have the following proposition:

Proposition2*: In case of one-way FDI, in the long run, an increase in the subsidy to northern firms has ambiguous effect on innovation rate and imitation rate. But an increase in the subsidy to southern firms would accelerate innovation rate by northern firms and increase the proportion of northern products on the market, but has ambiguous effect on imitation rate.*

Proof. see appendix D

Remark:

Bigger subsidy to innovation would immediately augment the innovation intensity and increase n_N, n_S would fall, the overall result of this change in proportion is shrink of labor demand in manufacture sector. So there would be excess labor in south absorbed by R&D sector and lead to a bigger scale of imitation, resulting in an rise of n_S and η. So we cannot know precisely how would n_S and η change in a new steady state only by intuition. The result shows the change of η is uncertain, but n_N would be smaller. The change of τ is uncertain, because the change of η is uncertain, if η is bigger in new steady state, the overall innovation intensity τ would be bigger since a bigger η implies in equilibrium there is bigger extent of innovation over southern products.

Bigger subsidy to imitation would raise the extent of imitation η at the beginning, resulting in more southern products and raise the labor demand in south. At the same time, a bigger η would raise the overall innovation intensity τ, resulting in more northern products, because in equilibrium the extent of imitation equals the extent of innovation over southern products, which is part of overall innovation. So in new steady state, both η and τ would be bigger, but the change of n_N is uncertain by intuition, and our comparative statics reveal that n_N would actually increase.

2.4 Equilibrium of two-way FDI

Due to strict intellectual property protection, firms in South can no longer imitate northern products in South, so they invest in north to build R&D centers there, in order to utilize the R&D human resource in the north. So both the northern firms and southern firms conduct R&D activities in the north, raising the demand for R&D labor significantly, while labor demand in the south shrinks. Now southern firms also participate in the R&D race with northern firms, but the innovation carried out by southern firms is not to have the quality increment one step ahead of current product, it is a type of second mover which aims at reaching the current best quality to take the market from current producer by its marginal cost advantage. So this kind of innovation is quite similar to the imitation conducted by southern firms in one-way FDI but is not imitation since it develops some new product features while the quality level is not higher than current best quality. This kind of innovation is widely seen in overseas research center of many Chinese firms like Haier, Lenovo etc., they do not hire the top engineers to do R&D because they are not trying to be the quality leader around the world, they are just trying to upgrade their quality level to catch up with the first class in the world. The reason we do not assume southern firm to be able to conduct innovation on the same level with northern leaders is that firstly southern firms do have a R&D gap with northern firms in the real world, secondly, if we assume away this technology gap there will be no difference between northern firms and southern firms and there is no point in analyzing the north-south product cycle.

We assume the resource requirement for northern leaders and northern followers to conduct R&D activity is respectively a_{NL} and a_{NF} , and $a_{NL} < a_{NF}$ as the leader can take advantage of its accumulated product-specific knowledge. The resource requirement for southern firms to conduct R&D activity is a_S. Here we need another two assumptions, $a_S > a_{NL}$ and $a_{NL} - a_{NF} + a_S > 0$. It means it incurs more cost for southern firm to do innovation than northern leader.

τ_{NN} denotes the R&D intensity of northern firms upgrading products currently produced by northern firms.

τ_{NS} denotes the R&D intensity of northern firms upgrading products currently produced by southern firms.

τ_S denotes the R&D intensity of southern firms.

The incentive for firms to undertake research activity is the future profit stream produced by devoting labor resources into R&D activity. So the maximization of future values requires the expected gains not exceed the R&D cost. For northern firms that target at southern products, it is

(28) $$v_{NS}\tau_{NS} \leq a_{NL}w_N\tau_{NS} \quad \text{with equality for } \tau_S > 0.$$

For northern firms that target at northern products, it is

(29) $$v_{NN}\tau_{NN} \leq a_{NF}w_N\tau_{NN} \quad \text{with equality for } \tau_N > 0$$

For southern firms, it is

(30) $$v_S\tau_S \leq a_S w_N\tau_S \quad \text{with equality for } \tau_S > 0$$

2.4.1 Steady state equilibrium

In case of two-way FDI, Let's define η as the extent of southern innovation, $\eta = \tau_S n_S$, and τ as the aggregate innovation rate, $\tau = \tau_{NN} n_{NN} + \tau_{NS} n_{NS}$.

In steady state, the number of products "flowing" out of the northern firms must equal to the number of products "flowing" into southern firms. So

$$\tau_S(n_{NN} + n_{NS}) = \tau_S n_S = \eta$$

$$(n_{NN} + n_{NS} = n_N)$$

within northern firms, the measure of products produced by northern firms that compete with another northern firm would face the risk of being copied by southern firms, the measure of these "outflow" of products must match the measure of northern products successfully innovated by these northern firms. That is,

$$\tau_{NS} n_{NN} = \tau_{NN} n_{NS}$$

Let's define the aggregate innovation rate $\tau = \tau_N n_N + \tau_S n_S = \tau_N n_N + \eta$.

Taking advantage of the two steady state condition we can derive that $\tau_S = \frac{\eta}{1-n_N}$, $n_{NN} = \frac{n_N(\tau-\eta)}{\tau}$, $n_{NS} = \frac{n_N \eta}{\tau}$ and $\tau_{NS} = \frac{\eta}{1-n_N}$, $\tau_{NN} = \frac{\tau-\eta}{n_N}$. So all these variables are functions of n_N, τ *and* η .

if we put subsidy into this analysis, governments of each country subsidize the R&D activity and imitation activity by burdening part of research cost(a fraction of $1 - s_D$ for north and $1 - s_M$ for south).

The profit function of the three types of firms are the same as in the case of one-way FDI . We then apply the profit function of the three types of firms. So the non-arbitrage conditions can be converted into:

(31) $$\frac{(\lambda - \theta w) e n_N}{\lambda a_{NL}} = (\rho n_N + \tau)(1 - s_D)$$

(32) $$\frac{(\lambda - 1) e n_N}{\lambda a_{NF}} = (\rho n_N + \tau)(1 - s_D)$$

(33) $$\frac{(\theta w - 1) e (1 - n_N)}{\theta w a_S} = (\rho(1 - n_N) + \eta)(1 - s_M)$$

labor market equilibrium produces another two equations. In case of two-way FDI, Norther labor force is devoted to R&D sector only, consisting of R&D carried out by northern leaders, northern followers and southern firms. So

(34) $$a_{NS}\tau_{NS}n_S + a_{NN}\tau_{NN}n_N + a_S\tau_S n_N = L_N$$

The labor demand of R&D by northern firms that innovate southern firms is $a_{NS}\eta$ (because the intensity of northern R&D targeting at southern products is $\tau_{NS} n_S$, which equals η, and the labor requirement per unit of R&D by such firms is a_{NS}. The labor demand of R&D by northern firms that innovate northern products is $a_{NS}(\tau-\eta)$, because the intensity of northern R&D targeting at northern products is $\tau_{NN}n_N$, which equals $\tau-\eta$, and the labor requirement per unit of R&D by such firms is a_{NF}. The labor demand of southern imitation is $a_S\eta$, because the aggregate extent of imitation is $\tau_S n_N$, which equals η, and the labor requirement per unit of imitation is a_S.

So the labor market clear equation in north can be expressed as

(35) $$L_N = a_{NL}\eta + a_{NF}(\tau-\eta) + a_S\eta$$

Compared with the case of one-way FDI, the labor demand in north expands by a new R&D sector from southern firms.

Southern labor force is devoted to manufacture of southern products and manufacture of northern products. The labor demand of manufacture by southern firms is $\frac{(1-n_N)e}{\theta}$. The labor demand of manufacture by northern followers is $\frac{n_N(\tau-\eta)e}{\lambda\tau\theta}$. The labor demand of manufacture by northern leaders is $\frac{n_N\eta ew}{\lambda\tau}$.

So the labor market clear equation of south is

(36) $$L_S = \frac{(1-n_N)e}{\theta} + \frac{n_N(\tau-\eta)e}{\lambda\tau\theta} + \frac{\eta n_N ew}{\lambda\tau}$$

Then the non-arbitrage condition for the three types of firms and the labor market equilibrium equations form a system of five equations with five endogenous variables, η, τ, n_N, e and w:

$$L_N = a_{NL}\eta + a_{NF}(\tau-\eta) + a_S\eta$$

$$L_S = \frac{(1-n_N)e}{\theta} + \frac{n_N(\tau-\eta)e}{\lambda\tau\theta} + \frac{\eta n_N ew}{\lambda\tau}$$

$$\frac{(\lambda-\theta w)en_N}{\lambda a_{NL}} = (\rho n_N+\tau)(1-s_D)$$

$$\frac{(\lambda-1)en_N}{\lambda a_{NF}} = (\rho n_N+\tau)(1-s_D)$$

$$\frac{(\theta w-1)e(1-n_N)}{\theta w a_S} = (\rho(1-n_N)+\eta)(1-s_M)$$

Relative wage w can be solved by equations () and (), it is the same as the case of one-way FDI. Then the endogenous variables are η, n_N, τ, e, the same as the case of one-way FDI.

