Democracy and Redistribution

Santanu Gupta
XLRI, Xavier School of Management
C. H. Area (East)
Jamshedpur 831035
Jharkhand, India
Email: santanu@xlri.ac.in

and

Raghbendra Jha
Arndt-Corden Department of Economics
Australian National University
Acton 2601
Australian Capital Territory
Australia
Email: r.jha@anu.edu.au

Abstract

In a probabilistic voting model with three jurisdictions and residents with different incomes, we analyze inefficiencies in local public good allocation that emerge from trying to satisfy the median voter. The median voter and the rich may gain but the poor lose out. We analyze a uniform tax rate and progressive two and three bracket tax structures. If the government extracts part of tax revenues as political rents and maximizes expected payoff there is a possibility of taxing away all private income with no allocation of public good, if electoral uncertainty is high, especially when the government is risk neutral.

JEL classifications: $H11; H50$
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1 Introduction

A major part of the interest in the Political Economy literature is why the poor despite being larger in number do not expropriate the rich. Meltzer and Richard (1981) show that if the median voter (voter with the median income) were to vote for the ideal tax rate, he/she would take into account the fact that an increase in tax rate may affect labor leisure choice and this may mean a lower pre tax income. So there is a limit to the extent of appropriation or the size of the government, where this tax revenues comes back as general transfers. Romer (1998), while trying to address the same issue argues that citizens vote over multidimensional policy issues. This vote of the large poor majority may be split since since some of the poor may vote for a party pretending to care about religious issue, than for a party that promises high taxes and transfers to poor. Alesina and Ferrara (2005) explain why it is not always the case that the poor would favor redistribution and the rich would oppose it. If today's poor expect to be wealthy tomorrow, they might oppose redistribution since they may be net losers in the process.

Keefer and Khemani (2004) point out that the poor in India receive poor public services. Since political parties cannot commit to provide quality public goods such as health, education and sanitation, the poor are attracted with sops of targeted transfer payments and subsidies which may not benefit them all that much. Stigler (1970) tried to explain Director's Law whereby public expenditures are made primarily for the benefit of middle class and are financed by taxes mainly collected from the poor and the rich. In the long run the middle class have been the beneficiaries, they were in coalition with the rich in the 19th century, and with extension of franchise have entered into coalition with the poor in the 20th century and today. Buchanan and Tullock (1962) discusses at length the problem of "tyranny of the majority" and ways to ensure that a particular section is not neglected.

From the perspective of public expenditures, Buchanan (1970, 1971) and Spann (1974) discuss the welfare gains and losses which occur when individuals with different incomes demand different levels of the public good. When there is collective consumption where all individuals consume the same level of the public good, under Lindahl taxation, the tax share of the richer individual is made higher and that of the poorer individual lower, till all individuals demand the same level of the public good. Under such circumstances there happens to be a transfer of income from the richer to the poorer individuals. There is, however, an unexplored possibility that there may be a transfer from the poor to the rich. This is in line with recent literature (e.g. Piketty (2014)) which argues that internal inequality within many countries is actually increasing over time. We explore this phenomenon in this paper. In our model by contrast, it is the poor who are always neglected in public good provision, and the rich are net gainers or losers depending how close their income is to the median income. We therefore try and explain why income rather than numbers may play a more central role in resource allocation.

In our model, individuals with different incomes live in different jurisdictions, they all contribute to taxes if mandated by law (in a progressive tax structure the poor need not pay taxes), but the decision on how much public goods they receive is with the Central government. We show that under these conditions, it is optimal for the Central government to neglect the jurisdiction where the poorest individual lives, and concentrate on spending tax revenues in jurisdictions inhabited by the richer individuals. Therefore, even in our situation, we observe a kind of "tyranny of the majority", but there is a redistribution of income in kind from the poor to the richest and the median income voter, or from the poor and the rich to the median income voter. Despite the fact that citizens lower their demands to the extent possible, the government always finds it optimal to satisfy the rich and the median voter in order to win elections.

Although there now exists a substantial literature now outlining that the median voter gets favored in democratic allocations, the debate on whether the rich or the poor are favored in majority voting rules is still unresolved. This paper tries to resolve this debate, with a model which shows that in a democracy, the median voter gets favored, followed by the rich. Wittman (1989) recognizes that the median voter allocations may be inefficient, but rent seeking may not lead to greater inefficiency: we identify these inefficiencies as over allocation of resources to the median voter and non allocation of resources to the poor voter. The problem of rent seeking may be serious to the extent of total appropriation of all resources by the government, if they maximize expected rents and are risk neutral. However, the problem may not be all that serious if governments are risk averse.This paper examines three institutional structures: a uniform one rate tax structure, a progressive two bracket tax structure as well as a three bracket tax structure and tries to evaluate which of these tax structure can best combat the inefficient resource allocation in a democracy.

Most countries have constitutional provisions to prevent blatant discrimination against any jurisdiction, governments do get past these provisions since there are provisions for discretionary grants under certain circumstances[1]. It is how these discretionary grants are spent, is the major concern in our paper. As pointed out by Buchanan and Tullock (1962) that even if there are constitutional provisions against blatant transfers, the majority coalition may exploit the minority through levying general taxes to provide special benefits, or through financing general benefits by special taxes. Our model is based on the first strategy, i.e. levying general taxes to provide special benefits.

The model developed here is a probabilistic voting model along the lines of Seabright (1996) and Gupta (2001). In both of these models, the incumbent government gets re-elected if the welfare provided to any jurisdiction net of the electoral uncertainty is greater than or equal to a reservation utility, which is, interpreted as the welfare expected from a rival political party, and is exogenously given in the model. Since the strategies of the opposition are not explicitly modelled in both these papers, and reservation utility is

[1] In India for instance, a part of the grants is decided by formula, however a large part of the grants are discretionary grants, for which the government is not accountable.

treated as being exogenous, we try and understand at what level reservation utility are set by citizens. Assuming that the the government in power does not represent any interest group, the opposition if it comes to power will behave in exactly the same manner as the incumbent. Knowing this, citizens strategically set their reservation utilities in order to attract the largest resources towards themselves. Since citizens are not aware of the incomes of competing voters in other jurisdictions, they drop their demands down to as low as possible. We recognize in this model that reservation utility of citizens depends on the acceptance of a tax rate that will be set by the government and a minimum demand for a local public good. Since we consider a quasi linear utility function, the reservation utility is therefore a private good equivalent of the value of services that are put forward as demand to representatives of the government in the jurisdiction and that need to be provided in order for the citizens to vote for the incumbent government. From this assumption it follows that the government's objective function is the maximization of the probability of re-election, we take on from this assumption, and extend it to one where the government might want to corner some of the tax resources; which it can do only at the expense of reducing its re-election prospects.

This paper is organized as follows. The next section describes the basic model as well as implications for resource allocation of local public goods to jurisdictions in a democracy in a one bracket tax structure where all citizens face the same tax rate. Section 3 describes the same resource allocation problem as in the previous section in a two bracket and three bracket progressive tax structure, where the poor are exempt from taxes. Section 4 analyzes the overall welfare which is the sum of welfare of all citizens in a one bracket, two bracket and three bracket tax structure and comments on the optimal tax structure to implement given the distortions present in a democracy. Section 5 discusses a situation where part of the tax revenues are extracted as political rent, which is not returned back to citizens as local public good in a situation when the government is risk neutral as well as risk averse.

Section 6 concludes.

2 The Model

We consider an economy with three[2] jurisdictions and with a single representative individual in each jurisdiction. We assume individuals with identical additively, separable utility function defined over a private and a local public good[3]. Individuals differ in their endowments or incomes. The central government decides on a uniform proportional tax rate and the amount of local public good to be supplied to jurisdictions. The voting model incorporates the notion of reservation utility as in Seabright (1996) and Gupta (2001). Individuals are assumed to be immobile across jurisdictions. The central government has to satisfy a majority of jurisdictions (in this case two) in order to get re-elected.

Jurisdictions and thereby the individuals living in the jurisdiction are represented by i where $i \in \{1, 2, 3\}$. Let there be one individual in each jurisdiction with income level y_i. The utility function of an individual in jurisdiction i is given by:

$$W_i = x_i + ln(c + g_i) \tag{1}$$

where $0 < c < 1$, if c is very close to zero, the dis-utility from non provision of public goods is very high, if c is very close to one, the dis-utility from non provision of any public good is not so high, citizens then may not be averse to no provision along with no taxation of incomes. x_i is the amount of private good consumed by the individual in jurisdiction i and

$$x_i = (1 - t_s)y_i \tag{2}$$

[2]The model can be easily extended to n jurisdictions where n is odd.
[3]These are simplifying assumptions which would help us highlight the result better.

where t_s is the uniform proportional tax rate levied by the central government, $0 < t_s < 1$. g_{si} is the amount of local public good provided to jurisdiction i by the central government.

The uncertainty regarding an incumbent government's re-election is captured by an electoral uncertainty ϵ, which is a random variable following a uniform distribution over the range $[-q, q]$[4] and a mean of zero. Let e_i denote the event that the individual is satisfied with the incumbent government and votes for it. The event e_i occurs when the welfare of an individual W_i in jurisdiction i, with income y_i net of electoral uncertainty ϵ is greater than a reservation utility R_i, which can be interpreted as the welfare expected from the current incumbent government in order to vote for it[5]. Since the utlity function is quasi concave, the reservation utility may be interpreted as the equivalent of welfare in terms of private good that must be provided by the government, either in form of lower taxes or larger public good allocation in order to vote for it, and this is conveyed to representatives of the incumbent government in the jurisdiction. A representative individual in jurisdiction i would be satisfied with the government if

$$W_i + \epsilon \geq R_i \tag{3}$$

where

$$R_i = (1 - t_{ai})y_i + ln(c + g_{di}) \tag{4}$$

where t_a is a tax rate that is acceptable to the citizen and g_{di} is the minimum local public good that is expected to be delivered by the incumbent government to a citizen in

[4] We assume that $q > (1 - t)y_h + ln(c + t\sum_i y_i)$, where $t = \frac{1}{y_h} - \frac{c}{\sum_i y_i}$. This ensures that the probability of re-election from any jurisdiction i derived in equation 6 is always less than 1.