Now total differentiation of the above equations with respect to η, n_N, τ, e, using L_S, L_N, s_M, s_D as shift parameters, let $\beta = E/\lambda\tau$, yields the following system:

(37) $$dL_S = (\beta n_N\frac{\theta w-1}{\theta})d\eta - \frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta]dn_N - \frac{\beta n_N\eta(\theta w-1)}{\theta\tau}d\tau + [\frac{1-n_N}{\theta} + \frac{n_N(\tau-\eta)}{\lambda\tau\theta} + \frac{n_N\eta}{\lambda\tau}]de$$

(38) $$\mathrm{dL_N} = (a_{NL} - a_{NF} + a_S)d\eta + a_{NF}d\tau$$

(39) $$\frac{\tau}{n_N}(1 - s_D)dn_N - (1 - \mathrm{s_D})\mathrm{d}\tau + \frac{\lambda - 1}{\lambda a_{NF}} n_N \mathrm{de} = -\mathrm{b_1} ds_D$$

(40) $$-(1 - \mathrm{s_M})\mathrm{d}\eta - \frac{\eta}{1 - n_N}(1 - s_M)dn_N + \frac{\theta w - 1}{\theta w a_S}(1 - n_N)de = -b_2 ds_M$$

So we have $A\begin{pmatrix} d\eta \\ dn_N \\ d\tau \\ de \end{pmatrix} = \begin{pmatrix} dL_S \\ dL_N \\ -b_1 ds_D \\ -b_2 ds_M \end{pmatrix}$

Where

$$A=\begin{pmatrix} \beta n_N \frac{\theta w-1}{\theta} & -\frac{\beta}{\theta}[(\lambda-1)\tau - (\theta w - 1)\eta] & -\frac{\beta n_N \eta(\theta w-1)}{\theta\tau} & \frac{1-n_N}{\theta} + \frac{n_N(\tau-\eta)}{\lambda\tau\theta} + \frac{n_N\eta w}{\lambda\tau} \\ a_{NL} - a_{NF} + a_S & 0 & a_{NF} & 0 \\ 0 & \frac{\tau}{n_N}(1 - s_D) & -(1 - s_D) & \frac{\lambda-1}{\lambda a_{NF}} n_N \\ -(1 - \mathrm{s_M}) & -\frac{\eta}{1-n_N}(1 - s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1 - n_N) \end{pmatrix}$$

And $\mathrm{b_1} = \eta + (1 - n_N)\rho$, $\mathrm{b_2} = \eta + n_N\rho$

We can derive that $|A| < 0$ (See appendix B and C)

2.4.2 Comparative Statics

By solving this system, we can have the following comparative statics:

$$\frac{\mathrm{d}\eta}{\mathrm{dL_S}} > 0$$

$$\frac{\mathrm{dn_N}}{dL_S} < 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{dL_S}} < 0$$

$$\frac{\mathrm{d}\tau_S}{\mathrm{dL_S}} > 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{dL_N}} > 0$$

So we have the following propositions:

Proposition 3: *In case of two-way FDI, in the long run an expansion of labor supply in South would slow down innovation by northern firms(*$d\tau/dL_S < 0$*),but accelerate innovation rate by southern firms(*$d\tau_S/dL_S > 0$*) and increase the aggregate extent of southern innovation(*$d\eta/dL_S > 0$*).*

Proof. see appendix D

Proposition 4*: In case of two-way FDI, in the long run an expansion of labor supply in North would* *accelerate the innovation rate($d\tau/dL_N > 0$), but has ambiguous effect on imitation rate(hence the length of product cycle) and the aggregate extent of imitation.*

Proof. see appendix D

Remark:

In case of two-way FDI, the south has only manufacture activity, it manufactures all products. So an expansion of southern labor force encourages manufacture activity through lower wage level, the shrink of manufacture cost increases future profit stream of each new product, thus provide bigger incentive on R&D activity for both northern firms and southern firms. But the labor supply in north is constant, so the expansion of R&D activity of either northern firms or southern firms would at the same time squeeze out the other party's R&D activity. If it is northern firms that expand its R&D activity and southern firm shrinks, the result is more northern products and less southern products in the short run, the effect on η is uncertain in new steady state, because in equilibrium $\tau_S n_N = \tau_{NS} n_S$, if northern firms increase the R&D intensity, there will be more northern products and n_S would be smaller, but imitation intensity τ_S would shrink. If it is southern firms that expand R&D activity, innovation intensity rises. As a result more southern products are produced. The effect on η could be in either way because for northern firms, there will be more southern products to innovation but R&D intensity falls, for southern firms, innovation intensity rises but there would be less northern products to innovate. The comparative statics show that what actually would happen is the latter case, i.e. southern firms expand R&D activity, and the effect on η is positive.

An expansion of northern labor supply would expand innovation by northern firms and southern firms, that would result in either more northern products or more southern products. The result is not so clear through intuition. If northern firms absorb a bigger bulk of labor resources in response to such a shock and expand its scale of R&D activity bigger than southern firms, this would result in bigger proportion of northern products on market and shrink labor demand in south according to the different labor requirement of producing northern and southern products($\frac{dL_N}{dn_N} < 0$). Then the falling wage in south would provide incentive to expand R&D activity of both firms, so both innovation intensity of both firm could possibly increase. And our comparative statics show that innovation intensity of northern firms rises in new equilibrium but the effect on innovation of southern firms is uncertain.

Effect of subsidy:

$$\frac{d\eta}{ds_D} > or < 0$$

$$\frac{dn_N}{ds_D} < 0$$

$$\frac{d\tau}{ds_D} > or < 0$$

$$\frac{d\tau_S}{ds_D} > or < 0$$

$$\frac{d\eta}{ds_M} > 0$$

$$\frac{dn_N}{ds_M} > or < 0$$

$$\frac{d\tau}{ds_M} < 0$$

$$\frac{d\tau_S}{ds_M} > or < 0$$

From the above results we get proposition5.

Proposition5: *In case of two-way FDI, in the long run, an increase in the subsidy to northern firms has ambiguous effect on innovation rate and imitation rate, but increases the proportion of northern products. An increase in the subsidy to southern firms would slow down innovation by northern firms and increases the extent of innovation by southern firms.*

Proof. see appendix D

Remark:

A bigger subsidy to innovation would raise the intensity of innovation, and result in more northern products. This would reduce southern wage, because adding one more product line of northern products would actually release some labor. Because the increase of northern products reduces southern products for the same scale, but the labor demand for one more southern product line is higher than northern product line(dL_S/dn_N<0). So the manufacture cost of both northern products and southern products falls, stimulating R&D activity of both types of firms, so n_N could change in either way in new steady state. Our comparative static result shows that the effect on $\eta, \tau,$ and τ_S is uncertain and the proportion of northern products falls.

A bigger subsidy to R&D by southern firm directly raises the level of southern innovation, and depresses northern innovation intensity because labor supply in north does not change. This would result in a bigger proportion of southern products and raise the wage level of south, then the higher manufacture cost would discourage both northern firms and southern firms from conducting R&D activity. So R&D intensity of both firms would go down, so in new equilibrium they could go either way. Then if R&D intensity of southern firms falls faster, resulting in a bigger proportion of northern products, southern wage would fall to balance demand and supply. In new steady state, result of comparative statics shows that the R&D intensity of northern firms shrinks but that of southern firms is uncertain. And the proportion of northern products is smaller.