[5] In Seabright (1996) and Gupta (2001) reservation utility is interpreted as the welfare expected from the rival political party. However since the opposition's strategies is not modelled in both these papers, and the opposition would also take the same decision were it to become the incumbent government, we interpret reservation utility as the welfare net of electoral shock expected from the current incumbent government in order to vote for it. Thus our reservation utility is in a way equivalent to that in Seabright (1996) or Gupta (2001)

jurisdiction i.

Therefore the event e_i occurs when

$$\epsilon \geq R_i - W_i \tag{5}$$

and the probability $p(e_i)$ of the individual being satisfied with the incumbent government and voting in its favor is given by

$$\begin{aligned} p(e_i) &= p(\epsilon \geq R_i - W_i) \\ &= \frac{q - (R_i - W_i)}{2q} \\ &= \frac{q + (t_{ai} - t_s)y_i - ln(c + g_{di}) + ln(c + g_i)}{2q} \end{aligned} \tag{6}$$

The sequence of events is as follows: Given a democratic structure, the central government has to win from any two of the three jurisdictions. Given that knowledge citizens in each jurisdiction i strategically announce to the government the tax rate t_{ai} they are willing to accept and the minimum local public good g_{di} they demand or expect from the incumbent government. These announcements determine the reservation utility R_i that is expected by citizens from each jurisdiction. The government then decides on the proportional tax rate t_s that it is going to impose on citizens, and on the allocation of the tax revenue as local public goods to different jurisdictions. The value of the electoral shock ϵ is realized and citizens decide whether or not to vote for the incumbent government.

The objective of the Central Government is to maximize the probability of winning from any two of the three jurisdictions. The objective function of the central government is thus given by

$$Z = p(e_1 \cap e_2 \cap -e_3) + p(e_1 \cap -e_2 \cap e_3) + p(-e_1 \cap e_2 \cap e_3) + p(e_1 \cap e_2 \cap e_3) \tag{7}$$

where $-e_i$ is the event of not satisfying jurisdiction i. The central government has to spend the taxes raised from individuals on allocation of local public goods to three jurisdictions. Therefore it is subject to the budget constraint

$$\sum_{i=1}^{3} g_i = t_s \sum_{i=1}^{3} y_i \tag{8}$$

The central government will set the tax rate and distribute resources for local public good to the jurisdictions in order to maximize the probability of getting re-elected from any of the two jurisdictions. This would depend not only on the endowment/incomes of the individuals in the jurisdictions, but also on the level of reservation utility of individuals. Given that the electoral uncertainty is perfectly correlated amongst all individuals in all jurisdictions, the probability of getting re-elected from any jurisdiction depends on the gap between the welfare experienced and the reservation utility of the representative citizen in the jurisdiction (see equation 6). The larger this gap, the greater is the probability of getting re-elected from any of the three jurisdictions. Therefore, the central government will always *ex ante* find it optimal to concentrate on the two jurisdictions with the largest gap and completely ignore a third jurisdiction in the allocation of the public good (see Appendix 1). This situation is similar to that discussed in Buchanan and Tullock (1962) of the tyranny of the majority, where, given the nature of democratic institutions, a minority of individuals will be discriminated against.

Can citizens alter their reservation utility to attract resources towards their jurisdiction? Yes they can, since the probability of getting re-elected from any jurisdiction would be dependent on $R_i - W_i$, the two jurisdictions to be favored would be the two with the least $R_i - W_i$. It is apparent from equation 6, that the probability of winning is higher in jurisdictions which agree to a very high tax rate and a very low demand for local public good. Let us assume that citizens in any jurisdiction are unaware of the incomes of citizens in other jurisdictions, so all citizens agree to a tax rate of $t_{ai} = 1$ and a local public good demand of zero implying $g_{di} = 0$ in the hope of attracting the maximum resources to themselves. Therefore for any tax rate set by the Central Government, $0 \leq t \leq 1$, the probability of winning is highest from the jurisdiction with the richest individual, followed by that with the individual with the median income and finally with the individual with the least income. Let $y_1 \geq y_2 \geq y_3$; since the probability of getting elected depends from any two jurisdictions is equal to the probability of getting elected from the jurisdiction with the individual with the median income (see Appendix 1), the local public good allocation is highest to the jurisdiction with the individual with the median income, followed by the jurisdiction with the highest income. The jurisdiction with the individual with the least income does not receive any local public good, therefore the allocation is $g_2 \geq g_1 \geq g_3 = 0$.

The Central Government acts as a Stakelberg leader and has to decide on an optimum tax rate t_s is one where no tax revenues accrue to the government in the form of gains (see Appendix 1 for the computation of the optimal tax rate). For any tax rate $t_s^* \neq 1$, the tax collection is divided between jurisdictions 1 and 2 to the two jurisdictions with higher income than jurisdiction 3. Therefore, the results can be summarized as:

Proposition 1 *In a situation where individuals living in different jurisdictions have different incomes, with all individuals having no demand for local public good from the government and agreeing to the highest tax rate to attract resources to themselves; the Central Government will find it optimal to allocate no local public goods to jurisdiction with the poorest individual and with the largest allocation to the individual with the median income. Redistribution of income is from the jurisdiction with the poorest individual to the ones with the two richer individuals, or from the jurisdictions with the poorest and the richest to the one with median income.*

This is exactly the opposite to what happens in the standard public goods provision model. In their work Buchanan (1970, 1971) and Spann (1974) discuss a whole range of models where people with different incomes demand different levels of public good, but the tax shares are so designed that in equilibrium individuals with different incomes demand the same level of public good (Lindahl taxation can achieve the same). In this situation, it follows that the rich pay higher amount as taxes than the poor for the same level of public good provided, and therefore there is a redistribution from the richer to the poorer individuals. Redistribution from the rich to the poor is usually the norm in a democracy, especially where the poor substantially outnumber the rich, the poor would then vote for higher taxes and higher redistribution towards themselves. Meltzer and Richard (1981) explain that with a rise in income inequality, there is a rise in mean income relative to the income of the decisive voter, and this increases taxes and redistribution. This situation is similar to the one discussed in Buchanan and Tullock (1962, chapter 11), when discriminatory transfers are prohibited by government provision, majority coalition can exploit the minority through levying general taxes to provide special benefits. Unproductive public projects whose total benefits are less than the cost imposed on society may be passed by simple majority voting rules, as long as the individual benefit from the public project exceeds the cost imposed on every individual in the dominant majority voting for the project. There exists empirical evidence that the rich stand to gain in redistributive politics. Le Grand (1982) finds that benefits of much of the expenditures on social services in United Kingdom such as health care, education, housing and transport accrue to people who can broadly be classified to being in the higher income groups. The middle class are more likely to get opportunities in education than the poor and are more likely to get opportunities in professional jobs. The poor according to Le Grand live in areas poorly endowed with social services and have to travel far to avail such services. Such gross discriminatory policies, against the poor may not be constitutionally legal in most democracies. Buchanan and Tullock (1962) mention that directly redistributive transfers would normally be prevented by constitutional transfers, however, even when such transfers are prohibited, the majority coalition may effectively exploit the minority only through levying general taxes to provide special benefits, or through financing general benefits by special taxes.

3 Scenario with Progressive Tax Structures

Such gross discriminatory policies, especially the one just outlined where the poor contribute to taxes yet receive no public good in return will not be permissible in most democracies. Most countries in the world have a progressive tax structure where, the poorest are either exempt from taxes or pay very low taxes. We will now investigate resource allocation in a two tier and a three tier tax bracket structure. We assume again that the representative individual in jurisdiction 1 is the richest and that in 3 is the poorest, i.e. $y_1 > y_2 > y_3$.

Let us now introduce a two bracket tax structure as follows:

$t_l = 0$ for $y \leq y_3$

$1 > t_h > 0$ for $y > y_3$

In this case the reservation utility R_i for citizens will be $R_3 = y_3 + ln(c + g_{d3})$ for jurisdiction 3 and $R_j = y_3 + (1 - t_{ahj})(y_j - y_3) + ln(c + g_{dj})$, for $j \in \{1, 2\}$ where t_{ahj} and g_{dj} are the tax rate accepted in the higher slab and the local public good demands by jurisdictions 1 and 2. In order to attract the maximum resources towards themselves, both these jurisdictions will agree to the tax rate in the higher slab as one and a local public good demand of zero. For similar reasons, g_{d3},the local public good demand by jurisdiction 3 is zero. Even in this situation it can be proved, that the government is best off concentrating on the two richest jurisdictions, in short, it will maximize the probability of winning from the jurisdiction with the individual with median income subject to this probability being less than or equal to the probability of winning from the jurisdiction with the individual with the highest income. Let $y_1 = y_2 > y_3$, the tax revenues will be equally distributed between jurisdictions 1 and 2. For $y_1 = y_2 > y_3$, the optimal tax rate t_h^{2*} and the optimal local public good allocations g_i^{2*} are worked out in Appendix 2. As in the case with a one bracket tax structure, as $y_1 > y_2$, $g_1^{2*} < g_2^{2*}$ and for a particular $y_1 = \overline{\overline{y_1}}$, $g_1^{2*} = 0$ and the constraint $p(e_2) \leq p(e_1)$ is met with strict equality. For $y_1 > \overline{\overline{y_1}}$, $g_1^{2*} = 0$ and the constraint $p(e_2) \leq p(e_1)$ is met with strict inequality (see Appendix 2 for the proof).