2.5 Welfare effect:

Based on our setup, we can calculate the welfare in both cases. In our model we do not distinguish utility of north from south, because they move in the same direction with our core variables. We focus on the utility of representative consumer. According to our budget constraint (3), we have $E = wL_N + L_S$, so expenditure is constant in steady state since relative wage w is constant in steady state. Through our calculation above, we find the steady state value of w is independent from all the other variables. And from (17) (18)and (31) (32), the relative wage w remains the same in both one-way FDI and two-way FDI. So E is the same in both cases as well. From (1)-(3) we can derive the indirect utility function as follows:

(40) $$U = \int_0^\infty e^{-\rho t}\left[logE - \int_0^1 log\frac{p_t(\beta)}{q_t(\beta)}d\beta\right]dt$$

Put $n_{NN} = \frac{n_N(\tau-\eta)}{\tau}$, $n_{NS} = \frac{n_N\eta}{\tau}$ into the above function we derive the indirect utility function as follows

(41) $$U = \int_0^\infty e^{-\rho t}[logE - \log\frac{(1-\theta w)\eta n_N}{\tau} + \theta w]dt$$

We use the upper superscript 1 to represent the steady state variables in case of one-way FDI, and superscript 2 to represent the steady state variables in case of two-way FDI.

(42) $$U^1 = \int_0^\infty e^{-\rho t}[logE^1 - \log\frac{(1-\theta w)\eta^1 n_N^1}{\tau^1} + \theta w]dt$$

(43) $$U^2 = \int_0^\infty e^{-\rho t}[logE^2 - \log\frac{(1-\theta w)\eta^2 n_N^2}{\tau^2} + \theta w]dt$$

We can convert (18) (19) (32) (33) into the following form, expressing τ and η with function of n_N and e:

(44) $$\tau^1 = n_N^1\left[\frac{(\lambda-1)e}{\lambda a_{NF}(1-s_D)} - \rho\right] = An_N^1$$

(45) $$\tau^2 = n_N^2\left[\frac{(\lambda-1)e}{\lambda a_{NF}(1-s_D)} - \rho\right] = An_N^2$$

(46) $$\eta^1 = (1-n_N^1)\left[\frac{(\theta w-1)e}{\theta a_S(1-s_M)} - \rho\right] = B_1(1-n_N^1)$$

(47) $$\eta^2 = (1-n_N^2)\left[\frac{(\theta w-1)e}{\theta a_S w(1-s_M)} - \rho\right] = B_2(1-n_N^2)$$

Then put these equations back into (20), and derive

(48) $$n_N^1 = \frac{L_N + (a_{NF}-a_{NL})B^1}{a_{NF}A + (a_{NF}-a_{NL})B^1}$$

Put them into (35) and derive

(49) $$n_N^2 = \frac{L_N - (a_{NL} + a_S - a_{NF})B^2}{a_{NF}A + (a_{NL} + a_S - a_{NF})B^2}$$

Through simple calculation we can see that $B_2 < B_1$ and $n_N^2 > n_N^1$

Put (44)-(47) into (42) and (43) we can convert the indirect utility function into:

(50) $$U^1 = \log(wL_N + L_S) - \log[\theta w - \frac{(\theta w - 1)B^1(1 - n_N^1)}{A}]$$

(51) $$U^2 = \log(wL_N + L_S) - \log[\theta w - \frac{(\theta w - 1)B^2(1 - n_N^2)}{A}]$$

So $U^1 > U^2$

The welfare of a representative consumer in case of one-way FDI is bigger than that in case of two-way FDI. Welfare can be decomposed into two parts, the income effect and price effect. The income effect works through the term $\log E$, the price effect works through the term $\log \frac{(1-\theta w)\eta n_N}{\tau} + \theta w$. The higher the income level, the higher welfare level, the lower the quality adjusted price, the higher the welfare level. Since $n_{NS} = \frac{\eta n_N}{\tau} = \frac{B(1-n_N)}{A}$, we can see that a bigger proportion of northern products that are innovated over southern product shrinks the scale of price effect, that is because the quality adjusted price of this group of products is the lowest among the three groups. In case of two-way FDI, the weighted average quality adjusted price level is higher than in case of one-way FDI. So the welfare level of one-way FDI is higher than two-way FDI. Fundamentally, that is because in case of one-way FDI, the proportion of northern products innovated from southern products is higher than in case of two-way FDI[4], the quality adjusted price level in one-way FDI is lower.

3.Conclusions

This paper extends the product cycle model of Grossman and Helpman(1991a) to study the effect of one-way FDI and two-way FDI on regime effect and on the way that relative size of north and south affecting innovation and technology transfer. Furthermore we use this framework to study the policy implication of subsidy to innovation and imitation.

Our result is very different from Grossman and Helpman(1991a), When there is no FDI, there is no connection between the labor market of north and south. The expansion of labor supply in either north or south would not generate an impact on the other country. So the result of Grossman & Helpman(1991b)'s model seems quite intuitive, that an expansion of northern labor supply stimulate northern innovation and an expansion of southern labor supply stimulates southern imitation. When there is FDI that moves

[4] Recall that $\frac{B^1(1-n_N^1)}{A} > \frac{B^2(1-n_N^2)}{A}$

manufacture from north to south, the labor market of both sides are linked by investment. For example, providing subsidy to northern firms would provide incentive for more R&D activity, and this rise of innovation intensity in north that generates more northern products would raise labor demand in south through manufacture sector. Then the expansion would change wage level in south and change the allocation of resources in manufacture sector and R&D sector. So in new equilibrium the impact of a change in relative size of north to south, or subsidy to innovation and imitation is quite complex in case of FDI and difficult to reason only through intuition. When there is two-way FDI, the link between two countries' labor market is different from the case of one way FDI and no FDI, so we need comparative statics to demonstrate the regime effect and policy effect.

Our results show that in case of one-way FDI, increasing the size of South would accelerate both innovation rate and imitation rate, but increasing the size of north would only accelerate innovation rate. But in case of two-way FDI, expanding the size of South would slow down innovation by northern firms and accelerate innovation by southern firms, while expanding the size of North would accelerate northern innovation but has ambiguous effect on southern innovation.

In terms of the effect of R&D subsidy, our result is somewhat contrary to intuition in that subsidy to innovation by northern firms do not necessarily promote innovation rate, and subsidy to R&D activity generates different impact on innovation and imitation in case of one-way FDI and two-way FDI. In case of one-way FDI subsidizing northern firms has ambiguous effect on innovation and imitation, but subsidizing southern firms would accelerate innovation but has ambiguous effect on imitation. In case of two-way FDI, subsidizing northern firms has ambiguous effect on innovation and imitation, the same as the case of one-way FDI, but subsidizing southern firms would slow down innovation by northern firms and increase the scale of innovation of southern firms. In other words, only in case of two-way FDI can southern government reach its policy target of promoting southern innovation by subsidizing southern firms, in case of one-way FDI, the effect of subsidy for southern firms would, on the contrary, promote innovation by northern firms.

Furthermore, we compared the welfare level in both cases and find that The welfare of a representative consumer in case of one-way FDI is bigger than that in case of one-way FDI. That is because the weighted average quality adjusted price level is higher in case of two-way FDI than in case of one-way FDI while the expenditure in both cases are the same.

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Appendix:

A.Steady state rate of innovation and imitation:

Our target is to express $\tau_N, \tau_S, \mu, n_S, n_{NN}, n_{NS}$ as functions of the three endogenous variables η, τ, n_N.

It is obvious that $n_S = 1 - n_N$

So $\tau_N = \frac{\tau-\eta}{n_N}$, $\tau_S = \frac{\eta}{n_S} = \frac{\eta}{1-n_N}$

From（4）, derive $\mu = \frac{\eta}{n_N}$, so $n_{NS} = \frac{\mu n_{NN}}{\tau_N} = \frac{\eta n_{NN}}{n_N \tau_N} = \frac{\eta n_{NN}}{\tau-\eta}$, put into $n_{NN} + n_{NS} = n_N$

We have $\left(\frac{\eta}{\tau-\eta} + 1\right) n_{NN} = n_N$, so $n_{NN} = \frac{n_N(\tau-\eta)}{\tau}$ and $n_{NS} = \frac{\eta n_{NN}}{\tau-\eta} = \frac{n_N \eta}{\tau}$.

So we have:

$$n_S = 1 - n_N$$

$$\tau_N = \frac{\tau - \eta}{n_N}$$

$$\tau_S = \frac{\eta}{1 - n_N}$$

$$\mu = \frac{\eta}{n_N}$$

$$n_{NN} = \frac{n_N(\tau - \eta)}{\tau}$$

$$n_{NS} = \frac{n_N \eta}{\tau}$$

B. How to get matrix A

In case of one-way FDI, equations(17)-(21) forms a system of η, n_N, τ, e as endogenous variables, and L_S, L_N, s_D, s_M as exogenous variables.

By total differentiating this system we can get a matrix A which satisfies that $A\begin{pmatrix} d\eta \\ dn_N \\ d\tau \\ de \end{pmatrix} = \begin{pmatrix} dL_S \\ dL_N \\ -b_1 ds_D \\ -b_2 ds_M \end{pmatrix}$,

so that we can conduct comparative analysis.

The calculating process is as follows:

$$\frac{dL_S}{d\eta} = a_S + \frac{e n_N w}{\lambda\tau}\left(1 - \frac{1}{\theta w}\right) = a_S + \frac{e n_N w}{\lambda\tau}\left(\frac{\theta w - 1}{\theta w}\right) = a_S + \beta n_N \frac{\theta w - 1}{\theta}$$

(we use $\beta = e/\lambda\tau$)

$$\frac{dL_S}{d\tau} = \frac{\eta n_N e}{\lambda\theta\tau^2} - \frac{\eta n_N e w}{\lambda\tau^2} = \frac{\eta n_N e}{\lambda\tau^2}\left(\frac{1-\theta w}{\theta}\right) = \frac{\beta n_N \eta(1-\theta w)}{\theta\tau}$$

$$\frac{dL_S}{dn_N} = -\frac{e}{\theta} + \frac{(\tau-\eta)e}{\lambda\tau\theta} + \frac{\eta e w}{\lambda\tau} = \frac{-e\lambda\theta + (\tau-\eta)e + ew\eta\theta}{\lambda\tau\theta} = \frac{-e}{\lambda\tau\theta}[(\lambda-1)\tau - (\theta w - 1)\eta]$$

$$= -\frac{\beta}{\theta}[(\lambda-1)\tau - (\theta w - 1)\eta]$$

$$\frac{dL_S}{de} = \frac{1-n_N}{\theta} + \frac{n_N(\tau-\eta)}{\lambda\tau\theta} + \frac{n_N\eta w}{\lambda\tau}$$

So we get

$$dL_S = \left[a_S + \beta n_N \frac{\theta w - 1}{\theta}\right] d\eta - \frac{\beta}{\theta}[(\lambda-1)\tau - (\theta w - 1)\eta] dn_N + \frac{\beta n_N \eta (1-\theta w)}{\theta\tau} d\tau + \left[\frac{1-n_N}{\theta} + \frac{n_N(\tau-\eta)}{\lambda\tau\theta} + \frac{n_N\eta w}{\lambda\tau}\right] de$$

$$\frac{dL_N}{d\eta} = a_{NL} - a_{NF}$$

$$\frac{dL_N}{d\tau} = a_{NF}$$

$$\frac{dL_N}{dn_N} = 0$$

$$\frac{dL_N}{de} = 0$$

So

$$dL_N = (a_{NL} - a_{NF})d\eta - a_{NF}d\tau$$

Total differentiating (25) We get

$$\frac{\tau}{n_N}(1-s_D)dn_N - (1-s_D)d\tau + \frac{\lambda-1}{\lambda a_{NF}} n_N de = -b_1 ds_D$$

Total differentiating (26) We get

$$-(1-s_M)d\eta - \frac{\eta}{1-n_N}(1-s_M)dn_N + \frac{\theta w - 1}{\theta a_S}(1-n_N)de = -b_2 ds_M$$

So we derive

$$A = \begin{pmatrix} a_S + \beta n_N \frac{\theta w - 1}{\theta} & -\frac{\beta}{\theta}[(\lambda-1)\tau - (\theta w - 1)\eta] & \frac{\beta n_N \eta (1-\theta w)}{\theta\tau} & \frac{1-n_N}{\theta} + \frac{n_N(\tau-\eta)}{\lambda\tau\theta} + \frac{n_N\eta w}{\lambda\tau} \\ a_{NL} - a_{NF} & 0 & a_{NF} & 0 \\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}} n_N \\ -(1-s_M) & -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w - 1}{\theta a_S}(1-n_N) \end{pmatrix}$$

In case of two-way FDI, we have the following system from (31)-(35), in which the endogenous variables are η, n_N, τ, e, and the exogenous variables are L_S, L_N, s_D, s_M.

$$L_S = \frac{(1-n_N)e}{\theta} + \frac{n_N(\tau-\eta)e}{\lambda\tau\theta} + \frac{\eta n_N e w}{\lambda\tau}$$

$$L_N = a_{NL}\eta + a_{NF}(\tau-\eta) + a_S\eta$$

$$\frac{(\lambda-1)e n_N}{\lambda a_{NF}} = (\rho n_N + \tau)(1-s_D)$$

$$\frac{(\theta w-1)e(1-n_N)}{\theta w a_S}=(\rho(1-n_N)+\eta)(1-s_M)$$

Like in the case of one-way FDI, we total differentiate this system to get a matrix A, the process is as follows:

$$\frac{\mathrm{dL_S}}{\mathrm{d\eta}}=\frac{en_Nw}{\lambda\tau}\left(1-\frac{1}{\theta w}\right)=\frac{en_Nw}{\lambda\tau}\left(\frac{\theta w-1}{\theta w}\right)=\beta n_N\frac{\theta w-1}{\theta}$$

(we use $\beta=e/\lambda\tau$)

$$\frac{\mathrm{dL_S}}{\mathrm{d\tau}}=\frac{\eta n_N e}{\lambda\theta\tau^2}-\frac{\eta n_N e w}{\lambda\tau^2}=\frac{\eta n_N e}{\lambda\tau^2}\left(\frac{1-\theta w}{\theta}\right)=\frac{\beta n_N\eta(1-\theta w)}{\theta\tau}$$

$$\frac{\mathrm{dL_S}}{\mathrm{dn_N}}=-\frac{e}{\theta}+\frac{(\tau-\eta)e}{\lambda\tau\theta}+\frac{\eta e w}{\lambda\tau}=\frac{-e\lambda\theta+(\tau-\eta)e+ew\eta\theta}{\lambda\tau\theta}=\frac{-e}{\lambda\tau\theta}[(\lambda-1)\tau-(\theta w-1)\eta]$$

$$=-\frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta]$$

$$\frac{\mathrm{dL_S}}{de}=\frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau}$$

$$\mathrm{dL_S}=\left(\beta n_N\frac{\theta w-1}{\theta}\right)d\eta-\frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta]dn_N-\frac{\beta n_N\eta(\theta w-1)}{\theta\tau}d\tau$$
$$+\left[\frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau}\right]\mathrm{de} \qquad (40)$$

$$\frac{\mathrm{dL_N}}{d\eta}=a_{NL}-a_{NF}+a_S$$

$$\frac{\mathrm{dL_N}}{d\tau}=a_{NF}$$

$$\frac{\mathrm{dL_N}}{dn_N}=0$$

$$\frac{\mathrm{dL_N}}{de}=0$$

$$\mathrm{dL_N}=(a_{NL}-a_{NF}+a_S)d\eta+a_{NF}d\tau \qquad (41)$$

Total differentiate (30),we get

$$\frac{\tau}{n_N}(1-s_D)dn_N-(1-\mathrm{s_D})\mathrm{d\tau}+\frac{\lambda-1}{\lambda a_{NF}}n_N\mathrm{de}=-\mathrm{b_1}ds_D \qquad (42)$$

Total differentiate (31),we get

$$-(1-\mathrm{s_M})\mathrm{d\eta}-\frac{\eta}{1-n_N}(1-s_M)dn_N+\frac{\theta w-1}{\theta w a_S}(1-n_N)de=-b_2ds_M \qquad (43)$$

So we can form matrix A from (40) (41) (42) (43),

$$A=\begin{pmatrix} \beta n_N \frac{\theta w-1}{\theta} & -\frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta] & -\frac{\beta n_N \eta(\theta w-1)}{\theta\tau} & \frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau} \\ a_{NL}-a_{NF}+a_S & 0 & a_{NF} & 0 \\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}}n_N \\ -(1-\mathrm{s_M}) & -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{pmatrix}$$

C. The value of A

In case of one-way FDI

$$A=\begin{pmatrix} a_S+\beta n_N \frac{\theta w-1}{\theta} & -\frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta] & -\frac{\beta n_N \eta(\theta w-1)}{\theta\tau} & \frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau} \\ a_{NL}-a_{NF} & 0 & a_{NF} & 0 \\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}}n_N \\ -(1-\mathrm{s_M}) & -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{pmatrix}$$