We also consider a three bracket tax structure as follows:

$t_l = 0$ for $y \leq y_3$

$1 > t_m > 0$ for $y_3 < y \leq y_2$

$1 > t_h > 0$ for $y > y_2$

In this case the reservation utility R_i for citizens will be $R_3 = y_3 + ln(c + g_{d3})$ for jurisdiction 3 and $R_2 = y_3 + (1 - t_{am2})(y_2 - y_3) + ln(c + g_{d2})$, and $R_1 = y_3 + (1 - t_{am1})(y_2 - y_3) + (1 - t_{ah1})(y_1 - y_2) + ln(c + g_{dj})$, where t_{am2} is the tax rate acceptable to individual in jurisdiction 2 for the bracket $y_3 < y \leq y_2$, and t_{am1} and t_{ah1} are the tax rates acceptable to individual 1 for tax brackets $y_3 < y \leq y_2$ and $y > y_2$ respectively. To attract resources towards themselves, all jurisdictions set their local public good demands, g_{d1}, g_{d2}, g_{d3} to zero, jurisdiction 2 accepts a tax rate of one for the second slab i.e. $t_{am2} = 1$, while jurisdiction 1 accepts tax rate for both slabs t_{am1} and t_{ah1} as one. As in earlier situations, the Central Government finds it advantageous to concentrate resources on jurisdictions 1 and 2 and the objective function is defined as follows:

Maximize $p(e_2) = \frac{1}{2q}[q - ln(c) + (1 - t_m)(y_2 - y_3) + ln(c + g_2)]$

subject to $g_1 + g_2 = 2t_m(y_2 - y_3) + t_h(y_1 - y_2)$

and subject to $p(e_2) \leq p(e_1)$

In this situation, it is the objective to maximize the probability of winning from the jurisdiction with the median income. Let us first consider the scenario, where $y_1 = y_2$. In this case there is no extra resources that can be squeezed out from the jurisdiction with the richest voter, nor is extra tax resources available from the jurisdiction with the poorest voter, it is best to give each jurisdiction the efficient level of local public good which is $1 - c$. Therefore if $y_1 = y_2$, $t_m = \frac{(1-c)}{y_2 - y_3}$. If $y_1 - y_2 \leq 2(1 - c)$, $t_h = 1$ and the residual amount is retrieved from the second bracket, that is $t_m = \frac{2(1-c)-(y_1 - y_2)}{2(y_2 - y_3)}$. If $y_1 - y_2 > 2(1 - c)$, then $t_m = 0$. A particular amount of welfare can be attained for the jurisdiction with the individual with the highest income by assigning an efficient allocation of local public good which is $(1 - c)$, and the rest of the tax revenue can be directed towards the jurisdiction with the individual with median income. In order to compensate the jurisdiction with the individual with the highest income for the lower public good to ensure $p(e_2) = p(e_1)$, $t_h < 1$, and t_h satisfies the the condition below:

$$ln(c + t_h(y_1 - y_2) - (1 - c)) = (1 - t_h)(y_1 - y_2) \quad (9)$$

Therefore the results can be summarized as:

Proposition 2 *In a progressive tax structure with individuals from three income brackets, where the individual with the least income is not taxed, the jurisdiction with the poorest individual is completely left out of the political system, it neither contributes to tax revenues nor receives any local public good. In a three bracket tax structure; the jurisdiction with the poorest individual receives no local public good, the one with the richest individual receives* $(1 - c)$ *of local public good. In a two bracket tax structure, the jurisdiction with the richest individual receives local public good only if its income is less than* $\overline{\overline{y_1}}$. *In both the two bracket and the three bracket tax structure, redistribution is from the jurisdiction with the richest to the one with the median individual.*

4 Analysis of Welfare

Given the nature of the utility function of individuals, it is apparent that the most efficient levels of local public good that will maximize the sum of welfare of all three jurisdictions will be an amount of $1 - c$ to each jurisdiction. Whenever local public good provision is higher than $1 - c$, there is over provision, whenever it is less than this amount, there is under provision. From the analysis so far, it is quite apparent that a democratic resource allocation is distortionary since the jurisdiction with the poorest individual receives no local public good in any of the tax structures. In all the tax structures, there is an over provision of local public good to the jurisdiction with the median individual. There is either and over provision or an under provision of local public good to the jurisdiction with the richest individual in the a one-bracket and two-bracket tax structures. Given the distortions prevalent in all tax structures, we look for the tax structure the highest aggregate welfare and therefore is the most desirable. Slemrod, Yitzhaki and Mayshar (1994) compared the desirability of a two bracket tax structure to a uniform tax rate. They find that a uniform tax bracket has lower administrative costs and if a two bracket structure is implemented, income in the higher bracket must be taxed at a lower rate.

With a three-bracket tax structure, the jurisdiction with the richest individual always receives an amount $1 - c$ of local public good which is the most efficient level, so in this context it is superior to the other two tax structures. However, a three bracket tax structure always ensures a complete expropriation of extra resources from the richest individual to ensure that its welfare is exactly equal that of the median income voter which may not be the case with a one-bracket or a two-bracket tax structure.

If $y_1 + y_2 - 2y_3 \leq 2(1 - c)$, a two-bracket or a three-bracket tax structure may suffer from an inadequate tax base implying that it may not be able to raise enough tax revenues to provide the efficient level of local public good to both jurisdictions 1 and 2. In such a situation, all tax rates are set at one, and $g_1^* = g_2^* = \frac{y_1 + y_2 - 2y_3}{2}$, and $g_3^* = 0$ in both structures and they yield identical aggregate welfare.

Table 1: Allocation and Overall Welfare with One, Two and Three Tax Bracket Structures: $y_1 + y_2 - 2y_3 < 2(1 - c)$

Scenario	Tax Regime	Taxes	c	g_1	g_2	$p(e_1)$	$p(e_2)$	$\sum_i W_i$
	I	$t = 0.0331$	1×10^{-16}	1.35	1.64	0.83	0.83	51.36
A	II	$t_h = 1$	1×10^{-16}	0.20	0.20	0.68	0.68	49.94
	III	$t_m = 1, t_h = 1$	1×10^{-16}	0.20	0.20	0.68	0.68	49.94
	I	$t = 0.0176$	0.7	0.65	0.94	0.65	0.65	89.25
B	II	$t_h = 1$	0.7	0.20	0.20	0.50	0.50	89.43
	III	$t_m = 1, t_h = 1$	0.7	0.20	0.20	0.50	0.50	89.43
$y_1 = 30.3$, $y_2 = 30.1$, $y_3 = 30$, $q = 100$, Objective Function $Z_1 = p(e_2)$								

Table 1, presents such a case where $y_1 + y_2 - 2y_3 < 2(1 - c)$. Scenario A discusses a situation where the welfare losses from non receipt of local public goods is heavy, that is where $c = 1 \times 10^{-16}$, and scenario B where the losses from non receipt of local public goods is much lower, that is $c = 0.7$. Tax regime I is one where there is a single tax rate, t for all income classes, tax regime II, is a two-bracket tax structure, where $t_l = 0$ for $y \leq y_3$, and $t_h > 0$ for $y > y_3$, and tax regime III, is a three bracket tax structure where $t_l = 0$ for $y \leq y_3$, $t_m \geq 0$ for $y_2 < y \leq y_3$ and $t_h > 0$ for $y > y_2$. In scenario A, in Tax Regime I, there is over provision of local public goods in a one-bracket tax structure to both jurisdictions 1 and 2, and in Tax Regime II and III there is under provision of local public goods even to favored jurisdictions, despite the highest tax rate for all brackets. We observe that the overall welfare, which is the sum of welfare in all three jurisdictions with the Tax Regime I is 51.36, which is larger than that obtained in Tax Regimes II or III which is 49.94. In contrast in scenario B, where losses from the non receipt of local public good is much lower, welfare from a one level tax bracket structure is at 89.25,which is lower than that obtained from a two-bracket and the three bracket structure at 89.43. Therefore, in Tax Regime I, inefficiencies from over-supply of local public goods to jurisdictions 1 and 2, more then offsets the inefficiencies from undersupply of local public goods to jurisdictions 1 and 2 under Tax Regimes II and III.

Table 2: Allocation and Overall Welfare with One, Two and Three Tax Bracket Structures $y_1 + y_2 - 2y_3 > 2(1 - c)$

Scenario	Tax Regime	y_2	Taxes	g_1	g_2	$p(e_1)$	$p(e_2)$	$\sum_i W_i$
A	I	30	$t = 0.026$	0.00	2.87	0.74	0.65	107.99
	II	30	$t_h = 0.856$	0.20	16.93	0.52	0.52	95.53
	III	30	$t_m = 0,\ t_h = 0.856$	0.20	16.93	0.52	0.52	95.53
B	I	32	$t = 0.024$	0.00	2.70	0.74	0.66	110.11
	II	32	$t_h = 0.464$	0.00	10.20	0.55	0.52	103.75
	III	32	$t_m = 0,\ t_h = 0.847$	0.20	15.04	0.52	0.52	99.30
C	I	43	$t = 0.0168$	0.00	2.06	0.75	0.72	121.54
	II	43	$t_h = 0.053$	0.00	1.74	0.59	0.57	121.75
	III	43	$t_m = 0,\ t_h = 0.75$	0.20	5.04	0.57	0.57	119.30
D	I	48.5	$t = 0.014$	0.00	1.85	0.75	0.74	127.18
	II	48.5	$t_h = 0.033$	0.00	1.28	0.60	0.59	127.51
	III	48.5	$t_m = 0,\ t_h = 0.68$	0.20	0.82	0.60	0.60	127.74
E	I	50	$t = 0.008$	0.50	0.50	0.75	0.75	129.30
	II	50	$t_h = 0.01$	0.20	0.20	0.60	0.60	129.38
	III	50	$t_m = 0,\ t_h = 0.68$	0.20	0.20	0.60	0.60	129.38
$y_1 = 50,\ y_3 = 30, c = 0.8,\ q = 100$, Objective Function: $Z_2 = p(e_2)$								

Table 2 discusses a situation where $y_1 + y_2 - 2y_3 > 2(1 - c)$, where the tax base is large enough to provide efficient levels of local public good to jurisdictions 1 and 2. We then vary the income of the median from the two extremes 30, the income of the poorest voter and 50, the income of the richest voter and try and analyze which Tax Regime emerges as the best in terms of overall welfare. The loss of welfare from non availability of local public goods is much lower than both examples in the previous table, it has been kept constant at $c = 0.8$ in this case. In all the situations discussed here, jurisdiction 1 is provided with the efficient level of local public good, that is $g_3^* = (1 - c) = 0.2$, in Tax Regime III and therefore it scores over Tax Regimes I and II, which display both under-supply and over-supply of local public goods to jurisdiction 1.

In scenario A, the individual with the median income has income equal to that of the individual with the least income, which renders Tax Regimes II and III identical. Despite the efficient supply of local public goods to jurisdiction 1 under Tax Regimes II and III, the overall welfare under these two institutions is at 95.53, and this is lower than that under Tax Regime I at 107.99. This is because, under Tax Regime III, the constraint $p(e_2) \leq p(e_1)$ is always met with equality, implying that there is always a complete expropriation of the rich to make their welfare exactly equal to that of individual with the median income. This leads to inefficiencies since the mode of transfer is through local public good to the jurisdiction where the individual with median income lives. Since marginal utility from a private good is one and that from a public good is less than one for a provision more than $1 - c$, such extraction and redistribution leads to lower welfare. Although complete extraction of the rich is an objective under all Tax Regimes, it is at times not feasible under Tax Regime I since all individuals are taxed at the same rate, and under Tax Regime II, the median income individual and the richest individual are charged the same tax rate. Additional tax burden if imposed, has to be imposed on the individual with median income, the extra resources gained as local public good may not compensate for the additional tax burden. Even if no local public good were to be provided to jurisdiction 1, at the optimum allocation, its welfare, implying the probability of winning from jurisdiction 1, may be strictly greater than that from jurisdiction 2 (see Appendix 1 and 2 for optimum local public good allocation in jurisdiction 1 and jurisdiction 2).

In scenario B and C, for the same reasons as outlined in scenario A, welfare under Tax Regime III is lower than that under Tax Regimes I and II. The only difference between these two scenarios is that, overall welfare is highest under Tax Regime I in scenario B, while it is highest under Tax Regime 2 in scenario C. The tax base under Tax Regime I is $(y_1 + y_2 + y_3)$ is larger than that under Tax Regime II which is $(y_1 + y_2 - 2y_3)$. As the tax base expands when we move from scenario B to scenario C, incentives for high tax rates are reduced since that would hurt more than help the jurisdiction with the individual with the median income. Higher welfare under Tax Regime II in scenario C is attributed to reducing the oversupply of local public goods to favored jurisdictions and compensating them with lower taxes instead.

In scenario D, as the median income increases to 48.5 or closer to the income of the richest individual at 50, Tax Regime III fares the best, and Tax Regime I fares the worst, in terms of overall welfare. The higher overall welfare under Tax Regime III can be mainly attributed to its efficient supply of local public good to jurisdiction 1, and to the non allocation of local public goods to jurisdiction 1 under Tax Regimes I and II.

Finally in scenario E, when the median income is as high as that of the highest income at 50, Tax Regimes II and III become identical with efficient levels of local public good allocation to both jurisdictions 1 and 2, and therefore the overall welfare is higher in both these cases than under Tax Regime I. Therefore the results can be summarized as:

Proposition 3 *When $y_1 + y_2 - 2y_3 < 2(1 - c)$, lower overall welfare under Tax Regimes II and III than under Tax Regime I may be explained due to inefficiencies from under-supply of local public good to jurisdictions 1 and 2. When $y_1 + y_2 - 2y_3 > 2(1 - c)$, if Tax Regime III does worse than Tax Regimes I or II, it must be in situations when the constraint $p(e_2) \leq p(e_1)$, is met with strict inequality under Tax Regimes I and II, where complete expropriation of the rich could not be achieved.*

5 The Disposition of Public Revenues

Until now we had assumed that the democratic government although going for a distortionary resource allocation in order to maximize its chances for re-election, spent all of tax revenues raised on local public goods. However, as pointed out by Brennan and Buchanan

(1980), there is also the question of disposition of public revenues which refer to the mix between the share of tax revenues used to provide public goods and that "devoted directly to the provision of perquisites (pecuniary and nonpecuniary) to the politicians-bureaucrats"[6]. Given that there are a part of resources that are not being returned to citizens, and citizens on their part may be unaware of the same due to rational ignorance (Downs 1957), the costs of acquiring such information is much more than the potential benefits to an individual voter. In an extreme situation, we have the model of "Leviathan", where, government may opt for maximization of their "surplus", to spend for discretionary use, which is the excess of government's revenue collection over spending on public goods. In this section, we discuss the situation where the government would still choose to spend some of the public money on public services in order to get re-elected. Therefore the government's objective function is to maximize their expected payoff which is: (Tax revenue collected less expenditure on public goods)$\times$probability of getting re-elected. Its problem is then to choose an appropriate tax rate and redistribute some of the tax revenues to two jurisdictions in order to maximize its expected payoff[7]. Citizens will have demand for local public good and acceptance of the tax rates in exactly the same way as in the earlier case. It will therefore be of interest to analyze, the tax rates levied by the government in this situation in each of the Tax Regimes, the public good allocation to jurisdictions, and the extraction of public resources by the government measure by $(T - G)$, where T is the total tax collection and $G = g_1 + g_2 + g_3$ is the total expenditure on local public goods, and $G \leq T$.

Table 3 and Table 4 report the allocation, overall welfare, and the extraction by the government, when the Government maximizes its expected payoff $p(e_2)(T - G)$ instead

[6]Quote from Brennan and Buchanan (1980)

[7]This situation is very similar to the one discussed in Brennan and Buchanan, The Power to Tax: Analytical Foundations of a Fiscal Constitution, page 27, where parties aim to maximize the expected returns from an election. In our case, it is only the incumbent government that has a policy choice, the opposition cannot announce its policy choice.

Table 3: Allocation, Overall Welfare and Rent Extraction with One, Two and Three Tax Bracket Structures: $y_1 + y_2 - 2y_3 < 2(1 - c)$

	TR	Taxes	c	g_1	g_2	$\sum_i W_i$	$p(e_2)$	$(T - G)$	$(T - G)p(e_2)$
	I	$t = 1$	1×10^{-16}	0.331	0.331	-39.06	0.68	89.74	60.90
A	II	$t_h = 1$	1×10^{-16}	0.002	0.002	40.18	0.65	0.40	0.26
	III	$t_m = 1, t_h = 1$	1×10^{-16}	0.002	0.002	40.18	0.65	0.40	0.26
	I	$t = 1$	0.7	0.000	0.000	-1.07	0.50	90.40	45.20
B	II	$t_h = 1$	0.7	0.000	0.000	88.93	0.50	0.40	0.20
	III	$t_m = 1, t_h = 1$	0.7	0.000	0.000	88.93	0.50	0.40	0.20
$y_1 = 30.3$, $y_2 = 30.1$, $y_3 = 30$, $q = 100$, Objective Function: $Z_3 = p(e_2)(T - G)$									

Table 4: Allocation, Overall Welfare and Rent Extraction with One, Two and Three Tax Bracket Structures: $y_1 + y_2 - 2y_3 > 2(1 - c)$