Let $\mathrm{a}_{14}=\frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau}>0$

$$|A|=-a_{14}\left[-(a_{NL}-a_{NF})(1-s_D)\frac{\eta}{1-n_N}(1-s_M)+a_{NF}(1-s_M)\frac{\tau}{n_N}(1-s_D)\right]$$
$$-\frac{\lambda-1}{\lambda a_{NF}}n_N\left[\frac{(a_{NF}-a_{NL})\eta}{1-n_N}(1-s_M)\frac{\beta\eta n_N}{\tau\theta}(1-\theta w)\right.$$
$$\left.+a_{NF}(1-s_M)\left(a_S+\beta n_N\frac{\theta w-1}{\theta}\right)\frac{\eta}{1-n_N}+\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\right]+\frac{\theta w-1}{\theta a_S}(1$$
$$-n_N)\{(a_{NF}-a_{NL})\frac{\beta\tau(\lambda-1)(1-s_D)}{\theta}-a_{NF}\left(a_S+\beta n_N\frac{(\theta w-1)}{\theta}\right)\frac{\tau}{n_N}(1-s_D)\ )$$

The first term on the right hand side

$$=-a_{14}\left[-(a_{NL}-a_{NF})(1-s_D)\frac{\eta}{1-n_N}(1-s_M)+a_{NF}(1-s_M)\frac{\tau}{n_N}(1-s_D)\right]$$
$$=-a_{14}\left[(a_{NF}-a_{NL})(1-s_D)\frac{\eta}{1-n_N}(1-s_M)+a_{NF}(1-s_M)\frac{\tau}{n_N}(1-s_D)\right]<0$$

(because $\mathrm{a_{NF}}>a_{NL}$, $(a_{NF}-a_{NL})(1-s_D)\frac{\eta}{1-n_N}(1-s_M)+a_{NF}(1-s_M)\frac{\tau}{n_N}(1-s_D)>0$)

The second term on the right hand side=

$$-\frac{\lambda-1}{\lambda a_{NF}}n_N\left[\frac{(a_{NF}-a_{NL})\eta}{1-n_N}(1-s_M)\frac{\beta\eta n_N}{\tau\theta}(1-\theta w)+a_{NF}(1-s_M)\left(a_S+\beta n_N\frac{\theta w-1}{\theta}\right)\frac{\eta}{1-n_N}\right.$$
$$\left.+\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\right]<0$$

(Because $\frac{(\mathrm{a_{NF}}-\mathrm{a_{NL}})\eta}{1-n_N}(1-s_M)\frac{\beta\eta n_N}{\theta\tau}(1-\theta\mathrm{w})+\mathrm{a_{NF}}(1-s_M)\beta n_N\frac{(\theta w-1)\eta}{\theta(1-n_N)}$

$$= (1-\mathrm{s_M})\frac{\eta\beta n_N(\theta w-1)}{(1-n_N)\theta}\left(a_{NF}-\frac{\eta}{\tau}(a_{NF}-a_{NL})\right)>0 \quad (recall\ that\ \theta w>1\ and\ \eta<\tau))$$

The third term on the right hand side=

$$\frac{\theta w-1}{\theta a_S}(1-n_N)\left\{(a_{NF}-a_{NL})\frac{\beta\tau(\lambda-1)(1-s_D)}{\theta}-a_{NF}\left(a_S+\beta n_N\frac{(\theta w-1)}{\theta}\right)\frac{\tau}{n_N}(1-s_D)\right\}<0$$

from (25) and (26), we have

$$a_{NL}=\frac{(\lambda-\theta w)e}{\lambda(\rho+\tau/n_N)(1-s_D)}$$

$$a_{NF}=\frac{(\lambda-1)e}{\lambda\left(\rho+\frac{\tau}{n_N}\right)(1-s_D)}$$

$$a_S=\frac{(\theta w-1)e}{\theta\left(\rho+\frac{\eta}{(1-n_N)}\right)(1-s_M)}$$

$$a_{NF}-a_{NL}=\frac{(\theta w-1)e}{\lambda\left(\rho+\frac{\tau}{n_N}\right)(1-s_D)}$$

So

$$(a_{NF}-a_{NL})\frac{\beta\tau(\lambda-1)(1-s_D)}{\theta}-a_{NF}\left(a_S+\beta n_N\frac{(\theta w-1)}{\theta}\right)\frac{\tau}{n_N}(1-s_D)$$

$$=\frac{(\theta w-1)e\beta\tau(\lambda-1)}{\lambda\left(\rho+\frac{\tau}{n_N}\right)\theta}-\frac{(\lambda-1)e\tau\beta(\theta w-1)}{\lambda\left(\rho+\frac{\tau}{n_N}\right)\theta}-\frac{a_{NF}a_S\tau(1-s_D)}{n_N}$$

$$=\frac{(\lambda-1)\tau e}{\lambda\left(\rho+\frac{\tau}{n_N}\right)\theta}[\beta(\theta w-1)-\beta(\theta w-1)]-\frac{a_{NF}a_S\tau}{n_N}(1-s_D)$$

$$=-\frac{a_{NF}a_S\tau(1-s_D)}{n_N}<0$$

In this way |A|<0

In case of two-way FDI

$$A=\begin{pmatrix}\beta n_N\frac{\theta w-1}{\theta} & -\frac{\beta}{\theta}[(\lambda-1)\tau-(\theta w-1)\eta] & -\frac{\beta n_N\eta(\theta w-1)}{\theta\tau} & \frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau}\\ a_{NL}-a_{NF}+a_S & 0 & a_{NF} & 0\\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}}n_N\\ -(1-\mathrm{s_M}) & -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N)\end{pmatrix}$$

Let $a_{14}=\frac{1-n_N}{\theta}+\frac{n_N(\tau-\eta)}{\lambda\tau\theta}+\frac{n_N\eta w}{\lambda\tau}$

Assume that $a_S>a_{NL}$, then $\frac{\rho+\frac{\tau}{n_N}}{\rho+\frac{\eta}{1-n_N}}>\frac{1-\frac{1}{\lambda}}{1-\frac{1}{\theta w}}>1(recall\ that\ \lambda>\theta w)$

That is $\frac{\tau}{n_N} > \frac{\eta}{1-n_N}$

$$|A| = -a_{14}\left[-(a_{NL}-a_{NF}+a_S)(1-s_D)\frac{\eta}{1-n_N}(1-s_M)+a_{NF}(1-s_M)\frac{\tau}{n_N}(1-s_D)\right]$$
$$-\frac{\lambda-1}{\lambda a_{NF}}n_N\left[\frac{(a_{NF}-a_{NL}+a_S)\eta}{1-n_N}(\ 1-s_M)\frac{\beta\eta n_N}{\tau\theta}(\theta w-1)\right.$$
$$\left.+a_{NF}(1-s_M)\left(\beta n_N\frac{\theta w-1}{\theta}\frac{\eta}{1-n_N}+\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\right)\right]$$
$$+\frac{\theta w-1}{\theta a_S}(1$$
$$-n_N)\left\{-(a_{NL}-a_{NF}+a_S)\left[(1-s_D)\frac{\beta\big(\tau(\lambda-1)-\eta(\theta w-1)\big)}{\theta}\right.\right.$$
$$\left.\left.+\frac{\beta\eta(\theta w-1)(1-s_D)}{\theta}\right]-a_{NF}\beta n_N\frac{(\theta w-1)\tau}{\theta n_N}(1-s_D)\right\}<0$$

Because the first term on the right hand side=

$$-a_{14}\left[-(a_{NL}-a_{NF}+a_S)(1-s_D)\frac{\eta}{1-n_N}(1-s_M)+a_{NF}(1-s_M)\frac{\tau}{n_N}(1-s_D)\right]$$
$$=-a_{14}(1-s_D)(1-s_M)\left[-(a_{NF}-a_S-a_{NL})\frac{\eta}{1-n_N}+a_{NF}\frac{\tau}{n_N}\right]<0$$
$$\left(-(a_{NF}-a_S-a_{NL})\frac{\eta}{1-n_N}+a_{NF}\frac{\tau}{n_N}>0\right)$$

the second term on the right hand side=

$-\frac{\lambda-1}{\lambda a_{NF}}n_N\left[\frac{(a_{NF}-a_{NL}+a_S)\eta}{1-n_N}(\ 1-s_M)\frac{\beta\eta n_N}{\tau\theta}(\theta w-1)+a_{NF}(1-s_M)\left(\beta n_N\frac{\theta w-1}{\theta}\frac{\eta}{1-n_N}+\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\right)\right]<0$, this is obvious.

the third term on the right hand side=

$$\frac{\theta w-1}{\theta a_S}(1-n_N)\left\{-(a_{NL}-a_{NF}+a_S)\left[(1-s_D)\frac{\beta\big(\tau(\lambda-1)-\eta(\theta w-1)\big)}{\theta}+\frac{\beta\eta(\theta w-1)(1-s_D)}{\theta}\right]\right.$$
$$\left.-a_{NF}\beta n_N\frac{(\theta w-1)\tau}{\theta n_N}(1-s_D)\right\}$$
$$=\frac{\theta w-1}{\theta a_S}(1-n_N)(1-s_D)\left[-(a_{NL}-a_{NF}+a_S)\frac{\beta\tau(\lambda-1)}{\theta}-a_{NF}\beta n_N\frac{(\theta w-1)\tau}{\theta n_N}\right]<0$$

So |A|<0.