	TR	y_2	Taxes	g_1	g_2	$\sum_i W_i$	$p(e_2)$	$(T - G)$	$(T - G)p(e_2)$
	I	30.0	$t = 1$	0	0	-0.67	0.5	110.0	55.00
A	II	30.0	$t_h = 1$	0	0	89.33	0.5	20.0	10.00
	III	30.0	$t_m = 1, t_h = 1$	0	0	89.33	0.5	20.0	10.00
	I	32.0	$t = 1$	0	0	-0.67	0.5	112.0	56.00
B	II	32.0	$t_h = 1$	0	0	89.33	0.5	22.0	11.00
	III	32.0	$t_m = 1, t_h = 1$	0	0	89.33	0.5	22.0	11.00
	I	43.0	$t = 1$	0	0	-0.67	0.5	123.0	61.50
C	II	43.0	$t_h = 1$	0	0	89.33	0.5	33.0	16.50
	III	43.0	$t_m = 1, t_h = 1$	0	0	89.33	0.5	33.0	16.50
	I	48.5	$t = 1$	0	0	-0.67	0.5	128.5	64.25
D	II	48.5	$t_h = 1$	0	0	89.33	0.5	38.5	19.25
	III	48.5	$t_m = 1, t_h = 1$	0	0	89.33	0.5	38.5	19.25
	I	50.0	$t = 1$	0	0	-0.67	0.5	130.0	65.00
E	II	50.0	$t_h = 1$	0	0	89.33	0.5	40.0	20.00
	III	50.0	$t_m = 1, t_h = 1$	0	0	89.33	0.5	40.0	20.00
$y_1 = 50$, $y_3 = 30, c = 0.8$, $q = 100$, Objective Function $Z_4 = p(e_2)(T - G)$									

of maximizing the probability of winning from two out of the three jurisdictions, which works out to be $p(e_2)$ or the probability of winning from the jurisdiction with the median income, for exactly the same scenarios as had been considered in the previous section. It is interesting to see that in all these situations, the tax rate is always equal to one in all situations. It should be noted that if the tax rate is set equal to one, there is no advantage in favoring the jurisdiction with the median income, if local public goods were to be provided, it would be of an equal amount to any two of the three jurisdictions. Another interesting aspect to observe is that in Table 3, in both situations A and B, Tax Regimes II and III are identical and score over Tax Regime I, whereas in a situation without rents, in Table 1, Tax Regimes II and III performed worse in situation A to Tax Regime I, but performed better in situation B. Similarly in Table 4, in all situations A, B, C, D and E, Tax Regimes II and III are identical and score over Tax Regime I, whereas in a situation without rents, in Table 2, Tax Regime I emerges out to be the best in situations A and B, Tax Regime II is best in situation C, and Tax Regime III is best in situations D and E in terms of overall welfare. Again, from the point of aggregate welfare of citizens, Tax Regimes II and III seem identical and better but there is lower rent extraction in these Tax Regimes. Appendix 3 discusses situations under which there is a complete extraction of resources by the government, which is that all of citizens' private income is taxed and no public good is delivered in any jurisdiction. This happens in situations when the electoral uncertainty q is very large, or the electoral loss to the government from non provision of local public good is low captured by a high c. In such situations a two bracket or a three bracket Tax Structure always scores over a one Bracket Tax Structure in terms of overall welfare. It should be noted that in this context, Persson, Tabellini and Trebbi (2003) have linked the accruing of government surplus or extraction of political rent to corruption and according to them electoral rules such as larger district magnitude and lower thresholds for representation, a larger share of representation elected on an individual ballot and a plurality rule in small districts are associated with less corruption.

The results can thus be summarized as:

Proposition 4 *In a situation where where the government extracts a surplus i.e. a part of the tax revenues is not returned to citizens in the form of public good, while trying to maximize its expected gain, government will go for complete expropriation of all private income if electoral uncertainty is high and loss from non provision of local public good is low. In particular, in a one bracket Tax Structure, if* $q > max[y_2, \frac{\sum_{i=1}^{3} y_i}{2c}]$, $t^* = 1$ *and* $g_i^* == 0$*; in a two bracket Tax Structure if* $q > max[(y_2 - y_3), \frac{y_1 + y_2 - 2y_3}{2c}]$, $t_m^* = 1$ *and* $g_i^* = 0$*, for a three bracket tax structure, for* $q > (y_1 + y_2 - 2y_3)max[1, \frac{1}{2c}]$, $t_h^* = t_m = 1$, $g_i^* = 0$.

The idea of complete extraction of resources with tax rates of unity and no allocation of local public good to any jurisdiction is extreme. This may not actually be the case if the government is risk averse and maximizes the objective function $p(e_2)(T - G)^\alpha$, where $1 > \alpha > 0$. In the risk neutral scenarios discussed in Table 3 and Table 4, where $\alpha = 1$ in all but one case (Table 3, scenario A, where $g_1, g_2 > 0$) we observe complete extraction, that is all taxes are equal to unity, and a zero allocation of local public good to all jurisdictions. If $\alpha = 0$, the government's objective function reduces to maximizing $p(e_2)$, which is exactly the government's objective in a situation with no rents. In the risk averse scenario, discussed in Table 5 and Table 6, with $\alpha = 0.01$ and other parameters exactly the same as in the risk neutral scenarios, we observe complete extraction of resources only in Table 5, scenario B, when the tax base is inadequate to provide efficient levels of local public good to favored jurisdictions, and the relative importance of local public good is low (given the high $c = 0.7$).

In Table 5, it is interesting to observe that even where the tax base is not large enough to provide efficient levels of local public goods to the favored jurisdictions, a single bracket tax structure scores worse in both scenarios A and B, despite favored jurisdictions receiving a higher amount of local public good in the single tax structure case. Although the inefficiency from non receipt of local public good is low in a single tax bracket case, it is the loss of welfare from higher tax collections, i.e. the non-receipt of private goods which accounts for the lower welfare in this case.

Table 5: Allocation, Overall Welfare and Rent Extraction with One, Two and Three Tax Bracket Structures: $y_1 + y_2 - 2y_3 < 2(1 - c)$

	TR	Taxes	c	g_1	g_2	$\sum_i W_i$	$p(e_2)$	$(T - G)$	$p(e_2)(T - G)^\alpha$
	I	$t = 0.0877$	1×10^{-16}	1.36	1.63	46.43	0.82	4.93	0.84
A	II	$t_h = 1$	1×10^{-16}	0.09	0.09	48.24	0.67	0.23	0.66
	III	$t_m = 1, t_h = 1$	1×10^{-16}	0.09	0.09	48.24	0.67	0.23	0.66
	I	$t = 0.0604$	0.7	0.66	0.94	85.38	0.65	3.87	0.65
B	II	$t_h = 1$	0.7	0.00	0.00	88.93	0.50	0.40	0.50
	III	$t_m = 1, t_h = 1$	0.7	0.00	0.00	88.93	0.50	0.40	0.50
$y_1 = 30.3$, $y_2 = 30.1$, $y_3 = 30$, $q = 100$, $\alpha = 0.01$, Objective Function $Z_5 = p(e_2)(T - G)^\alpha$									

An interesting question that arises is, whether the ideal tax structure in a situation with no rent extraction is also the ideal tax structure when there is rent extraction under a risk averse government. We describe an ideal tax structure as one which yields the highest aggregate welfare for citizens given by $\sum_i W_i$. Table 7 lists the ideal tax structure obtained from our simulation results from Table 1 to Table 6. We observe that if the government is risk neutral and is maximizing expected rents a two bracket or a three bracket tax structure would always yield a higher aggregate welfare than a one bracket tax structure. If governments are maximizing rents and are risk averse as is the situation in Table 6,where the tax base is not a constraint, the tax structure that yields the highest aggregate welfare exactly matches the situation in Table 4. Therefore with a large tax base to provide for efficient provision of local public goods to favored jurisdictions, distortions from rent seeking may not change the ideal tax structure from that in the no rent seeking

Table 6: Allocation, Overall Welfare and Rent Extraction with One, Two and Three Tax Bracket Structures: $y_1 + y_2 - 2y_3 > 2(1 - c)$

	TR	y_2	Taxes	g_1	g_2	$\sum_i W_i$	$p(e_2)$	$(T-G)$	$p(e_2)(T - G)^{\alpha}$
A	I	30.0	$t = 0.0692$	0.00	2.87	103.24	0.65	4.75	0.66
	II	30.0	$t_h = 0.8924$	0.20	7.81	94.08	0.51	9.84	0.52
	III	30.0	$t_h = 0.08924$	0.20	7.81	94.08	0.51	9.84	0.52
B	I	32.0	$t = 0.0652$	0.00	2.70	105.51	0.66	4.60	0.67
	II	32.0	$t_h = 0.8615$	0.00	8.87	94.87	0.51	10.08	0.53
	III	32.0	$t_m = 0$	0.20	6.80	97.83	0.52	8.97	0.53
			$t_h = 0.8873$						
C	I	43.0	$t = 0.0498$	0.00	2.06	117.48	0.71	4.07	0.72
	II	43.0	$t_h = 0.1391$	0.00	1.74	118.89	0.56	2.85	0.57
	III	43.0	$t_m == 0$	0.20	1.77	117.67	0.57	4.08	0.58
			$t_h = 0.8873$						
D	I	48.5	$t = 0.04481$	0.00	1.85	123.27	0.74	3.91	0.75
	II	48.5	$t_h = 0.0969$	0.00	1.28	125.06	0.59	2.45	0.59
	III	48.5	$t_m = 0.0341$	0.20	0.20	125.52	0.59	2.36	0.60
			$t_h = 1$						
E	I	50.0	$t = 0.0374$	0.50	0.50	125.44	0.74	3.86	0.75
	II	50.0	$t_h = 0.0694$	0.20	0.20	127.00	0.59	2.38	0.60
	III	50.0	$t_m = 0.0694$	0.20	0.20	127.00	0.59	2.38	0.60
$y_1 = 50$, $y_3 = 30, c = 0.8$, $q = 100$, $\alpha = 0.01$, Objective Function $Z_6 = p(e_2)(T - G)^{\alpha}$									

case. Therefore these results can be summarized as:

Proposition 5 *If governments are maximizing rents and are risk neutral, possibilities of complete expropriation are very high and a two or a three bracket tax structure would be preferred over a one bracket tax structure. If governments are risk averse and the tax base is not large enough for efficient provision of local public goods to favored jurisdictions (i.e.* $y_1 + y_2 - 2y_3 < 1 - c$*) , the ideal tax structure may not be the same as that in which the government collects no rents, due to additional inefficiencies from rent seeking.*

Table 7: Tax Bracket which yields the highest overall welfare with different government objectives