D.Comparative statics and the proof of propositions:

To prove these propositions we need to calculate the comparative statics, so we need to solve the adjugate matrix of |A| in case of one-way FDI and two-way FDI respectively:

One way FDI:

$$a_{11}^* = \begin{vmatrix} 0 & a_{NF} & 0 \\ \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}} n_N \\ -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= -a_{NF}\left[\frac{\tau(1-s_D)(\theta w-1)}{\theta a_S n_N}(1-n_N) + \frac{\eta(1-s_M)(\lambda-1)}{\lambda a_{NF}(1-n_{N})} n_N\right] < 0$$

$$a_{12}^* = \begin{vmatrix} a_{NL}-a_{NF} & a_{NF} & 0 \\ 0 & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}} n_N \\ -(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= -\left[\frac{(a_{NF}-a_{NL})(1-s_D)(\theta w-1)}{\theta a_S}(1-n_N) - \frac{(1-s_M)a_{NF}(\theta w-1)}{\theta a_S}(1-n_N)\right]$$

$$= \frac{(\theta w-1)(1-n_N)}{\theta a_S}[(a_{NF}-a_{NL})(1-s_D) - a_{NF}(1-s_M)] > 0r < 0$$

Because $a_{NL} < a_{NF}$ and the size of s_D and s_M is not predetermined.

$$a_{13}^* = \begin{vmatrix} a_{NL}-a_{NF} & 0 & 0 \\ 0 & \frac{\tau}{n_N}(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}} n_N \\ -(1-s_M) & -\frac{\eta}{1-n_N}(1-s_M) & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= (a_{NL}-a_{NF})\left[\frac{\tau(1-s_D)(\theta w-1)}{\theta a_S n_N}(1-n_N) + \frac{\eta(1-s_M)(\lambda-1)}{\lambda a_{NF}(1-n_{N})} n_N\right] < 0$$

$$a_{14}^* = \begin{vmatrix} a_{NL}-a_{NF} & 0 & a_{NF} \\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) \\ -(1-s_M) & -\frac{\eta}{1-n_N}(1-s_M) & 0 \end{vmatrix}$$

$$= -\left[\frac{(a_{NF}-a_{NL})(1-s_D)(1-s_M)}{(1-n_N)} + \frac{(1-s_M)a_{NF}\tau(1-s_D)}{n_N}\right] < 0$$

(because $\frac{\tau}{n_N} > \frac{\eta}{1-n_N}$)

$$a_{21}^* = -\begin{vmatrix} -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ \frac{\tau}{n_N}(1-s_D) & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}} n_N \\ -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= -\left[-\frac{\eta}{1-n_N}(1-s_M)\left[\frac{(1-\theta w)(\lambda-1)}{\lambda a_{NF}}\frac{\beta\eta n_N}{\theta\tau} + (1-s_D)a_{14}\right] + \frac{\theta w - 1}{\theta a_S}(1-n_N)(1-s_D)\left[\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} + \frac{\beta\eta(\theta w-1)}{\theta}\right]\right] > or < 0$$

$$a_{22}^* = \begin{vmatrix} a_S + \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ 0 & -(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}}n_N \\ -(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= -\left(a_S + \frac{\beta n_N(\theta w-1)}{\theta}\right)\frac{(1-s_D)(\theta w-1)}{\theta a_S}(1-n_N) - (1-s_M)\left[-\frac{\beta\eta n_N(\theta w-1)(\lambda-1)n_N}{\theta\tau\lambda a_{NF}} + (1-s_D)a_{14}\right] > or < 0$$

Because it is impossible to determine the sign of $\left[-\frac{\beta\eta n_N(\theta w-1)(\lambda-1)n_N}{\theta\tau\lambda a_{NF}} + (1-s_D)a_{14}\right]$

$$a_{23}^* = -\begin{vmatrix} a_S + \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & a_{14} \\ 0 & \frac{\tau}{n_N}(1-s_D) & \frac{\lambda-1}{\lambda a_{NF}}n_N \\ -(1-s_M) & -\frac{\eta}{1-n_N}(1-s_M) & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= -\left\{\left(a_S + \frac{\beta n_N(\theta w-1)}{\theta}\right)\left[\frac{\tau(1-s_D)(\theta w-1)}{\theta a_S n_N}(1-n_N) + \frac{\eta}{1-n_N}\frac{(1-s_M)(\lambda-1)}{\lambda a_{NF}}n_N\right] - (1-s_M)\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)](\lambda-1)n_N}{\theta\lambda a_{NF}} - \frac{(1-s_D)a_{14}\tau}{n_N}\right]\right\}$$

$$= -\left\{\left(a_S + \frac{\beta n_N(\theta w-1)}{\theta}\right)\left[\frac{\tau(1-s_D)(\theta w-1)}{\theta a_S n_N}(1-n_N) + \frac{\eta}{1-n_N}\frac{(1-s_M)(\lambda-1)}{\lambda a_{NF}}n_N\right] + (1-s_M)\left[\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)](\lambda-1)n_N}{\theta\lambda a_{NF}} + \frac{(1-s_D)a_{14}\tau}{n_N}\right]\right\} < 0$$

$$a_{24}^* = \begin{vmatrix} a_S + \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N}{\theta\tau}(\theta w-1) \\ 0 & \frac{\tau}{n_N}(1-s_D) & -(1-s_D) \\ -(1-s_M) & -\frac{\eta}{1-n_N}(1-s_M) & 0 \end{vmatrix}$$

$$= \frac{\tau}{n_N}(1-s_D)\left[-\frac{(1-s_M)\beta\eta n_N(\theta w-1)}{\theta\tau}\right]+(1-s_D)[-\left(a_S+\frac{\beta n_N(\theta w-1)}{\theta}\right)\frac{\eta(1-s_M)}{1-n_N} - (\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta})(1-s_M)\ ] < 0$$

$$a_{31}^* = \begin{vmatrix} -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ 0 & a_{NF} & 0 \\ -\frac{\eta}{1-n_N}(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= a_{NF}\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{\theta w-1}{\theta a_S}(1-n_N)+\frac{\eta}{1-n_N}(1-s_M)a_{14}\right] > 0r < 0$$

$$a_{32}^* = -\begin{vmatrix} a_S+\frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ a_{NL}-a_{NF} & a_{NF} & 0 \\ -(1-s_M) & 0 & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= -\{(1-s_M)a_{NF}a_{14}+\frac{\theta w-1}{\theta a_S}(1-n_N)[\left(a_S+\frac{\beta n_N(\theta w-1)}{\theta}\right)a_{NF}-\frac{(a_{NF}-a_{NL})\beta\eta n_N(\theta w-1)}{\theta\tau}]\} < 0$$

Because $\frac{\beta n_N(\theta w-1)}{\theta}a_{NF}-\frac{(a_{NF}-a_{NL})\beta\eta n_N(\theta w-1)}{\theta\tau} > 0$ $(recall\ that\ \frac{\eta}{\tau} < 1)$

$$a_{33}^* = \begin{vmatrix} a_S+\frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & a_{14} \\ a_{NL}-a_{NF} & 0 & 0 \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & \frac{\theta w-1}{\theta a_S}(1-n_N) \end{vmatrix}$$

$$= (a_{NF}-a_{NL})\left\{-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{\theta w-1}{\theta a_S}(1-n_N)+\frac{\eta}{1-n_N}(1-s_M)a_{14}\right\} > \text{or} < 0$$

$$a_{34}^* = -\begin{vmatrix} a_S+\frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & \frac{\beta\eta n_N}{\theta\tau} \\ a_{NL}-a_{NF} & 0 & a_{NF} \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & 0 \end{vmatrix}$$

$$= -\left\{(1-s_M)\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}a_{NF}+\frac{\eta}{1-n_N}(1-s_M)[\left(a_S+\frac{\beta n_N(\theta w-1)}{\theta}\right)a_{NF} + \frac{(a_{NF}-a_{NL})\beta\eta n_N(1-\theta w)}{\theta\tau}]\right\} < 0$$