Scenario	y_1	y_2	y_3	c	Government Objective		
					$Max\, p(e_2)$	$Max\, p(e_2)(T - G)$	$Max\, p(e_2)(T - G)^{\alpha}$
A	30.3	30.1	30	1×10^{-16}	I	II,III	II,III
	30.3	30.1	30	0.7	II, III	II,III	II,III
B	50	30	30	0.8	I	II,III	I
	50	32	30	0.8	I	II,III	I
	50	43	30	0.8	II	II,III	II
	50	48.5	30	0.8	III	II,III	III
	50	50	30	0.8	II,III	II,III	II,III
$\alpha = 0.01$, Scenario A: $y_1 + y_2 - 2y_3 < 2(1 - c)$, Scenario B: $y_1 + y_2 - 2y_3 > 2(1 - c)$							

6 Conclusion

This paper presents a model of political competition where citizens compete amongst themselves for the highest share of public resources. Thus this paper looks into another aspect of political competition from that in Persson and Tabellini (2000) which essentially looks upon competition between political parties for citizens votes. In the group of political income redistribution models discussed by Londregan (2006), our model falls in the class of models where there is competition over restricted income tax schedules which involve differential public goods allocation to citizens and a tax scheme leading to median voter being subsidized by the poor or by both the poor and the rich. The model here gives a theoretical explanation of Director's law as to why the median voter gains most in the context of a probabilistic voting model, and differentiates situations where the median voter gains at the expense of both the rich and the poor and between situations where both the median voter and the rich gain at the expense of the poor. Therefore income rather than the numerical numbers of rich, poor or the middle class is a determinant of resource allocation of local public goods.

Given that the poor lose out badly in a one bracket tax structure where they contribute to taxes yet receive no public good in return, a progressive two bracket or a three bracket tax structure where the poor are not taxed may be a good institutional mechanism to ensure that the poor are not discriminated completely. We then discuss the optimal number of tax brackets with three levels of income. All three tax brackets imply under-provision of local public goods to the poor and over-provision of public goods to the median voter. A three bracket tax structure would provide the optimal amount of local public good to the richest voter, while a one bracket or a two bracket tax structure may imply either over-provision or under-provision depending on difference in income between the richest and the median voter. However, even in progressive tax structures, the poor are completely ignored, they do not contribute to tax revenues and also receive no local public good.

It should be noted that in our model a higher probability of winning can only be ensured by delivering a higher level of welfare to citizens. In none of the models without or with rent extraction reported from Tables 1-6, is the probability of winning from the median jurisdiction highest with either a progressive two or three bracket tax structure. This implies that the median voter would always prefer a uniform tax structure to a progressive one and this finding goes against Snyder and Kramer's (1988) observation that that the middle class would always choose a progressive tax structure which imposes low or zero marginal tax rate on low incomes and a high rate on large incomes. Our model also concentrates on the difference in income between the richest voter and the median voter; if the difference is very large, a complete extraction of the richest voter may not be in the best interest of the median voter, and in such cases a one-bracket or a two-bracket structure may be preferred to a three-bracket tax structure, which, while promising optimal amount of public good to the richest voter, implies a complete extraction from the rich to make them just as well off as the median voter. In a situation where political rents are extracted, and the government maximizes the expected payoff rather than the probability of re-election there is the risk of complete extraction of all private income as taxes with no provision of public goods if electoral uncertainty is large, especially so if the government is risk averse.

In such circumstances, it is best to have a two bracket or a three bracket tax structure to minimize such extortions and have legal obligations of minimum provisions of local public goods to all jurisdictions.

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Appendix 1: Local public good allocation over jurisdictions and the optimal tax rate when all individuals have different incomes with a uniform (one bracket) tax rate

Let the individual in jurisdiction i have an income y_i. The probability of getting elected from any jurisdiction i is

$$p(e_i) = \frac{q - (R_i - W_i)}{2q} = \frac{q + (t_{ai} - t_s)y_i - ln(c + g_{di}) + ln(c + g_i)}{2q} \qquad (10)$$

The central government has to decide on local public good allocation for any tax rate t_s. The total resources at the disposal of the central government will be $\sum_{i=1}^{3} g_i = t_s \sum_{i=1}^{3} y_i$. Let us go for equal allocation of local public good across jurisdictions. Therefore the amount of local public good being given to a jurisdiction i; $i \in \{1, 2, 3\}$ is $g_m = \frac{1}{3} t_s \sum_{i=1}^{3} y_i$.

For an amount g_m of local public good going to every jurisdiction, let $p(e_1) \geq p(e_2) \geq p(e_3)$.

The central government has to win from two of the three jurisdictions, so it will maximize the probability of re-election from any two of the three jurisdictions, the objective function given by

$$Z = p(e_1 \cap e_2 \cap -e_3) + p(e_1 \cap -e_2 \cap e_3) + p(-e_1 \cap e_2 \cap e_3) + p(e_1 \cap e_2 \cap e_3) \qquad (11)$$

where $-e_i$ is the event of not satisfying jurisdiction i. The central government will maximize the above objective function subject to the budget constraint $\sum_{i=1}^{3} g_i = t_s \sum_{i=1}^{3} y_i$, to get the optimal resource allocation.

Given that $p(e_1) \geq p(e_2) \geq p(e_3)$, it implies $R_1 - W_1 \leq R_2 - W_2 \leq R_3 - W_3$. With a common electoral shock, the event e_i will occur, when

$$\epsilon \geq R_i - W_i \qquad (12)$$

Therefore when e_3 occurs, e_1 and e_2, necessarily occur, since $R_3 - W_3 \geq R_2 - W_2 \geq R_1 - W_1$. By similarly reasoning, when e_2 occurs, e_1 will definitely occur, which implies $p(e_1 \mid e_2) = 1$. Therefore

$$p(-e_1 \cap e_2 \cap e_3) = p(e_1 \cap -e_2 \cap e_3) = 0 \tag{13}$$

and the objective function reduces to

$$Z = p(e_1 \cap e_2 \cap -e_3) + p(e_1 \cap e_2 \cap e_3) = p(e_1 \cap e_2) = p(e_2).p(e_1 \mid e_2) = p(e_2) \tag{14}$$

Therefore, with equal allocation of local public goods across jurisdictions, the probability of getting re-elected is the probability of getting elected from the jurisdiction with the median probability of winning the elections. One should also note that with equal allocation of local public goods, $p(e_1) \geq p(e_2) \geq p(e_3)$. Therefore, one can do better, i.e. increase the probability of getting re-elected, by redistributing local public good allocation from the jurisdiction 3 to the other two jurisdictions. So the optimal allocation would be one where jurisdiction 3 receives no allocation of local public good, implying $g_3^* = 0$ and $p(e_2) = p(e_1)$.

Given the nature of allocation by the government, the fact that one jurisdiction will be discriminated against by not being allocated any local public good, it is in the interest of citizens to increase the probability of being re-elected from their jurisdiction. Since citizens are not aware of the incomes of citizens in other jurisdictions, the best offer they can offer to the government is accepting a tax rate of one and a zero demand for local public good. If this be the case the probability of getting elected from any jurisdiction if the government provides an allocation of local public good g_m to every jurisdiction will be:

$$p(e_i) = \frac{q - ln(c) + (1 - t_s)y_i + ln(c + g_m)}{2q} \tag{15}$$

It should be noted that even if all citizens were to adopt the same strategy of an acceptance of a tax rate of one and a local public good demand of zero, $p(e_1) > p(e_2) > p(e_3)$, will still hold if $t_s < 1$ and $y_1 \geq y_2 \geq y_3$. If we assume that $y_1 \geq y_2 \geq y_3$ still holds good, then the optimal allocation will be one were where $p(e_2) = p(e_1)$ and $g_2 \geq g_1 \geq g_3 = 0$.

The Central Government also has to decide on a tax rate t_s, for any tax rate $t_s \neq 1$, the tax collection to be divided between jurisdictions 1 and 2, the two jurisdictions with higher income than jurisdiction 3. In this case it acts as a Stackelberg leader and its objective function can thus be defined as;

Maximize $p(e_2) = \frac{q - ln(c) + (1-t_s)y_2 + ln(c+g_2)}{2q}$

subject to $g_1 + g_2 = t\sum_1^3 y_i$

and subject to $p(e_2) \leq p(e_1)$

For this particular problem, it is not possible to get explicit solutions of g_1^* and g_2^*, for all possible range of y_i, so we work out the solutions for extreme values. The minimum value of y_1 can be $y_1 = y_2$, if that be the case, the constraint $p(e_2) \leq p(e_1)$ will be satisfied with an equality at $g_1 = g_2$, the optimal value of g_2 will be given by solving for the following problem:

Maximize $p(e_2)$

subject to $g_2 = \frac{1}{2}t(2y_2 + y_3)$

Substituting the above constraint into the objective function, we get

$p(e_2) = \frac{q - ln(c) + (1-t)y_2 + + ln(c + \frac{1}{2}t(2y_2+y_3))}{2q}$

Maximizing the above function with respect to t, as first order condition for optimization the following equation is obtained.

$$\frac{\partial p(e_2)}{\partial t} = \frac{1}{2q}[-y_2 + \frac{1}{(c + \frac{1}{2}t(2y_2 + y_3))}\frac{(2y_2 + y_3)}{2}] = 0 \tag{16}$$

or

$$t^* = \frac{1}{y_2} - \frac{2c}{2y_2 + y_3} \tag{17}$$

and

$$g_2^* = g_1^* = \frac{1}{2}t^*(2y_2 + y_3) = 1 - c + \frac{y_3}{2y_2} \tag{18}$$

As y_1 increases with y_2 and y_3 remaining constant, the difference between y_1 and y_2 increases, $g_2^* > g_1^* > 0$, but the constraint $p(e_2) \leq p(e_1)$ is still met with equality till y_1 reaches a particular value $\overline{y_1}$, when $g_1^{**} = 0$, and $g_2^{**} = t^{**}(\overline{y_1} + y_2 + y_3)$. To find the optimal value of the tax rate, t^{**}, we substitute $g_2 = t(\overline{y_1} + y_2 + y_3)$ into the objective function.