$$a_{41}^* = -\begin{vmatrix} -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ 0 & a_{NF} & 0 \\ \frac{\tau(1-s_D)}{n_N} & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \end{vmatrix}$$

$$= -a_{NF}\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{(\lambda-1)n_N}{\lambda a_{NF}} - \frac{a_{14}\tau(1-s_D)}{n_N}\right] > 0$$

$$a_{42}^* = \begin{vmatrix} a_S + \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ a_{NL}-a_{NF} & a_{NF} & 0 \\ 0 & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \end{vmatrix}$$

$$= \left(a_S + \frac{\beta n_N(\theta w-1)}{\theta}\right)\frac{a_{NF}(\lambda-1)}{\lambda a_{NF}}n_N + (a_{NF}-a_{NL})\left[-\frac{\beta\eta n_N(\theta w-1)}{\theta\tau}\frac{(\lambda-1)n_N}{\lambda a_{NF}} + a_{14}(1-s_D)\right]$$

$$= a_S\frac{a_{NF}(\lambda-1)}{\lambda a_{NF}}n_N + \frac{\beta n_N(\theta w-1)}{\theta}\frac{(\lambda-1)}{\lambda}\left(1-\frac{(a_{NF}-a_{NL})\eta}{a_{NF}\tau}\right) + (a_{NF}-a_{NL})a_{14}(1-s_D) > 0$$

Because $1-\frac{(a_{NF}-a_{NL})\eta}{a_{NF}\tau} < 0$

$$a_{43}^* = -\begin{vmatrix} a_S + \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & a_{14} \\ a_{NL}-a_{NF} & 0 & 0 \\ 0 & \frac{\tau(1-s_D)}{n_N} & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \end{vmatrix}$$

$$= -(a_{NF}-a_{NL})\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{(\lambda-1)n_N}{\lambda a_{NF}} - a_{14}\frac{\tau(1-s_D)}{n_N}\right] > 0$$

$$a_{44}^* = \begin{vmatrix} a_S + \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & \frac{\beta\eta n_N}{\theta\tau}(1-\theta w) \\ a_{NL}-a_{NF} & 0 & a_{NF} \\ 0 & \frac{\tau(1-s_D)}{n_N} & -(1-s_D) \end{vmatrix}$$

$$= -\left(a_S + \frac{\beta n_N(\theta w-1)}{\theta}\right)\frac{a_{NF}\tau(1-s_D)}{n_N} + (a_{NF}-a_{NL})(1-s_D)\left[\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} + \frac{\beta\eta(\theta w-1)}{\theta}\right] > or < 0$$

Based on the above results we can calculate the comparative statics:

$$\frac{d\eta}{dL_S} = \frac{a_{11}^*}{|A|} > 0$$

$$\frac{d\eta}{dL_N} = \frac{a_{21}^*}{|A|} > or < 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{dL_S}} = \frac{a_{12}^*}{|A|} > \text{or} < 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{dL_N}} = \frac{a_{22}^*}{|A|} > \text{or} < 0$$

$$\frac{\mathrm{d\tau}}{\mathrm{dL_S}} = \frac{a_{13}^*}{|A|} > 0$$

$$\frac{\mathrm{d\tau}}{\mathrm{dL_S}} = \frac{a_{23}^*}{|A|} > 0$$

$$\frac{\mathrm{de}}{\mathrm{dL_S}} = \frac{a_{14}^*}{|A|} > 0$$

$$\frac{\mathrm{de}}{\mathrm{dL_N}} = \frac{a_{24}^*}{|A|} > 0$$

Policy implication:

$$\frac{\mathrm{d\eta}}{\mathrm{ds_D}} = -b_1 \frac{a_{31}^*}{|A|} > or < 0$$

$$\frac{\mathrm{d\eta}}{\mathrm{ds_M}} = -b_2 \frac{a_{41}^*}{|A|} > 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{ds_D}} = -b_1 \frac{a_{32}^*}{|A|} < 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{ds_M}} = -b_2 \frac{a_{42}^*}{|A|} > 0$$

$$\frac{\mathrm{d\tau}}{\mathrm{ds_D}} = -b_1 \frac{a_{33}^*}{|A|} > or < 0$$

$$\frac{\mathrm{d\tau}}{\mathrm{ds_M}} = -b_2 \frac{a_{43}^*}{|A|} > 0$$

$$\frac{\mathrm{de}}{\mathrm{ds_D}} = -b_1 \frac{a_{34}^*}{|A|} < 0$$

$$\frac{\mathrm{de}}{\mathrm{ds_M}} = -b_2 \frac{a_{44}^*}{|A|} > or < 0$$

two-way FDI:

$$\mathrm{a}_{11}^* = \begin{vmatrix} 0 & \mathrm{a_{NF}} & 0 \\ \frac{\tau(1-s_D)}{n_N} & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \\ -\frac{\eta(1-s_M)}{1-n_N} & 0 & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$= -a_{NF}\left[\frac{\tau}{n_N}(1-s_D)\frac{\theta w-1}{\theta w a_S}(1-n_N) + \frac{\eta}{1-n_N}(1-s_M)\left(\frac{\lambda-1}{\lambda a_{NF}}\right)n_N\right] < 0$$

$$a_{12}^* = -\begin{vmatrix} a_{NL} - a_{NF} + a_S & a_{NF} & 0 \\ 0 & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \\ -(1-s_M) & 0 & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$= -\left\{-\frac{(a_{NL} - a_{NF} + a_S)(1-s_D)(\theta w-1)(1-n_N)}{\theta w a_S} - \frac{(1-s_M)(\lambda-1)}{\lambda} n_N\right\} > 0$$

$$a_{13}^* = \begin{vmatrix} a_{NL} - a_{NF} + a_S & 0 & 0 \\ 0 & \frac{\tau(1-s_D)}{n_N} & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$= (a_{NL} - a_{NF} + a_S)[\frac{\tau}{n_N}(1-s_D)\frac{\theta w - 1}{\theta w a_S}(1-n_N) + \frac{\eta(1-s_M)}{1-n_N}\frac{(\lambda-1)n_N}{\lambda a_{NF}}] > 0$$

$$a_{14}^* = -\begin{vmatrix} a_{NL} - a_{NF} + a_S & 0 & a_{NF} \\ 0 & \frac{\tau(1-s_D)}{n_N} & -(1-s_D) \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & 0 \end{vmatrix}$$

$$= (-1)\left\{(a_{NL} - a_{NF} + a_S)\left[-\frac{\eta}{1-n_N}(1-s_M)(1-s_D)\right] + \frac{a_{NF}(1-s_M)\tau(1-s_D)}{n_N}\right\}$$
$$> or < 0$$

(recall that $\frac{\eta}{1-n_N} < \frac{\tau}{n_N}$, but $a_{NL} - a_{NF} + a_S$ may be bigger or smaller than a_{NF})

$$a_{21}^* = -\begin{vmatrix} -\frac{\beta[\tau(\lambda-1) - \eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ \frac{\tau(1-s_D)}{n_N} & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \\ -\frac{\eta(1-s_M)}{1-n_N} & 0 & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$= \frac{\eta(1-s_M)}{1-n_N}\left[-\frac{\beta\eta n_N(\theta w-1)}{\theta\tau}\frac{(\lambda-1)n_N}{\lambda a_{NF}} + (1-s_D)a_{14}\right]$$
$$- \frac{(\theta w-1)(1-n_N)}{\theta w a_S}\left[\frac{\beta[\tau(\lambda-1) - \eta(\theta w-1)]}{\theta}(1-s_D) + \frac{\tau(1-s_D)}{n_N}\frac{\beta\eta n_N(\theta w-1)}{\theta\tau}\right]$$
$$> or < 0$$

$$\mathrm{a}_{22}^{*}=\begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ 0 & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \\ -(1-s_M) & 0 & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$=-(1-\mathrm{s_D})\left[\frac{\beta n_N(\theta w-1)}{\theta}\frac{(\theta w-1)(1-n_N)}{\theta w a_S}+(1-\mathrm{s_M})\mathrm{a}_{14}\right]+\frac{(\lambda-1)n_N}{\lambda a_{NF}}(1-s_M)\frac{\beta\eta n_N(\theta w-1)}{\theta\tau}$$

$$=-(1-\mathrm{s_D})(1-\mathrm{s_M})\frac{\beta n_N(\theta w-1)}{\theta}\left[\rho(1-n_N)+\eta-\frac{\rho n_N\eta}{\tau}-\eta\right]-(1-\mathrm{s_D})(1-\mathrm{s_M})\mathrm{a}_{14}>or<0$$

$$\mathrm{a}_{23}^{*}=-\begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & a_{14} \\ 0 & \frac{\tau(1-s_D)}{n_N} & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$=-\frac{\beta n_N(\theta w-1)}{\theta}\left[\frac{\tau(1-s_D)}{n_N}\frac{(\theta w-1)(1-n_N)}{\theta w a_S}+\frac{\eta(1-s_M)}{1-n_N}\frac{(\lambda-1)n_N}{\lambda a_{NF}}\right]$$
$$+(1-s_M)\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{(\lambda-1)n_N}{\lambda a_{NF}}-\frac{\tau(1-s_D)}{n_N}a_{14}\right]<0$$

$$\mathrm{a}_{24}^{*}=\begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} \\ 0 & \frac{\tau(1-s_D)}{n_N} & -(1-s_D) \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & 0 \end{vmatrix}$$

$$=\frac{\beta n_N(\theta w-1)}{\theta}\left[-\frac{\eta(1-s_M)(1-s_D)}{1-n_N}\right]$$
$$-(1-s_M)(1-s_D)\left[\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}+\frac{\beta\eta(\theta w-1)}{\theta}\right]<0$$

$$\mathrm{a}_{31}^{*}=\begin{vmatrix} -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ 0 & a_{NF} & 0 \\ -\frac{\eta(1-s_M)}{1-n_N} & 0 & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$=\mathrm{a_{NF}}\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{(\theta w-1)(1-n_N)}{\theta w a_S}+\frac{\eta(1-s_M)}{1-n_N}a_{14}\right]>or<0$$

$$a^*_{32} = -\begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ a_{NL}-a_{NF}+a_S & a_{NF} & 0 \\ -(1-s_M) & 0 & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$= (-1)\left\{-(a_{NL}-a_{NF}+a_S)\left[-\frac{\beta\eta n_N(\theta w-1)}{\theta\tau}\frac{(\theta w-1)(1-n_N)}{\theta w a_S}\right] + \mathrm{a_{NF}}\left[\frac{\beta n_N(\theta w-1)(1-n_N)}{\theta w a_S}\frac{\beta n_N(\theta w-1)}{\theta} + (1-\mathrm{s_M})\mathrm{a_{14}}\right]\right\} < 0$$

$$a^*_{33} = \begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & a_{14} \\ a_{NL}-a_{NF}+a_S & 0 & 0 \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & \frac{(\theta w-1)(1-n_N)}{\theta w a_S} \end{vmatrix}$$

$$= -(\mathrm{a_{NL}}-\mathrm{a_{NF}}+\mathrm{a_S})\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{(\theta w-1)(1-n_N)}{\theta w a_S} + \frac{\eta(1-s_M)}{1-n_N}a_{14}\right] > or < 0$$

$$a^*_{34} = -\begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} \\ a_{NL}-a_{NF}+a_S & 0 & a_{NF} \\ -(1-s_M) & -\frac{\eta(1-s_M)}{1-n_N} & 0 \end{vmatrix}$$

$$= -(a_{NL}-a_{NF}+a_S)\frac{\eta(1-s_M)}{1-n_N}\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} - a_{NF}\left[\frac{\beta n_N(\theta w-1)}{\theta}\frac{\eta(1-s_M)}{1-n_N} + \frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}(1-\mathrm{s_M})\right] < 0$$

$$a^*_{41} = -\begin{vmatrix} -\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ 0 & a_{NF} & 0 \\ \frac{\tau(1-s_D)}{n_N} & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \end{vmatrix}$$

$$= -a_{NF}\left[-\frac{\beta[\tau(\lambda-1)-\eta(\theta w-1)]}{\theta}\frac{(\lambda-1)n_N}{\lambda a_{NF}} - a_{14}\frac{\tau(1-s_D)}{n_N}\right] > 0$$

$$a^*_{42} = \begin{vmatrix} \frac{\beta n_N(\theta w-1)}{\theta} & -\frac{\beta\eta n_N(\theta w-1)}{\theta\tau} & a_{14} \\ a_{NL}-a_{NF}+a_S & a_{NF} & 0 \\ 0 & -(1-s_D) & \frac{(\lambda-1)n_N}{\lambda a_{NF}} \end{vmatrix}$$

$$= (1 - \mathrm{s_D})[-(a_{NL} - a_{NF} + a_S)a_{14}] + \frac{(\lambda - 1)n_N}{\lambda a_{NF}}\left[\frac{\beta n_N(\theta w - 1)}{\theta}a_{NF} + (a_{NL} - a_{NF} + a_S)\frac{\beta\eta n_N(\theta w - 1)}{\theta\tau}\right] > or < 0$$

$$\mathrm{a}_{43}^* = -\begin{vmatrix} \frac{\beta n_N(\theta w - 1)}{\theta} & -\frac{\beta[\tau(\lambda - 1) - \eta(\theta w - 1)]}{\theta} & a_{14} \\ a_{NL} - a_{NF} + a_S & 0 & 0 \\ 0 & \frac{\tau(1 - s_D)}{n_N} & \frac{(\lambda - 1)n_N}{\lambda a_{NF}} \end{vmatrix}$$

$$= (a_{NL} - a_{NF} + a_S)\left[-\frac{\beta[\tau(\lambda - 1) - \eta(\theta w - 1)]}{\theta}\frac{(\lambda - 1)n_N}{\lambda a_{NF}} - a_{14}\frac{\tau(1 - s_D)}{n_N}\right] < 0$$

$$\mathrm{a}_{44}^* = \begin{vmatrix} \frac{\beta n_N(\theta w - 1)}{\theta} & -\frac{\beta[\tau(\lambda - 1) - \eta(\theta w - 1)]}{\theta} & \frac{\beta\eta n_N(\theta w - 1)}{\theta\tau} \\ a_{NL} - a_{NF} + a_S & 0 & a_{NF} \\ 0 & \frac{\tau(1 - s_D)}{n_N} & -(1 - s_D) \end{vmatrix}$$

$$= -(a_{NL} - a_{NF} + a_S)\left[\frac{\beta[\tau(\lambda - 1) - \eta(\theta w - 1)]}{\theta}(1 - \mathrm{s_D}) + \frac{\tau(1 - s_D)}{n_N}\frac{\beta\eta n_N(\theta w - 1)}{\theta\tau}\right] - a_{NF}\frac{\beta n_N(\theta w - 1)}{\theta}\frac{\tau(1 - s_D)}{n_N} < 0$$

Based on the above results we have the following comparative statics:

$$\frac{\mathrm{d}\eta}{\mathrm{dL_S}} = \frac{a_{11}^*}{|A|} > 0$$

$$\frac{\mathrm{d}\eta}{\mathrm{dL_N}} = \frac{a_{21}^*}{|A|} > or < 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{dL_S}} = \frac{a_{12}^*}{|A|} < 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{dL_N}} = \frac{a_{22}^*}{|A|} > \mathrm{or} < 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{dL_S}} = \frac{a_{13}^*}{|A|} < 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{dL_S}} = \frac{a_{23}^*}{|A|} > 0$$

$$\frac{\mathrm{de}}{\mathrm{dL_S}} = \frac{a_{14}^*}{|A|} > 0$$

$$\frac{\mathrm{de}}{\mathrm{dL_N}} = \frac{a_{24}^*}{|A|} > 0$$

$$\frac{\mathrm{d\tau_S}}{\mathrm{dL_S}} > 0$$

$$\frac{\mathrm{d\tau_S}}{\mathrm{dL_N}} > or < 0$$

Policy implication:

$$\frac{\mathrm{d}\eta}{\mathrm{ds_D}} = -b_1 \frac{a_{31}^*}{|A|} > or < 0$$

$$\frac{\mathrm{d}\eta}{\mathrm{ds_M}} = -b_2 \frac{a_{41}^*}{|A|} > 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{ds_D}} = -b_1 \frac{a_{32}^*}{|A|} < 0$$

$$\frac{\mathrm{dn_N}}{\mathrm{ds_M}} = -b_2 \frac{a_{42}^*}{|A|} > \text{or} < 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{ds_D}} = -b_1 \frac{a_{33}^*}{|A|} > or < 0$$

$$\frac{\mathrm{d}\tau}{\mathrm{ds_M}} = -b_2 \frac{a_{43}^*}{|A|} < 0$$

$$\frac{\mathrm{de}}{\mathrm{ds_D}} = -b_1 \frac{a_{34}^*}{|A|} < 0$$

$$\frac{\mathrm{de}}{\mathrm{ds_M}} = -b_2 \frac{a_{44}^*}{|A|} < 0$$