$p(e_2) = \frac{q - ln(c) + (1-t)y_2 + + ln(c + t(\overline{y_1} + y_2 + y_3))}{2q}$

Maximizing the above function with respect to t, as first order condition for optimization the following equation is obtained.

$$\frac{\partial p(e_2)}{\partial t} = \frac{1}{2q}[-y_2 + \frac{1}{c + t(\overline{y_1} + y_2 + y_3)}(\overline{y_1} + y_2 + y_3)] = 0 \tag{19}$$

or

$$t^{**} = \frac{1}{y_2} - \frac{c}{(\overline{y_1} + y_2 + y_3)} \tag{20}$$

and

$$g_2^{**} = t^{**}(\overline{y_1} + y_2 + y_3) = 1 - c + \frac{\overline{y_1} + y_3}{y_2} \tag{21}$$

To get the optimal value of $\overline{y_1}$, we substitute for t^{**} and g_2^{**} from equations 20 and 21 into the constraint $p(e_2) = p(e_1)$. We will then have:

$$
\begin{aligned}
&\frac{q - ln(c) + (1 - \frac{1}{y_2} + \frac{c}{(\overline{y_1} + y_2 + y_3)})y_2 + ln(c + 1 - c + \frac{\overline{y_1} + y_3}{y_2})}{2q} \\
&\qquad = \frac{q - ln(c) + (1 - \frac{1}{y_2} + \frac{c}{(\overline{y_1} + y_2 + y_3)})y_1 + ln(c)}{2q}
\end{aligned} \tag{22}
$$

or

$$
(1 - \frac{1}{y_2} + \frac{c}{(\overline{y_1} + y_2 + y_3)})(\overline{y_1} - y_2) + ln(c) - ln(1 + \frac{\overline{y_1} + y_3}{y_2}) = 0 \tag{23}
$$

Solving for $\overline{y_1}$ from equation 23, we get $\overline{y_1} = \overline{y_1}(y_2, y_3, c)$. Therefore when y_1 increases from y_2 to $\overline{y_1}$, the tax rate increases from $(\frac{1}{y_2} - \frac{2c}{2y_2 + y_3})$ to $(\frac{1}{y_2} - \frac{c}{(\overline{y_1} + y_2 + y_3)})$, and the local public good allocation increases from $(1 - c + \frac{y_3}{2y_2})$ to $(1 - c + \frac{\overline{y_1} + y_3}{y_2})$.

At $y_1 = \overline{y_1}$, $p(e_2) = p(e_1)$, as y_1 increases further with y_2 and y_3 constant, all tax proceeds can still go as public good allocation to jurisdiction 2, as was the case when $y_1 = \overline{y_1}$, provided the constraint $p(e_2) \leq p(e_1)$ is met with strict inequality. This will be the case if $\frac{\partial p(e_1)}{\partial y_1} > \frac{\partial p(e_2)}{\partial y_2}$ at $y_1 = \overline{y_1}$. Since $p(e_i) = \frac{q - (R_i - W_i)}{2q}$, and since $R_i = ln(c)$, the inequality would hold if $\frac{\partial W_1}{\partial y_1} > \frac{\partial W_2}{\partial y_1}$ at $y_1 = \overline{y_1}$. Therefore

$$
\frac{\partial W_1}{\partial y_1}|_{y_1 = \overline{y_1}} = (1 - t^{**}) - \overline{y_1}\frac{\partial t^{**}}{\partial y_1} \tag{24}
$$

$$
\begin{aligned}
\frac{\partial W_2}{\partial y_1}|_{y_1 = \overline{y_1}} &= -y_2\frac{\partial t^{**}}{\partial y_1} + \frac{1}{c + g_2^{**}}(\frac{1}{y_2}) \\
&= -y_2\frac{\partial t^{**}}{\partial y_1} + \frac{1}{\overline{y_1} + y_2 + y_3}
\end{aligned} \tag{25}
$$

$$
\frac{\partial W_1}{\partial y_1}|_{y_1 = \overline{y_1}} - \frac{\partial W_2}{\partial y_1}|_{y_1 = \overline{y_1}} = (1 - t^{**}) - \frac{1}{\overline{y_1} + y_2 + y_3} - (\overline{y_1} - y_2)\frac{\partial t^{**}}{\partial y_1} \tag{26}
$$

Substituting the value of t^{**} and $\frac{\partial t^{**}}{\partial y_1}$ into equation 26, we get

$$\frac{\partial W_1}{\partial y_1}|_{y_1 = \overline{y_1}} - \frac{\partial W_2}{\partial y_1}|_{y_1 = \overline{y_1}} = 1 - \frac{1}{y_2} + \frac{c}{(\overline{y_1} + y_2 + y_3)} - \frac{1}{\overline{y_1} + y_2 + y_3} - (\overline{y_1} - y_2)\frac{c}{(\overline{y_1} + y_2 + y_3)^2} \quad (27)$$

or

$$\frac{\partial W_1}{\partial y_1}|_{y_1 = \overline{y_1}} - \frac{\partial W_2}{\partial y_1}|_{y_1 = \overline{y_1}} = 1 - \frac{1}{y_2} - \frac{1}{\overline{y_1} + y_2 + y_3} + \frac{c}{(\overline{y_1} + y_2 + y_3)}[1 - \frac{\overline{y_1} - y_2}{\overline{y_1} + y_2 + y_3}] \quad (28)$$

It should be noted that $0 < \frac{\overline{y_1} - y_2}{\overline{y_1} + y_2 + y_3} < 1$, so the expression $\frac{c}{(\overline{y_1} + y_2 + y_3)}[1 - \frac{\overline{y_1} - y_2}{\overline{y_1} + y_2 + y_3}] > 0$. For $(\frac{\partial W_1}{\partial y_1}|_{y_1 = \overline{y_1}} - \frac{\partial W_2}{\partial y_1}|_{y_1 = \overline{y_1}}) > 0$, it must be the case that

$$\frac{1}{y_2} + \frac{1}{\overline{y_1} + y_2 + y_3} < 1 + \frac{c}{(\overline{y_1} + y_2 + y_3)}(1 - \frac{\overline{y_1} - y_2}{\overline{y_1} + y_2 + y_3}) \quad (29)$$

The expression $\frac{1}{\overline{y_1} + y_2 + y_3}$ reaches its maximum value when $\overline{y_1} = y_2$ and $y_3 = 0$ and for equation 29 to hold, it must be the case that $y_2 > \frac{3}{2} - \frac{c}{2}$ in this situation. Therefore, $y_2 > \frac{3}{2} - \frac{c}{2}$ is a sufficient condition for the constraint $p(e_2) \leq p(e_1)$ to be satisfied in strict inequality for all values of $y_1 > \overline{y_1}$.

Appendix 2: Local public good allocation over jurisdictions with different incomes with a two bracket tax structure

Let us now introduce a two bracket tax structure as follows:

$t_l = 0$ for $y \leq y_3$

$1 > t_h > 0$ for $y > y_3$

In this case the reservation utility R_i for citizens will be $R_3 = y_3 + ln(c + g_{d3})$ for jurisdiction 3 and $R_j = y_3 + (1 - t_{ahj})(y_j - y_3) + ln(c + g_{dj})$, for $j \in \{1, 2\}$ where t_{ahj} and g_{dj} are the tax rate accepted in the higher slab and the local public good demands by jurisdictions 1 and 2. In order to attract the maximum resources towards themselves, both these jurisdictions will agree to a the tax rate in the higher slab as one and a local public good demand of zero. For similar reasons, g_{d3},the local public good demand by jurisdiction 3 is zero.

Even in this situation it can be proved the local public good allocation will be only for jurisdictions with the median income and the richest income voter and the objective function will reduce to maximizing the probability of getting elected from the jurisdiction with the median income.

In such a situation, the objective for the Central Government will be as follows:

Maximize

$$p(e_2) = \frac{1}{2q}[q - ln(c) + (1 - t_h)(y_2 - y_3) + ln(c + g_2)] \tag{30}$$

subject to $g_1 + g_2 = t_h[(y_2 - y_3) + (y_1 - y_3)] = t_h[y_1 + y_2 - 2y_3]$

subject to $p(e_2) \leq p(e_1)$

Even in this situation the solution is along the same lines as in the last situation with uniform one tax bracket structure. Explicit solutions of the tax rate, local public good allocation to jurisdictions 1 and 2 are not possible so we work out the optimal values in the boundary situations, when $y_1 = y_2$ to $y_1 = \overline{\overline{y_1}} > y_2$, when the constraint $p(e_2) \leq p(e_1)$ is met with equality and allocation of local public good to jurisdiction one is zero. If $y_1 = y_2$, $p(e_2) = p(e_1)$ and $g_2 = t_h(y_2 - y_3)$. Substituting the value of g_2 into equation 30 we get

$$p(e_2) = \frac{1}{2q}[q - ln(c) + (1 - t_h)(y_2 - y_3) + ln(c + t_h(y_2 - y_3))]$$

Maximizing the above function with respect to t_h, we get

$$\frac{\partial p(e_2)}{\partial t_h} = \frac{1}{2q}[-(y_2 - y_3) + \frac{1}{c + t_h(y_2 - y_3)}(y_2 - y_3) = 0 \tag{31}$$

or $t_h^* = \frac{1}{(y_2 - y_3)} - \frac{c}{(y_2 - y_3)}$ and $g_1^* = g_2^* = (1 - c)$. As in the case of the single tax bracket structure, as y_1 increases with y_2 and y_3 remaining constant, for a particular value of $y_1 = \overline{\overline{y_1}} > y_2$, $g_1^{**} = 0$ and $g_2^{**} = t_h^{**}(\overline{\overline{y_1}} + y_2 + y_3)$. To find the optimal value of the tax rate t_h^{**}, we substitute $g_2 = t_h(\overline{\overline{y_1}} + y_2 + y_3)$ in equation 30 to get

$$p(e_2) = \frac{1}{2q}[q - ln(c) + (1 - t_h)(y_2 - y_3) + ln(c + t_h(\overline{\overline{y_1}} + y_2 + y_3))]$$

Maximizing the above function with respect to t_h, we get

$$\frac{\partial p(e_2)}{\partial t_h} = \frac{1}{2q}[-(y_2 - y_3) + \frac{1}{c + t_h(\overline{\overline{y_1}} + y_2 + y_3)}(\overline{\overline{y_1}} + y_2 + y_3) = 0 \quad (32)$$

or $t_h^{**} = \frac{1}{(y_2 - y_3)} - \frac{c}{(\overline{\overline{y_1}} + y_2 + y_3)}$ and $g_2^{**} = 1 - c + \frac{\overline{\overline{y_1}} - y_3}{(\overline{\overline{y_1}} + y_2 + y_3)}$. To get the optimal value of $\overline{\overline{y_1}}$, we substitute for the optimal values of t_h^{**} and g_2^{**} into the constraint $p(e_2) = p(e_1)$. We will then have

$$(1 - t_h^{**})(\overline{\overline{y_1}} - y_2) + ln(c) - ln(c + g_2^{**}) = 0 \quad (33)$$

Solving for $\overline{\overline{y_1}}$ from equation 33 we get $\overline{\overline{y_1}} = \overline{\overline{y_1}}(y_2, y_3, c)$. At $y_1 = \overline{\overline{y_1}}$, $p(e_2) = p(e_1)$, as y_1 increases further, with y_2 and y_3 constant, all tax proceeds can go as public good allocation to jurisdiction 2, as was the case when $y_1 = \overline{\overline{y_1}}$, provided the constraint $p(e_2) \leq p(e_1)$ is met with a strict inequality. This will be the case if $\frac{\partial p(e_1)}{\partial y_1} > \frac{\partial p(e_2)}{\partial y_2}$ at $y_1 = \overline{\overline{y_1}}$. Since $p(e_i) = \frac{q - (R_i - W_i)}{2q}$, and since $R_i = y_3 + ln(c)$, the inequality would hold if $\frac{\partial W_1}{\partial y_1} > \frac{\partial W_2}{\partial y_1}$ at $y_1 = \overline{\overline{y_1}}$. Therefore

$$\frac{\partial W_1}{\partial y_1}|_{y_1 = \overline{\overline{y_1}}} = (1 - t_h^{**}) - (\overline{\overline{y_1}} - y_3)\frac{\partial t_h^{**}}{\partial y_1} \quad (34)$$

$$\frac{\partial W_2}{\partial y_1}|_{y_1 = \overline{\overline{y_1}}} = -(y_2 - y_3)\frac{\partial t_h^{**}}{\partial y_1} + \frac{1}{c + g_2^{**}}\frac{\partial g_2^{**}}{\partial y_1} \quad (35)$$

$$\frac{\partial W_1}{\partial y_1}|_{y_1=\overline{\overline{y_1}}} - \frac{\partial W_2}{\partial y_1}|_{y_1=\overline{\overline{y_1}}} = 1 - \frac{1}{y_2 - y_3} + \frac{c}{\overline{\overline{y_1}} + y_2 - 2y_3} - (\overline{\overline{y_1}} - y_2)\frac{c}{(\overline{\overline{y_1}} + y_2 + y_3)^2} + \frac{(\overline{\overline{y_1}} + y_2 - 2y_3)}{(y_2 - y_3)^2} \tag{36}$$

or

$$\begin{aligned}\frac{\partial W_1}{\partial y_1}|_{y_1=\overline{\overline{y_1}}} - \frac{\partial W_2}{\partial y_1}|_{y_1=\overline{\overline{y_1}}} = 1 &- \frac{1}{y_2 - y_3} + \frac{c(\overline{\overline{y_1}} + y_2 - 2y_3)}{(\overline{\overline{y_1}} + y_2 - 2y_3)^2} \\ &- \frac{c(\overline{\overline{y_1}} - y_2)}{(\overline{\overline{y_1}} + y_2 + y_3)^2} \\ &+ \frac{(\overline{\overline{y_1}} - y_3)}{(y_2 - y_3)^2} + \frac{1}{y_2 - y_3} \\ = 1 + \frac{2c(y_2 - y_3)}{(\overline{\overline{y_1}} + y_2 - 2y_3)^2} + \frac{(\overline{\overline{y_1}} - y_3)}{(y_2 - y_3)^2}\end{aligned} \tag{37}$$

Since the above expression is positive, the constraint $p(e_2) \leq p(e_1)$ is met with strict inequality and all the tax revenues collected go as local public good allocation to jurisdiction 2 only for $y_1 \geq \overline{\overline{y_1}}$.

Appendix 3: Optimal Tax Rate and Local Public Good allocation with positive rent extraction

A government that would siphon off a part of the tax revenues for its own agenda would maximize its expected payoff $EP = (T - G)Z$, where T is the total tax revenues collected and G is the amount of expenditure on local public goods to jurisdictions and Z is the probability of winning from any two jurisdictions. Since the sequence of events of citizens putting their demands on local public good and acceptance of a tax rate is the same as in the earlier situation, citizens not knowing the incomes in other jurisdictions will set a zero demand for local public good and acceptance of a tax rate of one in order to attract the maximum resources towards themselves. In such a situation, the probability of winning the election becomes equal to the probability of getting elected from the jurisdiction with the median income, and again the central government will find it advantageous to provide the local public good only to jurisdictions with the highest and the median income. With a single bracket uniform tax structure the objective function of the government is defined as:

$Maximize\, EY = (T - g_1 - g_2)p(e_2)$

$subject\, to\, g_1 + g_2 = t\sum_i y_i$

$and\, p(e_2) \leq p(e_1)$

From Tables 3 and 4, we observe that in most cases, the optimal values of t and g happen to be $t^* = 1$ and $g^* = 0$. It will be interesting to see under what circumstances the same will hold good. For that we need to calculate the marginal returns from t, g_1, g_2.

$$\frac{\partial EY}{\partial t} = \frac{q - ln(c) + ln(c + g_2) + (1 - t)y_2}{2q}(\sum_{i=1}^{3} y_i) - [t\sum_{i=1}^{3} y_i - g_1 - g_2]\frac{y_2}{2q} \tag{38}$$

The above equation when evaluated at $t = 1$, $g_1 = 0$, $g_2 = 0$, give give

$$\frac{\partial EY}{\partial t}|_{t=1,\, g_1=g_2=0} = 0.5\sum_{i=1}^{3} y_i(1 - \frac{y_2}{q}) \tag{39}$$

The above expression will be positive for $q > y_2$. Therefore, the optimal value of the tax rate is one if $q > y_2$ if $g_1 = g_2 = 0$.

It is apparent that $\frac{\partial EY}{\partial g_1} = -p(e_2)$, for any value of t, g_1, g_2. Therefore there should be no public good allocation to jurisdiction 1. The only reason why public good is allocated to jurisdiction 1 is the ensure the constraint $p(e_2) \leq p(e_1)$ is satisfied. For $t = 1$, the least amount of public good that needs to be given to jurisdiction 1 to satisfy the constraint is $g_1 = g_2$, so the objective function of the government can be re-written as

$$EY|_{t=1} = [\sum_{i=1}^{3} y_i - 2g_2][\frac{q - ln(c) + ln(c + g_2)}{2q}] \quad (40)$$

The marginal return from public good to jurisdiction 2,evaluated at $t = 1$ will be

$$\frac{\partial EY}{\partial g_2}|_{t=1} = -2[\frac{q - ln(c) + ln(c + g_2)}{2q}] + [\sum_{i=1}^{3} y_i - 2g_2]\frac{1}{2q(c + g_2)} \quad (41)$$

The above expression evaluated at $t = 1$ and $g_2 = 0$ will be

$$\frac{\partial EY}{\partial g_2}|_{t=1\, g_2=0} = -1 + \frac{\sum_{i=1}^{3} y_i}{2cq} \quad (42)$$

Therefore for $q > \frac{\sum_{i=1}^{3} y_i}{2c}$, the above expression will be negative and the optimum value of local public good to jurisdiction 2 at $t = 1$ will be zero. Therefore if $q > max[y_2, \frac{\sum_{i=1}^{3} y_i}{2c}]$, $t^* = 1$ and $g_1^* = g_2^* = 0$ implying that there is a complete expropriation of all resources of citizens by the government.

By similar reasoning it can be argued that for $q > max[(y_2 - y_3), \frac{y_1 + y_2 - 2y_3}{2c}]$, $t_m^* = 1$ and $g_i^* = 0$ in a two bracket Tax Structure. For a three bracket tax structure, the marginal return from t_h is always positive, by similar reasoning for $q > (y_1 + y_2 - 2y_3)max[1, \frac{1}{2c}]$, $t_h^* = t_m = 1$, $g_i^* = 0$.